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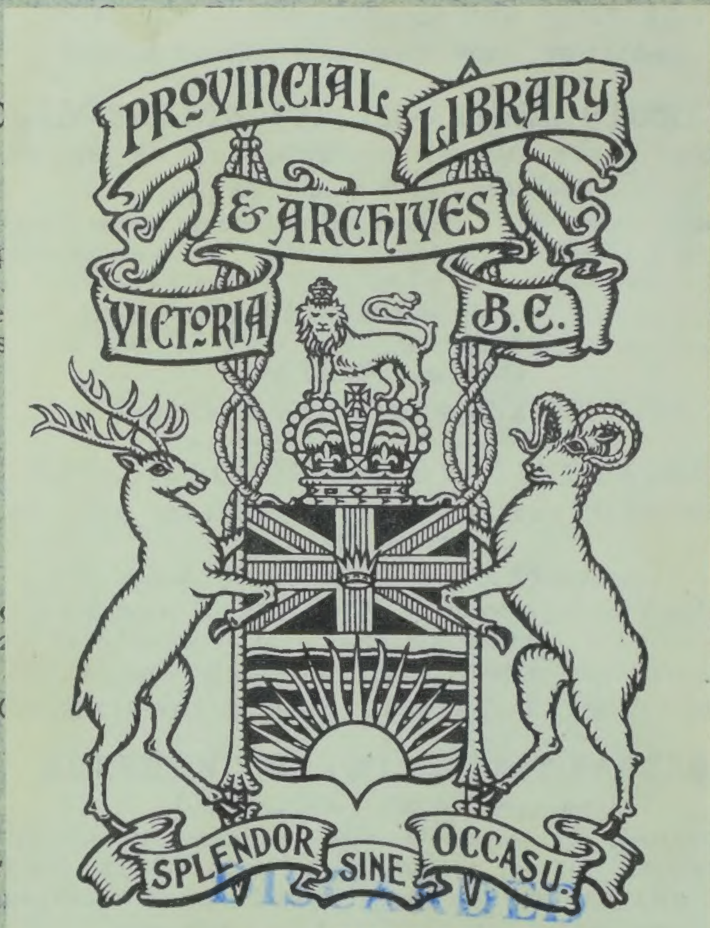

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PREFACE TO THE SIXTEENTH EDITION

OF the last edition, the 15th, as many as eighty-four thousand copies were sold.

The Department has listened to the representations of the Art Masters, and has now come back to the original scheme of examination, with the addition of a few problems. These extra problems—comprising those on the Parabola, Hyperbola, Spirals, Ionic Volute, Cycloids, forms of Arches and Mouldings,—are now given. They are to be found in Chapters XVI. and XVII. The book now forms a complete Text-Book for the Second-grade Examinations.

Owing to an affliction which now affects my right hand, I have to express my deep indebtedness to one of my sons, Mr. F. Rutherfurd Rawle, B.A., for most invaluable assistance which he has rendered me in completing this work.

J. S. R.

LONDON, *Jan.*, 1896.

PREFACE TO THE FIFTEENTH EDITION.

SO great has been the favour with which the previous editions of this book have been received—as many as one hundred and forty thousand copies having been sold—that it has been deemed desirable to prepare the present edition, which is revised throughout, and considerably enlarged.

When expedient, certain Problems which are based upon the same principle of construction, have been grouped together.

One special feature claimed for this work, is the addition of a great number of Notes—nearly four hundred in number—appended to the Problems. These, together with the General Notes at the end of each Chapter, are given with the conviction that the Student's interest will be more thoroughly aroused, and that the subject will be better understood, than is the case when the Problems are stated as so many facts, unsupported by any illustration.

This work is designed for use as a Text-Book for Government and other Examinations. It covers more than is required for the Elementary Examinations of the Department of Science and Art. Candidates are, however, strongly advised not to confine their attention solely to the requirements of any Syllabus, but to endeavour to gain as complete a knowledge of the subject as possible.

Technical Instruction has, of late years, occupied much attention; and as a necessary consequence, Practical Geometry has become an important branch of general education. A knowledge of Scale-Drawing and of Solid Geometry is of the greatest service to all who are engaged in any of the Constructive Arts. Therefore, the sections of this work treating upon Scales, and the Projections of Solids, have been entirely remodelled and considerably augmented. No attempt has been made to deal with what is technically termed Descriptive Geometry; because some twenty-five years' experience in teaching has convinced me that this subject can be studied more successfully *after* the Student has gone through a course, similar to that laid down in this book.

Many additional Problems are given, relating to Inscribed and Circumscribed Figures, and Geometrical Pattern Drawing. Such Problems are useful to Architectural Draughtsmen, and to all who are occupied in any kind of Decorative Work.

The CLASS-SHEETS have been prepared for use in Class-Teaching. By their adoption, uniformity of work is secured throughout the class.

It is hoped that the alterations in this book may broaden its field of usefulness, and that in its new form it may meet with a reception similar to that accorded to the previous editions. Also, that it may awaken such an interest in the study of Geometry that the Student may be encouraged to follow the subject still further, in its more advanced stages.

My hearty acknowledgments and thanks are due to Professor Hulme, for many valuable suggestions, given to me during the progress of this work; and also to my son, Mr. George D. Rawle, for the valuable assistance he has rendered in the revision of the manuscript.

J. S. R.

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EXAMINATIONS IN GEOMETRY.

Since the syllabus of the DEPARTMENT OF SCIENCE AND ART is modified from time to time, it is unadvisable to indicate, definitely, any Problems as being more important than others; but Masters and Students, with the requirements of each year before them, as given in the Directory, can make from the Text-Book, the necessary selection for themselves.

Students are reminded that "The examination questions will be given with the object of testing the candidate's knowledge of the **principles** of the subject, . . . it will not be sufficient to have merely learned a few problems by heart." (*See Directory of the Department of Science and Art.*) These remarks apply with equal force to the Examinations for the ARMY, held at WOOLWICH and SANDHURST: etc. (*See Preface to the 15th Edition, 4th Paragraph.*)

PRACTICAL GEOMETRY.

CHAPTER I. INTRCDUCTION.

"WHATSOEVER THY HAND FINDETH TO DO, DO IT WITH THY MIGHT."—*Ecclesiastes*, ix. 10.

"THERE IS (GENTLE READER) NOTHING (THE WORD OF GOD ONELY SET APART) WHICH SO MUCH BEAUTIFIETH AND ADORNETH THE SOULE AND MINDE OF MAN, AS DOTH THE KNOWLEDGE OF GOOD ARTES AND SCIENCES." —*Sir Henry Billingsley.* 1570.

A KNOWLEDGE of Practical Geometry is of service to all who are studying Art, in any of its branches. It is absolutely necessary to the Engineer and the Architect, and to the majority of those who are engaged in any occupation requiring precision of hand-work, either in drawing, or in construction, such as Joiners, Cabinet-makers, Masons, Smiths, Plumbers, Designers, Decorators, &c. To the general student in Art it is of great use, as a training for the eye, in the correct perception of form, and as an important help in the study of Perspective. The present course is a practical one—no theoretical proofs are attempted.

2.—Geometry is an exact science, requiring the greatest *accuracy* in work. No one can expect to study the subject with advantage, if not prepared to do the work thoroughly and conscientiously. All slovenliness and carelessness must, from the first, be avoided. The student in Geometry must go over the same ground FREQUENTLY, until every step is properly mastered. The problems must be worked in many *different positions*, before it can be said that the work has been done efficiently. All the NOTES and EXERCISES should be carefully worked out, to insure a thorough understanding of the subject.

3.—**DRAWING INSTRUMENTS, etc.**—Each student should be provided with the following instruments and materials :—

Pencils, at least two, **F** and **HH**; the latter for drawing the *lines of construction*, and the former for darkening the lines of the *figures*

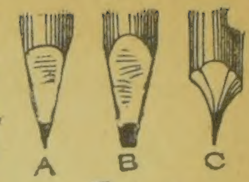


FIG. 1.

obtained. For Architectural, Geometrical, and Mechanical Drawing, the pencils should be sharpened flatly, like a chisel, as shewn in the two views at A and B (Fig. 1). In sharpening a pencil cut off long thin shavings—never cut a point in the clumsy manner shewn at C. Small pencils, called "Engineers' pencils," should be used for bow-pencil compasses. Use a piece of No. 0 glass-paper, or a very fine file, for sharpening the lead.

4.—Compasses.—1st. DIVIDERS, with two steel points for measuring. Some compasses have a lengthening-bar, for drawing large circles. 2nd. BOW-PENCIL COMPASSES, with a steel point, and a leg with a holder for a pencil. The bow-pencil is used for drawing circles. 3rd. BOW-PEN COMPASSES, with a steel point, and a pen, which is used for drawing circles in ink. In addition to the above, each student should possess a set of spring bow-compasses, which are made expressly for drawing very small circles. No architectural or engineering student should be without a set. The joints of the compasses should be sufficiently firm, else the pencil or pen will change its position, and cause inaccuracies. The points should be kept very fine. In working, always hold the compasses by the head. Do not let the fingers touch the sides, as this may alter the distance, or radius.

5.—A Ruling-Pen is for ruling lines in ink. It has a screw to regulate the thickness of the lines. Always clean the pens after using them: use a piece of wash-leather. Do not put pens away with the nibs closed, as this weakens the springs. When a ruling-pen gets out of order, it can, for a small charge, be re-set by a mathematical instrument maker.

6.—Straight-edges, or Rulers. Use a 6in. or a foot rule, with a bevelled edge, and divided into inches, and parts of inches. Never use the PARALLEL-RULER, which consists of two flat rulers fastened together by a pair of equal brass links, as its action is very imperfect. For drawing parallel or perpendicular lines, use a set-square and a straight-edge, as described further on.

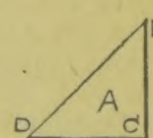


FIG. 2.

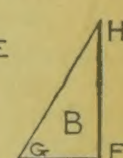


FIG. 3.

7.—Set-squares are triangular pieces of thin wood, or ebonite, &c. There are two shapes generally used. 1st. The set-square of FORTY-FIVE DEGREES (45°), A (Fig. 2), has a *right angle* (90°) at C, and the remaining angles, at D and E, are each equal to *forty-five* degrees. 2nd. The set-square of SIXTY DEGREES (60°), B (Fig. 3), has a *right angle* at F, an angle of *sixty* degrees at G, and an angle of *thirty* degrees (30°) at H. (*The measurement of angles is explained in* §§ 48 *to* 53.)

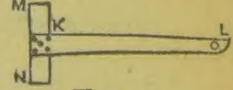


FIG. 4.

8.—THE T-square is made of two pieces of wood, fastened together in the shape of the letter T (Fig. 4). The *blade* K L is attached to the *stock*, or *hilt*, M N, and the upper edge of the blade is at *right angles* to the stock. The best form of construction for the T-square is that in which the stock is fastened *under* the blade, as shewn in the diagram, as it allows more freedom of action in the use of the set-squares.

9.—THE Drawing-Board should be made of thoroughly well-seasoned wood, and all the corners should be right angles. Projections or uneven portions on the edges interfere with the accurate working of the T-square. The most convenient sizes for ordinary work are half-imperial (23 by 16 inches), and full-imperial (31 by 23 inches).

10.—**Drawing-Pins** are for fastening the corners of the drawing-paper on to the drawing-board. Those having fine points, and large thin tops, or heads, with bevelled edges, are the most convenient for use.

11.—The **Protractor** is intended for *measuring angles*. It is either made in the form of a semi-circle, of brass, or horn; or in the shape of a thin flat rule. The *flat protractor* is the more useful of the two, and it usually has marked upon it a number of scales. Diagrams and explanations are given in Chapter II.

12.—**French Curves** are thin pieces of wood cut out into various curved forms. When inking in some curved line, we can often find some portion of a French curve which will correspond with a portion of the curve we are drawing; we can then ink in that part with the ruling-pen, by passing it round that portion of the French curve which coincides. It requires great care so as not to shew any unsightly joins. The most useful shape is somewhat similar to Fig. 5. This shape is generally serviceable in the drawing of ellipses.

FIG. 5.

13.—**Drawing-Paper.**—The paper should be hard, and not spongy in texture. The surface should not be too rough. The best kinds of paper for geometrical drawing, are Whatman's medium, and some descriptions of hard cartridge. The most useful size is "imperial," which can be cut in half, or quarter sheets, for smaller work. When stretching the paper on the drawing-board, the *back* of the paper, only, should be damped.

14.—**Indian or China Ink** is used for inking in all black lines in Geometrical drawings. It can be had in sticks, or in a liquid state; the former is more convenient. In grinding stick ink do not use a rough surface, as the ink will then clog the pen. It should be ground gradually in a smooth saucer. Do not use too much water at first, add drop by drop, and when ground to a paste-like consistency, dilute with more water. Keep it free from dust. If the paper is at all greasy, add a little OX-GALL to the ink. This preparation is invaluable, when mixed with ink, or a tint, enabling us to work upon a soiled surface. All the forms or figures obtained should be inked in Indian ink. The lines of construction had better be ruled in carmine, crimson lake, or Prussian blue. Never use common writing-ink of any description, as it corrodes the drawing-pens. The nibs of the ruling-pen should be filled with a brush.

15.—There are many other instruments of service to the Engineer and the Architect, which need not be described here,—such as the *proportional compasses*, the *trammel*, the *eidograph*, the *pantagraph*, the *centrolinead*, &c. Those students who wish to gather more information about mathematical instruments, than can possibly be given in this work, are strongly recommended to procure Hulme's "*Mathematical Drawing Instruments, and How to Use Them*," which is a most useful book, especially for those who are engaged as mechanical or architectural draughtsmen.

GENERAL INSTRUCTIONS.

16.—Keep all the Instruments in good working order. Keep the straight-edges, set-squares, and T-square free from notches. Keep the paper, India-rubber, set-squares, &c., perfectly clean. Use stale bread, in preference to India-rubber, for a finished drawing.

17.—Do not make unsightly holes with the points of the compasses in the drawing-paper. Use the drawing-pins at the corners and edges, and not in the middle of the drawing-board: the latter proceeding makes pitfalls into which the points of the compasses enter, and make large holes in the paper.

18.—The problems should be worked on the "CLASS-SHEETS FOR PRACTICAL GEOMETRY," which have been specially prepared for this purpose. If worked on plain paper, the diagrams should be made twice or three times the size of those in this book.

19.—Shew all the lines of construction. When proficiency is attained, then some of these lines may be omitted in actual practice. The lines of construction should be made finer than those of the figures obtained. All the lines should be sufficiently distinct, but not too thick. When thick lines cross each other it is impossible to determine accurately the point of intersection.

20.—Occasionally, for the purpose of making the method of construction clearer, some of the angles in the diagrams are engraved in solid black, or distinguished by a black dot, &c.; but it is not necessary to treat the angles in this manner in the drawn diagrams. Students may, however, with advantage, surround a given point by a small circle, thus ⊙, neatly drawn with spring-bow compasses.

21.—Make a practice of lettering or figuring the points immediately they are obtained.

22.—In joining points by straight lines, do so as accurately as possible. Suppose we wish to join points A and B (Fig. 6) by a straight line. First, rest the point of the pencil on B, then place the edge of the ruler against the point of the pencil, in the position of $x\ x$. Next, adjust the straight-edge, gradually, until point A is reached; then, keeping the rule steady, remove the pencil from B, and draw the line A B, from left to right In joining two or more lines to a point, it is frequently advisable to let each line go a little beyond the point, as then the latter can be more easily distinguished.

FIG. 6.

23.—When a straight line or arc has to be drawn, from which a certain distance is to be measured off, always make the line or arc of sufficient length, so that it will not be necessary to prolongate it.

24.—All arcs used in the *construction* should be made as large as possible. When the construction lines are drawn to a small scale, the work is more likely to be inaccurate.

25.—When arcs have to cut each other in the construction, let them, if possible, intersect at wide angles, as the point of intersection can be seen more distinctly at O than at P (Fig. 7).

26.—When inking-in arcs and straight lines in combination, always do the arcs first, as then the juncture can be made with greater accuracy. For instance, the arc Q R should be inked in first, and then the straight line from R to S (Fig. 8).

FIG. 7. FIG. 8.

26a.—When using a hard pencil do not press too heavily on the drawing-paper, because this makes a groove in the paper, which renders it impossible to ink-in the lines, neatly, afterwards.

CHAPTER II. LINES AND ANGLES.

CLASS-SHEETS 1 AND 2.

⁂ *The* DEFINITIONS *at the commencement of* EACH *Chapter should be well studied before beginning the Problems, and they should also be read, occasionally, while the Problems are being worked.*

DEFINITIONS :—27.—A Point marks position only, as at the beginning or end of a line, or where a line touches or intersects another line. Mathematically defined—a point has *no magnitude*. The smallest dot has some size, and must therefore be a *surface*, but for convenience we call it a point. A *given* point means one of fixed position. (*Euclid* I. *Definitions*, 1, 3.)

28.—A Line has *length* or *direction*, but no thickness. The finest stroke must have some breadth, though for convenience we call it a line. When lines cross each other they are said to *intersect*, at what is called the *point of intersection*. Two straight lines cannot have more than one common point of intersection. (*Euclid* I. *Def.* 2.)

29.—A Straight Line is the shortest distance between two points. A straight line is also called a **Right Line**. A *finite* line means one of a given fixed length. When the length of the line is not known, nor given, it is said to be of *indefinite* length. To *produce* a line means to lengthen it at either extremity. In geometrical drawing the lines of construction are frequently inked-in in *dotted*, or *occult* lines, thus ...., or in *link-lines*, or *chain-lines*, thus, — — — —, or in *link-dotted lines*, thus, — · — · —, or — ·· — ·· —. Whenever the word "line" is mentioned by itself, in this book, a "straight or right line" is meant. (*Euclid* I. *Def.* 4.)

30.—A Horizontal Line is perfectly level.

31.—A Vertical Line is perfectly upright, like a plumb-line when hanging at rest.

32.—An Oblique Line is a line drawn in any direction which is neither horizontal nor vertical.

33.—A Surface has length and breadth, but not thickness. A surface may be FLAT, like the top of a table; or CURVED, like the *concave* surface of the inside of a cup, or the *convex* surface of a sphere. A surface is also called a **Superficies.** When a surface is perfectly flat, so that a straight-edge, throughout its entire length, lies evenly upon it, it is termed a **Plane.** The surface of water, when at rest, is a plane. The boundaries of planes are lines. PLANE GEOMETRY deals only with plane surfaces or figures. (*Euclid* I. *Defs.* 5, 6, 7.)

34.—Parallel Lines are two or more lines situated in the same plane, and throughout their entire length they are the same distance apart from one another; therefore, if they are produced the greatest imaginable distance either way, they will never meet (Fig. 9). Only one line can be drawn parallel to a given line, through one given point. Straight lines which are parallel to the same straight line are parallel to one another. (*Euclid* I. *Def.* 35.)

FIG. 9.

35.—An Angle is formed by the inclination of two lines which meet in a point called the *angular point*, or *vertex*. The lines which form the angle are called the *arms* or *limits*, and they are said to *contain* or *include* the angle. We speak of an angle as having a certain size or magnitude, according to the inclination of its arms or sides. The lengthening or shortening of these lines does not alter the size

of the angle. If the lines forming the angle are straight, and in one plane, we have a **plane rectilineal angle**. Whenever the word "angle" is used by itself in this book, a "plane rectilineal angle" is intended. (*Euclid* I. *Def.* 9.)

36.—**Right Angles.**—When one line meets another in such a manner that the *adjacent angles* formed at the point of junction are *equal* to each other, the two lines are said to be **Perpendicular** to each other and each of the equal angles is called a **Right angle.** Thus, in Figs. 10 and 11, the line A B meets the line C D in such a way that the angles are equal. Then angles C B A and A B D are *right angles*. All right angles are equal to one another. A perpendicular line is not necessarily a vertical line, as is frequently supposed; thus, a vertical line is perpendicular to a horizontal line, and a horizontal line is perpendicular to a vertical one. Lines perpendicular to each other are sometimes said to be *square* with each other. Hence the names *set-square* and T-*square*. (*Euclid* I. *Def.* 10. *Axiom* XI.)

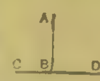


FIG. 10.

FIG. 11.

37.—**An Obtuse Angle** is *greater*, or blunter, than a right angle. An **Acute angle** is *less*, or sharper than a right angle. Thus, in Fig. 12, G F E, E F H, are two right angles; then G F I is an *obtuse angle*, and I F H is an *acute angle*. Obtuse and acute angles are also called *oblique angles*. In naming an angle, the central letter, or numeral, always indicates the angular point; thus, we say the "angle J K L" (Fig. 13), and not the "angle J L K;" or, for brevity, we use the letter at the angular point, only, and say the "angle K." (*Euclid* I. *Defs.* 11, 12.)

FIG. 12.

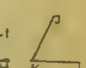


FIG. 13.

38.—**A Circle** (Fig. 14) is a plane figure contained by a continuous curved line, called the **Circumference,** every portion of which is equidistant from a certain point within it, called the **Centre,** A. The half of a circumference is a *semi-circumference*. When we draw a circle, we are said to *describe* it, and the same term is often used for the construction of other plane figures. (*Euclid* I. *Defs.* 15, 16.)

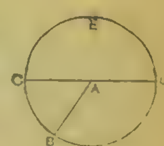


FIG. 14.

39.—**A Radius** (plural *radii*) is any straight line, as A B (Fig. 14), drawn from the centre of a circle to the circumference. All radii of the same circle are of equal length. (*Euclid* I. *Def.* 15.)

40. **A Diameter of a Circle** is a straight line C D (Fig 14), drawn through the centre, and which is terminated at each end by the circumference. It is therefore twice the length of a radius, and is the longest line that can be drawn in a circle. All diameters of the same circle are of equal length. A diameter divides a circle into two equal figures called **Semi-circles**. (*Euclid* I. *Defs.* 17, 18, 19.)

41.—**An Arc of a Circle** is a portion of the circumference, as D B, B C, or B C E D (Fig. 14). When we draw an arc we are said to *describe* or *strike* it.

42.—**A Sector** is *any* portion of a circle enclosed by an arc and two radii. Some sectors which have fixed proportions to the entire circle are distinguished by certain names. That portion which is exactly the *fourth* part of any circle is termed a **Quadrant,** as at G F H (Fig. 15). The *sixth* part is called a **Sextant,** as at J K F; and the *eighth* part is termed an **Octant,** as at L M F. The portions J F M, J F L, K F H, &c., are sectors. (*Euclid* III. *Def.* 10.)

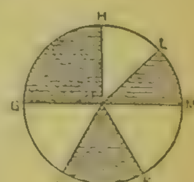


FIG. 15.

On the Use of the Set-squares and T-square.

43.—Set-squares are used in the following manner: Draw a straight line X Y. Let the straight-edge be made to coincide with X Y, then let the 45° set-square D E C (Fig. 16) be placed as shewn, resting upon the straight-edge. Then any line drawn along the edge D E makes an angle of 45° with the line X Y. Move the set-square to the left, or to the right, and any other line *d e*, also makes an angle of 45° with X Y. These two lines D E, *d e*, are PARALLEL to each other. A line drawn along the edge C E is PERPENDICULAR, or at RIGHT ANGLES, to X Y. By moving the set-square we can get another line *c e*, PERPENDICULAR to X Y. All such lines as C E, *c e*, are parallel to each other.

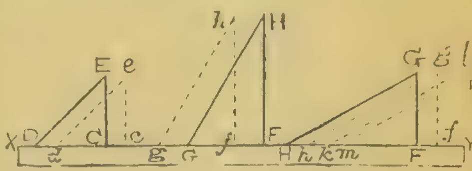


FIG. 16. FIG. 17. FIG. 18.

If we place the set-square of 60°, as at G H F (Fig. 17), with the angle of 60° at G, then a line drawn along the edge G H makes an angle of 60° with X Y. By shifting the set-square we can get other such lines as *g h* making the same angle with X Y. Lines G H, *g h* are PARALLEL to each other. The lines F H, *f h* are PARALLEL to each other, and are each at RIGHT ANGLES to X Y.

44.—If the 60° set-square be turned round with the angle of 30° at H (Fig. 18), then such lines as H G, *h g*, *k l*, *m n*, make angles of 30° with X Y. The lines F G, *f g*, are perpendicular to X Y. The rule X Y is not obliged to be *horizontal*, as drawn in the diagram, but may be placed in *any required position*, and then the set-squares used as already explained. Thus, suppose we wish to draw lines parallel to any given line A B (Fig. 19). Make *any edge* of *either set-square* to coincide with A B, as edge A C. Then place the *straight-edge* against the side A D; and by moving the set-square, the lines *e f*, *g h*, will be *parallel* to A B.

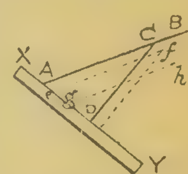


FIG. 19.

45.—The set-squares can be used in another way. Thus, suppose we have some parallel lines, K L, *m n*, *o p* (Fig. 20), and we wish to draw other lines at right angles to them. Use the 60° set-square, and place its longest edge against the line K L. Place the straight-edge X Y as shewn, then turn the set-square round as indicated, and draw the lines Q R, *s t*, *v w*, which are at right angles to K L, *m n*, *o p*. Remember, that if the 60° angle be used at L, the 30° angle must be used at Q, and *vice versâ*. The 45° set-square may be used in a similar manner. In all the foregoing operations it is necessary that the straight-edge be kept *steadily in a fixed position*. In the notes, appended to some of the problems, methods are shewn by which certain problems can be worked with the set-squares, only. These methods are given as practical illustrations, and should not be used in an *Examination*, unless asked for.

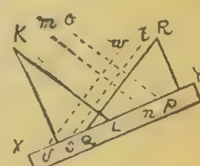


FIG. 20.

46.—The **T-square** is used for drawing horizontal lines. All lines drawn along the edge D C (Fig. 21) are parallel. To obtain vertical lines, use the edge L K of either set-square, resting on D C. Whilst engaged upon the same drawing, never use the **T**-square on more than one edge of the drawing-board, as if the latter be not perfectly true in shape, it will cause inaccuracies in the work.

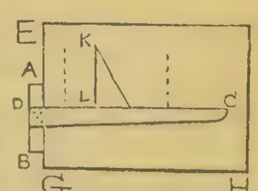


FIG. 21.

47.—As it is absolutely necessary that straight-edges and set-squares should be perfectly true, if accurate work is to be done, the following methods are given for testing them. For the *straight-edge*:—Place it in position I J (Fig. 22), and draw a line along its upper bevelled edge. Then turn the straight-edge round, in the position K L, so that the same bevelled edge may cover the line already drawn, and draw another line. If the edge is true, the two lines will coincide exactly, as in the diagram. If there is any deviation whatever, the ruler is useless. For the *set-square*:—Place a true straight-edge in position M N, and the 60° set-square, as at O P Q (Fig. 23). Draw the line O Q. Reverse the set-square, to position O R Q. Draw another line O Q, and if it coincides exactly with the first line drawn, then O Q must be a perpendicular, and the angle O Q P must be a right angle (See § 36). If there is any deviation, as in Fig. 24, the set-square is useless. Adopt a similar test for the set-square of 45°.

FIG. 22.

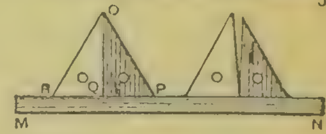
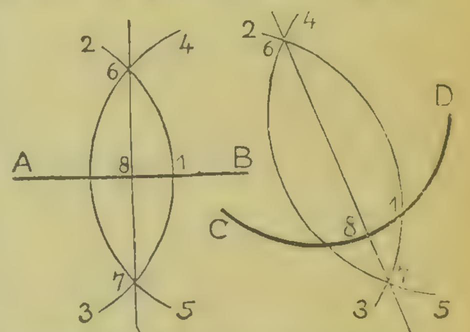


FIG. 23. FIG. 24.

PROBLEM 1.—To bisect, that is, to divide into two equal portions, a given straight line A B; or a given arc C D.



1.—Use the *bow-pencil compasses*. Place the point end on **A**, and open out the pencil end until it stretches some distance *beyond* the centre of the line, as to **1**. With centre **A**, and the distance or radius **A 1**, draw or *describe* the *arc* **2 3**.

2.—With centre **B**, and the *same* radius as just used, describe another arc **4 5**, cutting the first arc at points **6** and **7**.

3.—Draw the straight line **6 7**, and the line **A B** is BISECTED at point **8**, that is, divided into two equal parts.

4.—Adopt the same method for the bisection of the arc **C D**. Each of the lines **6 7** is called a *bisector*.

Note (a).—Lines A B and 6 7 are perpendicular to each other. Points 6 and 7 are equidistant from A and B, and from C and D.

FIG. 25.

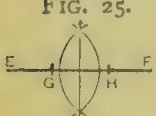


FIG. 26.

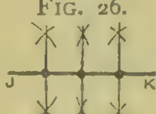


FIG. 27.

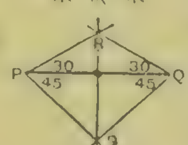


FIG. 28.

(b).—When the line to be bisected is a very long line, proceed thus:—Let **E F** (Fig. 25) be the given line. Mark off two equal distances, **E G**, **F H**, and bisect **G H**, as shewn.

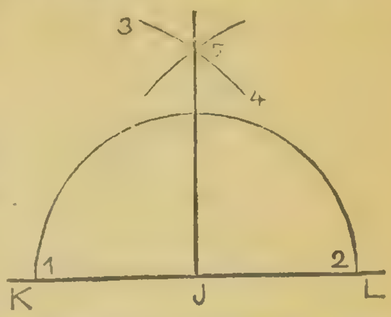
(c).—By Problem 1, a line J K (Fig. 26) may be divided into 2, 4, 8, 16, 32, &c., equal parts, by first bisecting the whole line, then each half, then each quarter, &c. Adopt a similar plan for an *arc*.

(d).—When bisecting a line, by trial with compasses, mark off two equal distances, L M, N O, from L and N (Fig. 27), as nearly as can be guessed, half the length of L N. Then the compasses can be more easily adjusted, in the next trial, to get the exact bisection. Adopt a similar method when bisecting an *arc*, by trial with compasses.

(e).—To bisect a STRAIGHT LINE **P Q** (Fig. 28), by the use of the SET-SQUARES, only. On one side of **P Q**, draw lines from the extremities, making equal angles with **P Q**, and intersecting at **R**. On the other side of **P Q**, draw lines at equal angles, intersecting at **S**. Join **R S**, which is the *bisector* of **P Q**. The equal pairs of angles may be of 30°, 45°, or 60°. When bisecting an ARC, join the extremities by a straight line, and proceed as above.

(f).—Line 6 7, which bisects the straight line A B, is called a *perpendicular* bisector, and whenever the word "bisector," in reference to a straight line, is used alone, a *perpendicular* bisector is meant.

PROBLEM 2.—From a given point J, in line K L, to draw a line perpendicular to K L; that is, at right angles to it.



1.—With **J** as centre, and the greatest convenient radius, mark points **1** and **2**, *equidistant* from **J**.

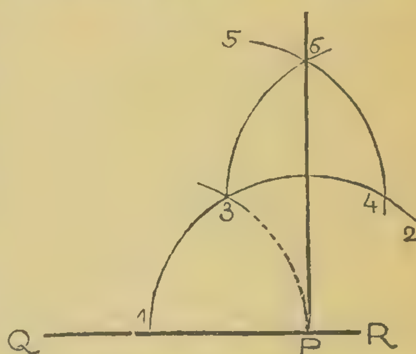
2.—With **1** as centre, and the greatest convenient radius, describe arc **3 4**. With **2** as centre, and the *same* radius, cut arc **3 4** at **5**.

3.—Join **5 J**, by a straight line, and it is PERPENDICULAR to **K L**.

Note (a).—It is not necessary to draw the *whole* of the semi-circle. It is quite sufficient if points 1 and 2 are marked equidistant from J.

(b).—The perpendicular J 5, can be obtained by using the *set-square*, as already explained.

PROBLEM 3.—To draw a perpendicular from a point P, at or near the end of a line Q R.



1.—With centre **P**, and the greatest convenient radius, describe an arc **1 2**. (N.B. Keep the *same* radius until the problem is finished.)

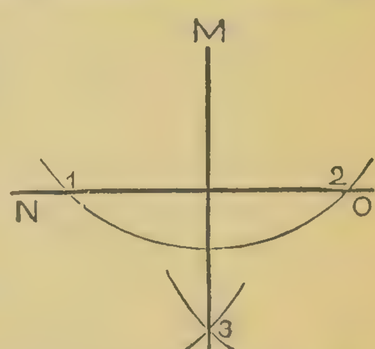
2.—With centre **1**, describe arc **3**. With centre **3**, describe arc **4 5**. With centre **4**, describe arc **3 6**.

3.—Draw the line **6 P** which is PERPENDICULAR to, or at RIGHT ANGLES to **Q R**.

Note (a).—The arcs which cut at 6 are not obliged to be of the *same* radius as the previous arcs. It is *safer* to use the *same* radius throughout, as it is easier to remember the construction.

(b).—This method requires the greatest accuracy, and it should only be employed when the *geometrical construction* is asked for in an examination. Observe, as a general rule, that for all *practical* purposes, a *geometrical construction* should not be resorted to when better results can be obtained by the use of the *set-squares*.

PROBLEM 4.—From point M, to draw a line perpendicular to a straight line N O.



1.—With **M** as centre, and any sufficient radius, describe an arc cutting **N O** at **1** and **2**.

2.—With points **1** and **2** as centres, and any radius, describe arcs cutting at **3**.

3.—Draw the line **M 3**, which is PERPENDICULAR to **N O**.

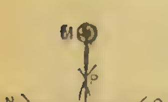
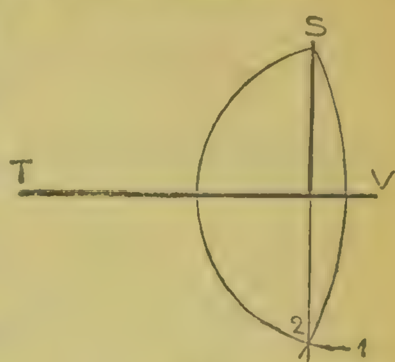



FIG. 29

Note (a).—The lines M 3, N O, are PERPENDICULAR to *each other*. The line M 3 can be obtained, at once, by the use of the *set-square*.

(b).—If there is not room on the drawing-paper to obtain the intersecting arcs as at 3, they may be drawn as shewn at P (Fig. 29). The line drawn from M through P is perpendicular to N O.

PROBLEM 5. To draw a perpendicular line from a point S, nearly opposite to the end of a line T V. 1st Method.



1.—With centre **V**, and the distance **V S**, describe arc **S 1**.

2.—With centre **T**, and radius **T S**, describe arc **S 2**.

3.—Draw the line **S 2**, which is PERPENDICULAR to **T V**.

Note (a).—If point S is to the right of point V, proceed in the same way as above. *Any* two points on T V will do for the centres of the arcs.

2nd Method.—**Let A be the given point and B C the given line.** (Fig. 30).

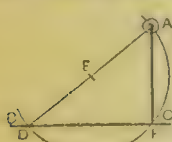


Fig. 30.

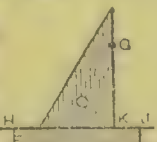


Fig. 31.

1.—Mark any convenient point **D**. Join **A D**, and bisect it at **E**.

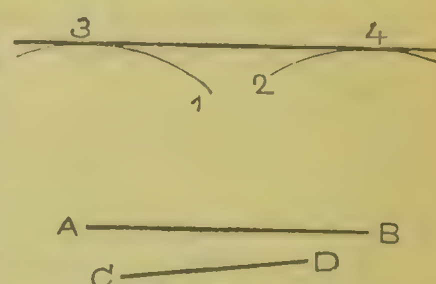
2.—With centre **E**, and radius **E A**, or **E D**, describe the arc, cutting **B C** at **F**.

3.—Draw **A F**, which is PERPENDICULAR to **B C**. (*See* § 81.) The first method is better than the second method, because it is simpler in construction.

(b).—By the second method, a perpendicular may be raised from point **F**. With *any* convenient point **E**, as centre, and radius **E F**, describe the arc, cutting **B C** at **D**. Through **E**, draw **D A**. Join **F A**, which is perpendicular to **B C**.

(c). Fig. 31 shews how we can draw G K perpendicular to H J, with a *set-square* only.

PROBLEM 6.—To draw a line parallel to A B, at a given distance, C D, from it.



1.—With centres **A** and **B**, and the distance **C D** as radius, describe arcs **1** and **2**.

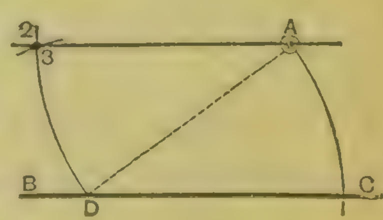
2.—Draw the line **3 4**, resting upon and just touching arcs **1** and **2**.

3.—The line **3 4** is PARALLEL to **A B**, and the distance **C D** from it.

Note (a).—*Any* two points in A B may be used as centres for arcs 1 and 2.

(b).—The problem may be worked thus:—Raise perpendiculars each equal to C D, at A and B, either geometrically or with a set-square. Join the upper extremities.

PROBLEM 7.—Through a given point A, to draw a line parallel to a given line B C.



1.—With *any* convenient point, **D**, as centre, and **D A**, as radius, describe the arc **A 1**.

2.—With **A**, as centre, and the *same* radius, describe the arc **D 2**. With centre **D**, and radius **1 A**, cut arc **D 2** at point **3**.

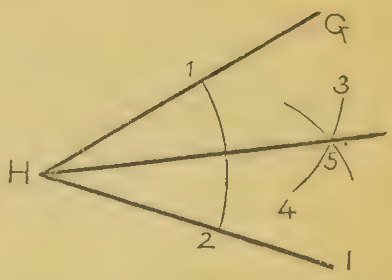
3.—Draw the straight line **A 3**, which is PARALLEL to **B C**.

FIG. 32.

Note (a).—If we join A D, the angle C D A is equal to the angle D A 3.

(b).—By the use of the SET-SQUARE, to draw a line through point **E**, parallel to **F G** (Fig. 32). Adjust the straight-edge and either of the set-squares, so that *any* edge of the set-square coincides with **F G**. Move the set-square, until this edge covers point **E**. Then line **H J** is parallel to **F G**. Adopt this method in preference to the geometrical one, in all practical work.

PROBLEM 8.—To bisect any given angle G H I; that is, to divide it into two equal angles.



1.—With **H** as centre, and the greatest convenient radius, describe an arc **1 2**.
2.—With point **1** as centre, and any radius, describe arc **3 4**. With centre **2**, and the *same* radius, cut arc **3 4** at **5**.
3.—Join **H 5**, and line **H 5** BISECTS the ANGLE **G H I**; that is, the angle **G H 5** is equal to the angle **5 H I**. Line **H 5** is called the *bisector* of the angle **G H I**.

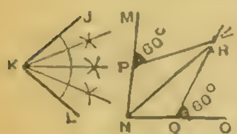
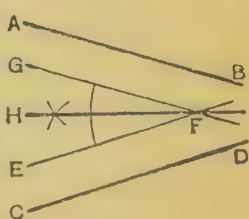


FIG. 33. FIG. 34.

Note (a).—By this problem an angle J K L (Fig. 33) can be divided into 2, 4, 8, 16, &c., equal angles, by first bisecting the angle, then bisecting each half, &c.

(b).—To bisect an angle **M N O** (Fig. 34), with a SET-SQUARE. With the straight-edge, mark off two equal distances, **N P, N Q**. With any corner of either set-square, draw lines from **P** and **Q**, making equal angles, and intersecting at **R**. Join **N R**, which bisects angle **M N O**.

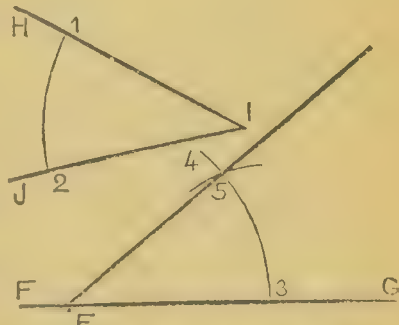
PROBLEM 9.—To bisect the angle which would be made by any two converging lines, A B, and C D, when the angular point is not accessible.



1.—By Problem 6, at any convenient distance, draw **E F** parallel to **C D**; and at the *same* distance draw **G F** parallel to **A B**.
2.—Bisect the angle **G F E** by the line **H F**. Then if the line **H F** is produced, it bisects the angle made by the lines **A B, C D**, *produced*.

Note.—E F, G F, must be drawn so that point F comes in a convenient position.

PROBLEM 10.—At a given point E, in the line F G, to make an angle equal to the angle H I J.



1.—With centre **I**, and the greatest convenient radius, describe an arc **1 2**.
2.—With centre **E**, and the *same* radius, describe arc **3 4**.
3.—With centre **3**, and the distance **1 2**, cut off **3 5** equal to **1 2**.
4.—Draw the line **E 5**, and the ANGLE **5 E G** is EQUAL to the angle **H I J**.

Note (a).—To make an angle at E, say two or three times the size of H I J,—mark off on the arc 3 4, two or three times the length of 1 2.

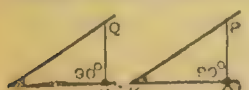


FIG. 35.

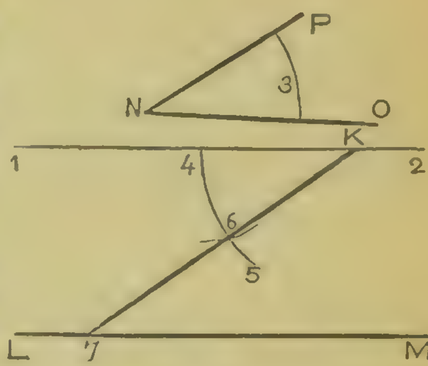
(b).—With a SET-SQUARE to make an angle at **K**, in line **K L**, equal to angle **M** (Fig. 35), mark two equal portions, **M N, K O**. At **N** and **O** draw equal angles with a set-square,—say of 90° each. Make **O P = N Q**. Join **K P**. Then **L K P = N M Q**.

PROBLEM 11.—From a given point K, to draw a line, meeting the line L M, and making an angle with it equal to the given angle P N O.

1.—Through point **K** (*by Problem* 7), draw the line **1 2** parallel to **L M.**

2.—With centre **N**, and any convenient radius, describe arc **3.** With centre **K**, and same radius, describe arc **4 5.** Cut off **4 6**, *equal* to the length of the arc **3.**

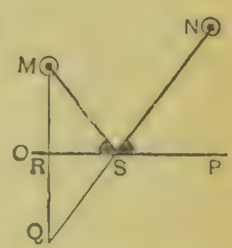
3.—Through **6**, draw the line **K 7.** Then the ANGLE **K 7 M** is EQUAL TO the angle **1 K 7**, which was made equal to **N.**



PROBLEM 12.—From two given points, M and N, draw lines to meet upon a given line O P, at equal angles.

1.—Draw **M Q**, perpendicular to **O P**, making **R Q** equal to **M R.**

2.—Join **N Q.** Draw **M S.** Then **M S, N S**, make EQUAL ANGLES with **O P.**



Note (a).—The perpendicular may be drawn from N, instead of M.

(b).—The lines M S, N S, make together the *shortest* path that can be drawn from M and N, to meet O P.

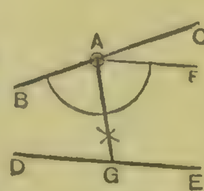
FIG. 36.

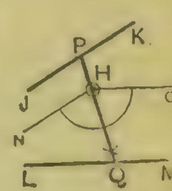
(c).—When the given line **T U** lies between the given points **V, W** (FIG. 36), from **W**, draw **W X** perpendicular to **T U**, making **Y X = Y W.** From **V**, through **X**, draw **V Z.** Draw **W Z.** The lines **V Z, W Z**, make equal angles with **T U.** The perpendicular may be drawn from **V** instead of **W.**

PROBLEM 13.—From a given point A, in a line B C, to draw a line making equal angles with the two inclined lines B C, D E.

1.—Draw **A F** parallel to **D E.**

2.—Bisect the angle **B A F**, by the line **A G.** Then **A G** is inclined at equal angles to **B C, D E.**

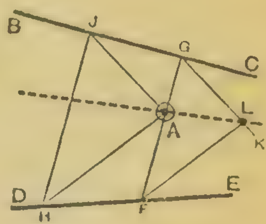




Note (a).—When the given point **H** is between two inclined lines **J K, L M**, draw **H N** parallel to **J K**, and **H O** parallel to **L M.** Bisect angle **N H O** by the required line **P Q.** If the given point is *outside* either of the given lines, proceed in a similar way.

(b).—If the given lines are parallel, no matter how the given point may be placed, draw through it a line perpendicular to the given parallel lines.

PROBLEM 14.—Through a given point A, to draw a line which shall converge towards the same point to which two lines B C, D E, converge,—when their point of intersection is inaccessible.



1.—Through **A** draw **F G** at any inclination to **B C**, **D E**.

2.—On either side of point **A**, draw a line **H J**, parallel to **F G**. Join **H A** and **J A**.

3.—Draw **G K** parallel to **J A**, and **F L** parallel to **H A**. Through **A** draw the chain-line **A L**, which converges to the same point as lines **B C**, **D E**.

Note (a).—If we draw H J on the right of A, point L comes on the left of A.

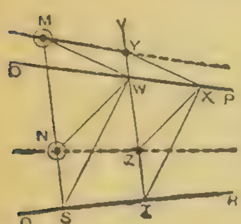


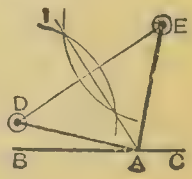
FIG. 37.

(b).—When two points, **M** and **N**, are given,—one between, the other outside two given lines **O P**, **Q R** (FIG. 37), through **N** draw **M S**. On either side of **M S** draw **T V** parallel to **M S**. Join **S W**, **W M**. Draw **T X**, **X Y**, parallel to **S W**, **W M**. Draw the chain-line **M Y**, which is one of the required lines. Join **W N**. Draw **X Z** parallel to **W N**. Draw the chain-line **N Z**, which is the other required line.

(c).—Adopt the same principle, no matter how many points are given. Thus, if a third point is given, either between or outside O P, Q R, draw lines to the point from S and M, then other lines parallel to these lines from T and Y, and the intersection of the latter lines gives the point through which to draw the third required line.

(d).—Triangles J H A and G F L, M S W and Y T X, N S W and Z T X are *similar* triangles. (*See* § 80.)

PROBLEM 15.—To find a point A, in a given line B C, which shall be equidistant from two given points, D and E.



1.—Draw **D E**, and bisect it by the line **1 A**.

2.—Then **A** is the required POINT, and if we join **A D** and **A E**, the lines **A D**, **A E** are of EQUAL LENGTH.

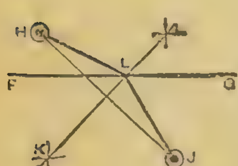


FIG. 38.

Note (a).—When the given line **F G** lies between the given points, **H**, **J** (FIG. 38), join **H J**. Bisect **H J**, by the bisector **K L**. Then point **L**, in the line **F G**, is equidistant from **H** and **J**.

(b).—If the lines D E, or H J, be perpendicular to B C, or F G, such points as A and L, cannot be obtained.

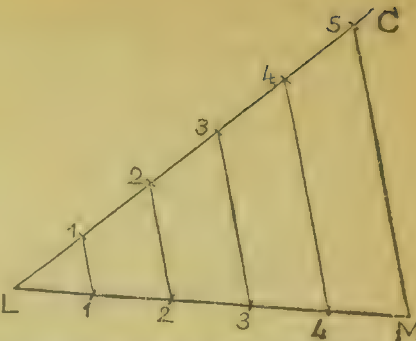
PROBLEM 16.—To divide a given straight line into any number of equal parts. Say, divide the straight line L M into five equal parts.

1.—Draw a line **L C**, of *any* length, and making *any* angle with **L M**.

2.—Mark off *five equal* spaces, of *any* convenient length, from **L** on **L C**.

3.—From the end of the last equal space draw a line to **M**, as **5 M**.

4.—From the remaining divisions between **L** and **5**, draw lines to **L M**, *parallel* to **5 M**. Then **L M** is divided into FIVE EQUAL PARTS.



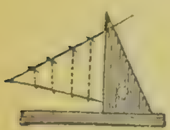


FIG. 39.

FIG. 40.

Note (a).—In such problems as the above, where several parallel lines have to be drawn, use a SET-SQUARE and STRAIGHT-EDGE, as shewn in Fig. 39.

(b).—L 1 1, L 2 2, &c., are *similar* triangles. If we draw *parallel* lines, *equidistant* from one another, and then draw lines in *any* direction across them, as at A, B (Fig. 40), lines A, B, are divided by the parallels into equal parts, as in Problem 16.

The Measurement of Angles.

48.—The Division of a Circle into Degrees.—We suppose the circumference of every circle to be divided into **360** equal parts, which are called DEGREES (°). All ANGLES are measured according to the number of degrees they contain. The fourth part of a circle, or *right angle*, A X C, or A X D (Fig. 41), contains the fourth part of 360°, that is 90°. A semi-circle, D A C, contains two right angles, or 180°. The angles B X E, E X C—each the half of a right angle—contain 45° each. By problem 18, obtain angle B X F, the trisection of the right angle D X B, then B X F = 30°, or the 12th part of a circle, and D X F = 60°, or the 6th part of a circle. Angle A X E = 135° (90° + 45° = 135°), and E X F = 75° (45° + 30° = 75°), and C X F = 120° (45° + 45° + 30° = 120°). If we divide the whole circumference into 360 equal parts, and draw lines from each division to the centre X, we obtain 360 equal angles of 1° each.

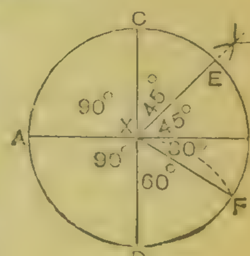


FIG. 41.

49. All the angles which can be made within a right angle at the angular point, added together, = 90°. Thus, in C X B (Fig. 41), 45° + 45° = 90°, and in B X D, 30° + 60° = 90°. Again, all the angles which can be made in a semi-circle, at its centre, added together, = 180°, or two right angles; thus, in D A C, 90° + 90° = 180°, and in the semi-circle C B D, 45° + 45° + 30° + 60° = 180°. Again, the sum, or total, of *all* the angles that can possibly be made at one point, as at the centre X, must = 360°; thus, in the diagram we see that 90° + 90° + 45° + 45° + 30° + 60° = 360°. (*Euclid* I. 15, *Cor.* 1, 2.)

50.—It does not matter what size a circle may be, the circumference is always supposed to be divided into 360 equal parts. Let A C B (Fig. 42), be an angle of 120°, then if we describe any number of circles, larger or smaller, with the same centre C, the angle made by the radii C A, C B, remains unaltered. Now, 120° happens to be ⅓ of 360°, so the distance A B is an arc ⅓ the length of the circumference of the large circle, and in the same way, D E is ⅓ the length of the circumference of the small circle.

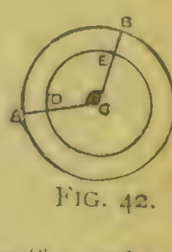


FIG. 42.

51.—Each degree is divided into 60 equal parts called *minutes* (′); and each minute is divided into 60 equal parts called *seconds* (″). So when great precision is required, we may have an angle of, say, 25 degrees, 47 minutes, 19 seconds—written thus, 25° 47′ 19″. A circle contains 360°, or 21,600 minutes, or 1,296,000 seconds.

52.—The Protractor is used for measuring angles. The arc of the SEMI-CIRCULAR PROTRACTOR is divided into 180 equal *degrees* (Fig. 43). The divisions on the FLAT PROTRACTOR (Fig. 44) are obtained by first marking 180 equal divisions on the arc of a semi-circle, which has the lower edge of the protractor for its diameter. Lines are then drawn from these equal divisions to cut three edges of the flat protractor. In each protractor the degrees are numbered from left to right and right to left. A little notch on each protractor indicates the position of point D, the centre of the semi-circle. The protractors are used in the following manner:—Suppose at point C, in line A B (see Figs. 43 and 44), we wish to draw an angle of 90°, or a *right angle.* Place the lower edge of either protractor on A B, so that the central notch D coincides with C. On the upper edge of the protractor, mark on the paper a point E, where the division 90 is shewn. Draw the line C E, which is at 90°, or at right angles, to A B. This is a useful way for obtaining perpendiculars. For an angle of 40°, mark where division 40 comes on the protractor. Draw C F, then the angle A C F = 40°. In the same way get an angle A C G = 120°, &c.

FIG. 43.

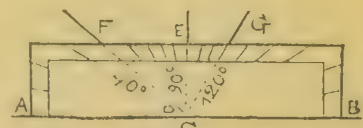


FIG. 44.

53.—To measure an angle, *already drawn*, say, a given angle A C F (Figs. 43 and 44). Place the lower edge of the protractor on A C, with D resting on C, and note where C F cuts the outer edge, where the stamped numerals give the size of the angle—in this case 40°. The measurement of angles by the SCALE OF CHORDS is explained in Chapter IX.

PROBLEM 17.—To make certain angles of a given number of degrees, by geometrical construction. At point A, make an angle of 60°. At B, an angle of 30°. At C, an angle of 15°.

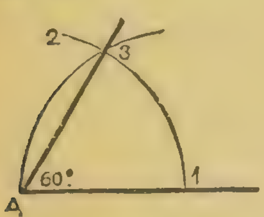


FIG. 45.

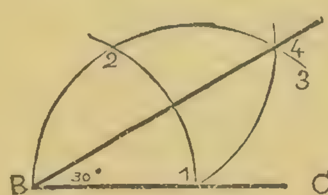


FIG. 46.

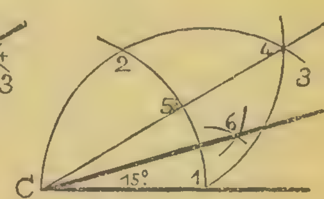


FIG. 47.

Fig. 45.—With centre **A,** and any radius, describe arc **1 2,** and with centre **1,** and *same* radius, describe arc **A 3.** Draw line **A 3,** and the angle **1 A 3** is one of **60°.**

Note (a).—The *radius* of any circle can be marked exactly *six* times round the *circumference* of the circle to which it belongs. And, if lines are drawn from the six divisions to the centre, the circle is divided into six equal *sectors*, the two *radii* of each sector forming an angle of 60°. Thus, each sector contains one-sixth of 360°—that is, 60°. The angle at A is obtained on this principle.

Fig. 46.—With centre **B,** describe arc **1 2.** With centre **1,** and *same* radius, describe arc **B 3.** With centre **2,** and *same* radius, get arc **1 4.** Join **B 4,** then angle **1 B 4** equals **30°** (*i.e., half of* 60°. *See Fig.* 45).

Fig. 47.—Obtain **1, 2, 3, 4,** as in Fig. 46. Bisect angle **5 C 1,** by line **6 C.** Then angle **1 C 6 = 15°** (*i.e., half of* 30°. *See Fig.* 46).

At D, make an angle of 45°. At E, an angle of 75°; and at F, an angle of 120°.

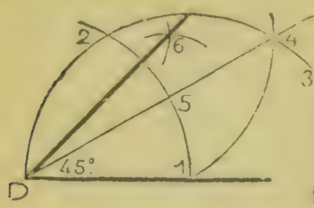


FIG. 48.

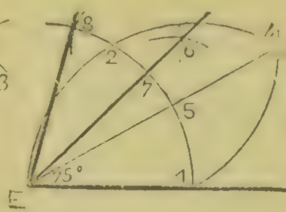


FIG. 49.

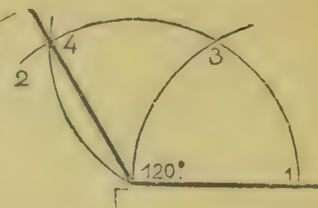


FIG. 50.

Fig. 48.—Obtain **1, 2, 3, 4, 5,** as in Fig. 47. With centres **2** and **5** describe equal arcs cutting at **6.** Join **D 6.** Then the angle **1 D 6 = 45°** (*i.e., three-fourths of* 60°. *See Fig.* 45).

Fig. 49.—Obtain points **1** to **6,** as in Fig. 48. Draw **E 6.** Make **2 8 = 2 7.** Draw **E 8.** Then angle **1 E 8 = 75°.**

Fig. 50.—With centre **F,** and any radius, describe arc **12.** With centre **1,** and *same* radius, describe **F 3.** With centre **3** and *same* radius, obtain point **4.** Draw **F 4.** Then angle **1 F 4 = 120°.** (*The radius has been marked off twice—from* 1 *to* 3, *and from* 3 *to* 4—*giving us twice* 60°, *or* 120°.)

Note (b).—Many other angles may be obtained by similar modes of geometrical construction. Thus, for 20°, divide 1 3 (Fig. 45) into 3 = parts. For 10°, bisect an angle of 20°. For 40°, divide 1 4 (Fig. 50) into 3 = parts. For 50°, add 30° to 20°.

(c).—In mechanical and architectural drawing, obtain angles by the use of the protractor and set-squares, except when the geometrical method may be more convenient.

(d).—**To make certain angles with the set-squares**:—The set-squares give us, direct, angles of 30°, 45°, 60°, and 90°. Let X Y, in the annexed diagrams, be a straight line. Place the 45° set-square as at Fig. 51, and the exterior angle G = 135°. In Fig. 52, the 60°

FIG. 51.

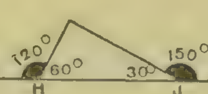


FIG. 52.

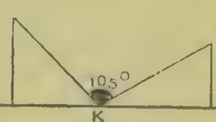


FIG. 53.

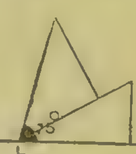


FIG. 54.

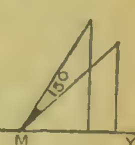
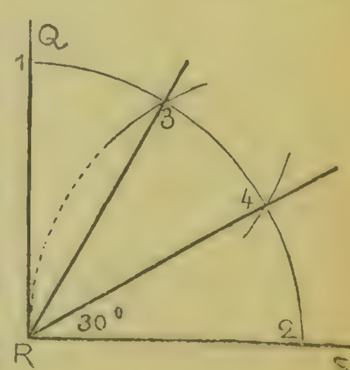


FIG. 55.

set-square gives an exterior angle of 120° at H, and 150° at J. Fig. 53 shews how an angle of 105° is obtained at K. Fig. 54 gives us 75° at L (30° + 45° = 75°); and Fig. 55 gives us 15° at M (60° − 45° = 15°).

PROBLEM 18.—To trisect a right angle Q R S, that is, to divide it into three equal angles.



1.—With centre **R,** and the greatest convenient radius, describe arc **1 2.**

2.—With centres **1** and **2,** and the *same* radius, describe arcs **3** and **4.**

3.—Draw **R 3** and **R 4,** and the right angle (90°), is TRISECTED, that is, divided into *three equal angles* of 30° each.

Note (a).—The *arc* 1 2 is trisected at 3 and 4. The *quadrant* is divided into 3 equal *sectors* 1 R 3, 3 R 4, and 4 R 2.

(b).—If we bisect the angles 2 R 4, 4 R 3, 3 R 1, we divide the right angle into *six* equal angles. Then if we bisect each of these six angles, we get *twelve* equal angles, and so on.

(c).—We can trisect a right angle by using the SET-SQUARE of 60°. Thus, make angle S R 3=60°, and angle S R 4=30°.

(d).—There is no direct *geometric* method for trisecting *any* given angle. Fig. 48 shews how an angle of 45° may be geometrically trisected. Angle 6 D 5=15°, i.e., ⅓ of 45°.

General Notes about Lines and Angles.

54.—Two straight lines cannot enclose a space. (*Euclid* I. *Axiom* X.)

55.—The perpendicular distance from a point to a straight line, is the shortest distance between the point and the line.

56.—Only one perpendicular can be drawn to a given straight line from a given point outside the line, or at a point in the line. And not more than one straight line can be drawn parallel to a given straight line, through a given point.

57.—The shortest distance between two parallel straight lines is the length of a perpendicular, common to both, enclosed between them.

58.—When two straight lines intersect each other, the vertically opposite angles are equal; that is, A X C=D X B, and A X D=C X B (Fig. 56). Thus, if the magnitude of one of the angles, A X C, is said to be 45°, then D X B must =45°. Again, by § 49, we know if C X A = 45°, that A X D must = 135°, and the sum of the four angles at X=360°. (*Euclid* I. 15.)

59.—Two angles which have the same apex E (Fig. 57), and one side, E F, common to both, are called *adjacent* angles.

FIG. 56.

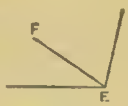


FIG. 57.

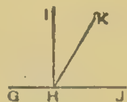


FIG. 58.

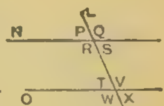


FIG. 59.

60.—If two angles, taken together, are equal to *one* right angle, they are said to be *complementary*, and either angle is termed the *complement* of the other. Thus, in Fig. 58, I H K+K H J=I H J, which is a right angle, then I H K is the *complement* of K H J, and *vice versâ*. If two angles, taken together, are equal to *two* right angles, they are said to be *supplementary*. Thus, G H K is the *supplement* of K H J.

61.—When a straight line, L, crosses two other straight lines, N, O (Fig. 59), the angles made by these lines have certain names. Thus, angles P, Q, W, X, are called *exterior* angles; R, T, S, V, are *interior* angles; the pairs S, T and R, V, are *alternate* angles; and the pairs P, T; R, W; Q, V; S, X, are *corresponding* angles.

62.—When the lines N, O (Fig. 59), are *parallel*, then the corresponding angle P = corresponding angle T, and in the same way R=W, Q=V, and S=X. The alternate angle R=V, and T=S. The two interior angles on the same side of L are, together, equal to two right angles, thus R+T=180°, and S+V=180°. The line L is called a *transversal*. (*Euclid* I. 27, 28, 29.)

Work out the Exercises on LINES AND ANGLES.

CHAPTER III. TRIANGLES.

SCALES AND SCALE-DRAWING. (PART I.)

CLASS-SHEETS 3 AND 4.

DEFINITIONS, etc.:—63.—A Rectilineal or Rectilinear Figure is a plane figure bounded by *straight lines*. When drawing a rectilineal figure, we are said to *construct* it. (*Euclid* I. *Def.* 20.)

64.—An Equilateral Figure has all its sides of equal length.

65.—An Equiangular Figure has all its angles of the same size.

66.—The term EQUILATERAL, or EQUIANGULAR, is applied to *any* plane rectilineal figure, no matter how many sides, or angles, respectively, the figure may have, provided the *sides*, or *angles*, as the case may be, are equal. A figure having both equal sides and equal angles, is said to be *equilateral and equiangular*.

67.—**A Triangle** is a plane rectilineal figure bounded by three straight lines, and it has three angles. It is also called a **Trilateral Figure** (*i.e.*, *three-sided*), or a **Trigon** (*i.e.*, *three-angled*). There are six varieties of triangles. Three are named with reference to the comparative lengths of their SIDES; and three are named with regard to the sizes of their ANGLES. (*Euclid* I. *Def.* 21.)

(A). *The three kinds of* TRIANGLES, *named with reference to their* SIDES, *are described in sections* 68, 69, 70 :—

68.—**An Equilateral Triangle,** A B C (Fig. 60), has *three equal sides.* The three angles are also equal, and each angle contains 60° : thus, this figure is likewise *equiangular.* (*Euclid* I. *Def.* 24.)

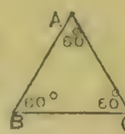


FIG. 60.

FIG. 61.

FIG. 62.

69.—**An Isosceles Triangle** has only *two equal sides,* D E, D F (Fig. 61). The two equal sides may be longer or shorter than the third side. The unequal side is called the *base*, and the angle opposite to the base is called the *apex*, or *vertical angle*, no matter in what position the triangle may be placed. The angles, E and F, at the base, are equal to each other. Thus, if two angles of a triangle are equal, the sides opposite to these angles are also equal to each other. (*Euclid* I. *Def.* 25.)

70.—**A Scalene Triangle,** G H I (Fig. 62), has *three unequal sides,* and consequently three unequal angles. (*Euclid* I. *Def.* 26.)

(B). *The three kinds of* TRIANGLES, *named with reference to their* ANGLES, *are described in sections* 71, 72, 73 :—

71.—**A Right-angled Triangle** has one *right angle*, as at K (Fig. 63), and two acute angles, as at J and L. The sides J K, K L, may or may not be equal. The side *opposite* to the right angle is called the **Hypotenuse,** and it is the longest side of the triangle. The remaining sides are called the BASE and PERPENDICULAR. (*Euclid* I. *Def.* 27.)

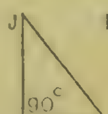


FIG. 63.

FIG. 64.

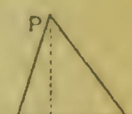


FIG. 65.

72.—**An Obtuse-angled Triangle,** M N O (Fig. 64), has one of its angles, N, an *obtuse angle.* (*Euclid* I. *Def.* 28.)

73.—**An Acute-angled Triangle,** P Q R (Fig. 65), has *three acute angles.* (*Euclid* I. *Def.* 29.)

74.—From the above definitions we see that a right-angled triangle, or an obtuse-angled triangle, can be also an isosceles or a scalene triangle; and an acute-angled triangle can be also an equilateral, an isosceles, or a scalene triangle.

75.—*Any* two sides of a triangle, added together, are greater in length than the remaining side. Thus, we cannot construct a triangle having sides of 2 in., 3 in., and 5 in., because 2 and 3 are not greater than 5. (*Euclid* I. 20.)

76.—The *three angles* of any triangle, added together, are always equal to **180°**. Thus, if two angles of a triangle are 60° and 90°, the remaining angle must be **30°**, as in the case of the set-square of 60°. If two angles are 47° and 29°, the remaining angle is 104°, since 47° + 29° + 104° = 180°. (*Euclid* I. 32.)

77.—The **Base** of a triangle is the side on which the triangle stands. The angle opposite to the base is called the **Vertex** (*plural*, VERTICES), **Vertical angle,** or **Apex** (*plural*, APEXES *or* APICES). Each angle, or side, may in turn become the vertex, or base, respectively, with two exceptions, *viz.*, the isosceles and the right-angled triangle. (*See* §§ 69 and 71.)

78.—The **Altitude** of a triangle is the perpendicular, drawn from the apex to the base, as line P S (Fig. 65); or to the base produced, as line M T (Fig. 64). The altitude of a scalene triangle varies, according to which side is assumed to be the base. (*Euclid* VI. *Def.* 4.)

79.—A Perimeter is the total length of *all* the sides of ANY *rectilineal* figure. Thus, in the case of a triangle, having sides of 3, 5, and 6 inches, we say the PERIMETER is 14 inches. Or, we say that the SUM of its sides = 14 inches.

80.—Similar Triangles are those which have their *corresponding* ANGLES *equal to each other*. Thus, in Fig. 66, A B C, D E F, are *similar* triangles, because angle A is the same size as the corresponding angle D, and B is the same as E, and C the same as F. The *sides* of A B C are not the same length as those of D E F, but they bear the *same proportion* to one another in A B C, as they do in D E F. Thus, if A C is twice B C, in length, then the corresponding side D F is twice the length of E F; &c. If the corresponding *sides*, as well as the corresponding *angles*, are equal to each other, the triangles are said to be EQUAL AND SIMILAR. (*Euclid* VI. *Def.* 1; *Prop.* 4.)

81.—The angles in a *semi-circle* are all *right angles*. Thus, in a semi-circle (Fig. 67), if we draw lines from V, W, the extremities of the arc, to any pts. X, Y, Z, in the arc, the angles V X W, V Y W, V Z W, are right angles. (*Euclid* III. 31.)

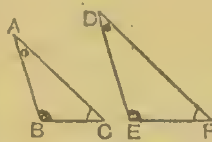


FIG. 66.

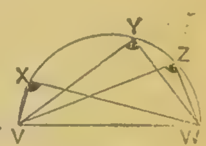


FIG. 67.

SCALES AND SCALE-DRAWING.—PART I.

Some elementary exercises on SCALES *are introduced here, as it is essential that a knowledge of these should be acquired before proceeding further in the course.*

82.—By the use of scales we can draw a diagram to any size, *greater* or *less* than a given diagram or object, &c. Thus, we can draw the plan of a house to such a scale, that every dimension of a $\frac{1}{4}$ inch on the plan represents 1 foot actually. This plan is then drawn to the "*scale of a $\frac{1}{4}$ inch to the foot.*" We may decide upon *any* distance to represent either feet, yards, or miles, &c. Thus, in a map, an *inch* may represent a *mile*. The map is then drawn to the "*scale of* 1 *in. to a mile.*" If we wish to make a drawing larger than the thing represented, as in the case of diagrams of microscopic objects, we may decide that 1 in. shall represent the *one thousandth* part of an actual inch in the object. Such a diagram would be drawn to a "*scale of an inch to the $\frac{1}{1000}$th of an inch.*" In every case, the real length is called the *natural distance*, and the enlarged or reduced length, used upon any given scale, is called the *artificial distance*.

83.—In geometrical drawing, the following signs are used to indicate feet and inches:—*one* accent (′) after a numeral signifies "*foot*" or "*feet*"; and *two* accents (″) signify "*inch*" or "*inches.*" Thus, 4′ 10″ means 4 feet 10 inches, either in the real dimensions, or in the given dimensions by scale, as the case may be.

84.—The construction of simply divided Scales.—When required to make a diagram to a given SCALE, say, a "**scale of 1 in. to 1 ft.,**" draw parallel lines, **A D** (Fig. 68), and from **A**, mark off *any* number of inches,

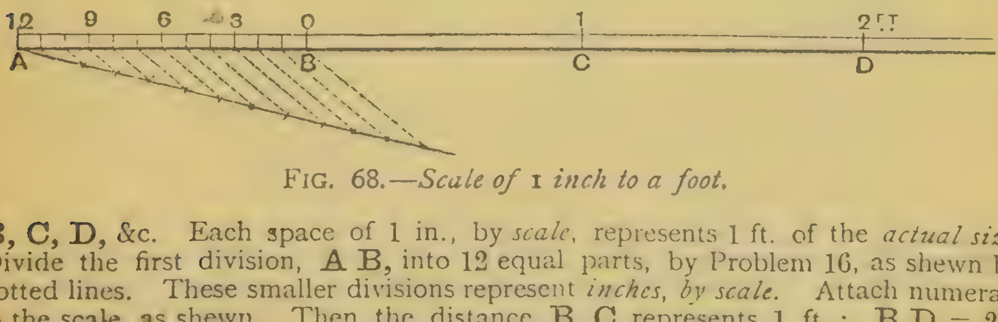


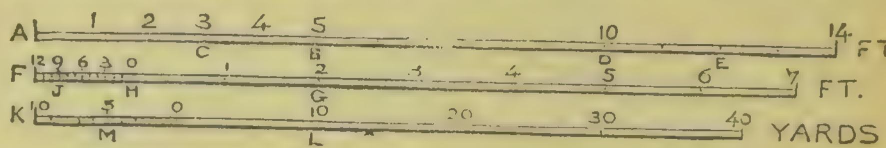
FIG. 68.—*Scale of* 1 *inch to a foot.*

B, C, D, &c. Each space of 1 in., by *scale*, represents 1 ft. of the *actual size*. Divide the first division, **A B,** into 12 equal parts, by Problem 16, as shewn by dotted lines. These smaller divisions represent *inches, by scale*. Attach numerals to the scale, as shewn. Then the distance **B C** represents 1 ft.; **B D** = 2′; **B 3** = 3″; **B 9** = 9″; **C 6** = 1′ 6″; **D 9** = 2′ 9″; and so on.

85.—Unfortunately, there is no distinct understanding, as to the method of describing a scale. The "*scale of* 1 *in. to* 1 *ft.*" is frequently stated as a "*scale of* 1 *ft. to* 1 *in.*" It must be remembred that both statements have, by custom, the same meaning. It is better, in all cases, to let the *first* quantity refer to the *scale* measurement; and the *second* quantity, to the *actual size*; thus, we say a "*scale of* 1 *in. to* 1 *ft.*"—or, as it is sometimes stated, "*scale* 1 *in.* = 1 *ft.*"

86.—To construct a scale of 1 inch to 5 feet, shewing 14 feet:—Draw the parallel lines from **A** (Fig. 69). Make **A B** = 1″. Divide **A B**

FIG. 69. FIG. 70. FIG. 71.



into 5 equal parts, and mark off 14 of these equal parts, which represent feet, by *scale*. Thus, **A C** represents 3′; **A D** = 10′, and **A E** = 12′, by scale. If the sub-divisions of inches are required, proceed as in Fig. 68, and divide A 1 into 12 equal parts. (*Simply divided scales are also called* **Plain Scales.**)

87.—Construct a scale of 1″ to 3′, to measure 7′, shewing feet and inches:—Draw the parallel lines from **F** (Fig. 70). Make **F G** = 1″. Divide **F G** into 3 equal parts, to represent feet, by scale. From **H**, mark off 7′, by scale. Divide **F H** into 12 equal parts, to represent inches. Then, from **G** to **H** represents 2′, and **G J** = 2′ 9″.

88.—Draw a scale shewing 20 yards to an inch, to measure up to 40 yards:—Draw the parallel lines from **K** (Fig. 71). Make **K L**=1″. Bisect **K L** at **O**. From **L**, mark off the distances to **20, 30, 40,** making them each equal to **L O**. Divide **K O** (the space of half an inch) into 10 equal parts; then these 20ths of an inch represent YARDS, by *scale*. We have thus constructed a scale of 1 in. to 20 yds. From **O** (*zero*) to **40** = 40 yards; **L M** = 15 yds.; **5** to **20** = 25 yds.—by *scale*.

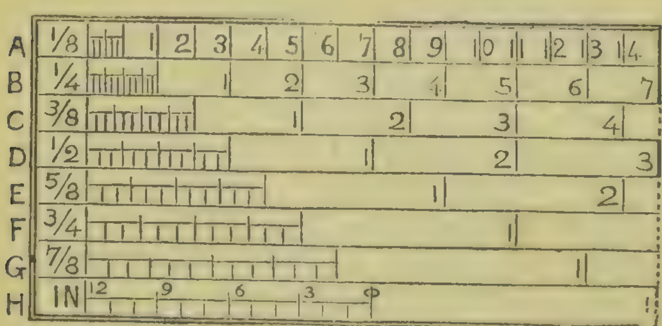


FIG. 72.

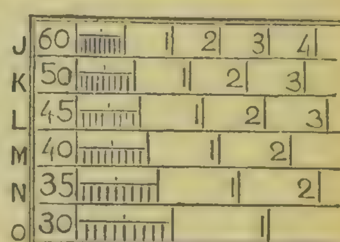


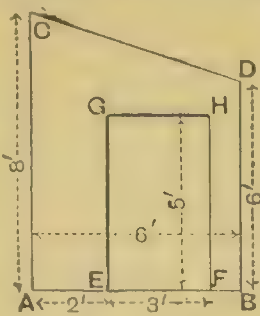
FIG. 73.

89.—Figs. 72, 73, represent scales which are usually stamped upon the flat protractor. Those from A to H (Fig. 72) are called DUODECIMAL SCALES, because each unit of measurement is divided into *twelve* equal parts. Scale H is an *inch scale*, which has been described in § 84. Scale G is a $\frac{7}{8}$″ scale; *i.e.*, a distance, equal to *seven-eighths of an inch*, is taken as the *unit* of measurement, to represent FEET. One of these units is divided into 12 equal parts, to represent INCHES. In the same way, F is a $\frac{3}{4}$″ scale, D a $\frac{1}{2}$″ scale, and A an $\frac{1}{8}$″ scale; &c. All the scales in Fig. 72 are used as explained in §§ 84 and 87. A scale of 1 inch to a foot is frequently called a "one-inch scale"; and $\frac{1}{2}$″ to 1 foot, a "half-inch scale"; &c.

90.—The scales shewn in Fig. 73 are called DECIMAL SCALES, because the unit of measurement, in each case, is divided into *ten* equal parts. In scale J, an inch is divided into 6 equal parts, and one of these units is subdivided into *tenths*. With this scale we can use the larger divisions to make a drawing $\frac{1}{6}$th the real size, or we may use the smaller divisions, to make a drawing $\frac{1}{60}$th the real size.

Scale K shews 50ths of an inch ; L, 45ths ; M, 40ths ; N, 35ths ; and O, 30ths of an inch. These scales may be used in the same manner as scale J.

91. — Exercises in Scale - Drawing.—Fig. 74 is the end view of a shed. It is drawn to a scale of $\frac{1}{8}$ inch to a foot. (*See scale* A, *Fig.* 72.) That is, every dimension of $\frac{1}{8}$ inch on this diagram represents 1 foot of the actual size. Thus, the line **A C** is marked as 8 feet high, and if we measure **A C** on the diagram, we find it is 8 feet by the *scale* A (Fig. 72), and it *represents* 8 feet in *actual length.*



Scale $\frac{1}{8}$ inch to a foot.
FIG. 74.

92.—Required an *enlarged* copy of Fig. 74, to a scale of a $\frac{1}{4}$ inch to a foot. For this purpose, construct a scale similar to scale B (Fig. 72), which is a scale of a $\frac{1}{4}$ inch to a foot. Refer to Fig. 74 for the given dimensions. Draw **J K** (Fig. 75), and make it 6 feet in length, by the $\frac{1}{4}$ inch scale. With a set-square, raise the perpendiculars **J L** = 8 feet, and **K M** = 6 feet, by scale. Join **L M.** Make **J N**=2 feet, and **N O**=3 feet, by scale. From **N** and **O** raise perpendiculars **N P, O Q,** each 5 feet, by scale. Draw **P Q.** Then the enlarged diagram is completed, and is drawn to a scale of a $\frac{1}{4}$ inch to a foot. Notice that in Fig. 75 the lines are twice the length of the corresponding lines in Fig. 74, because a $\frac{1}{4}$" scale is double the length of an $\frac{1}{8}$" scale. For practice, make another copy of Fig. 74, to a scale of $\frac{3}{8}$" to a foot.

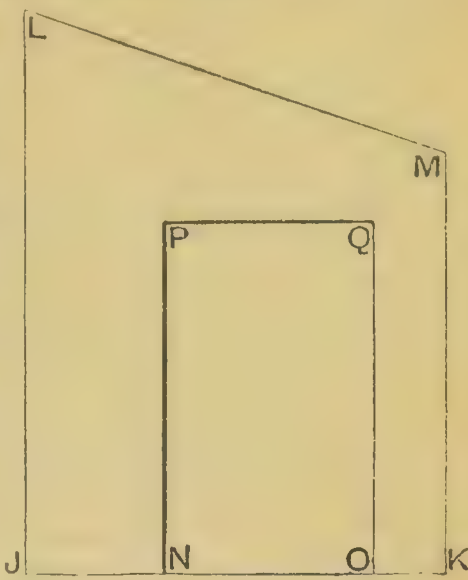


FIG. 75. *Scale $\frac{1}{4}$ inch to a foot.*

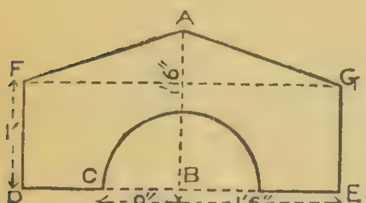


FIG. 76.—*Scale $\frac{3}{8}$" to a foot.*

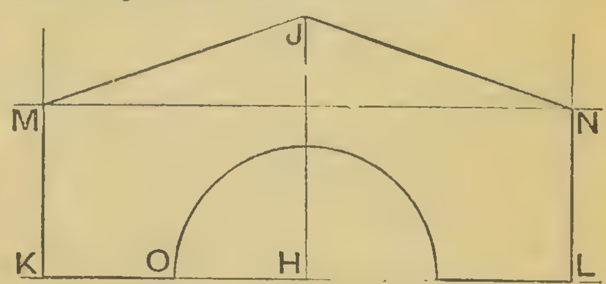


FIG. 77.—*Scale $\frac{5}{8}$" to a foot.*

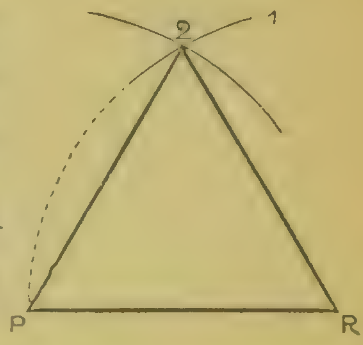
93.—Fig. 76 is a diagram drawn to a scale of $\frac{3}{8}$ths inch to a foot. See scale C, Fig. 72. By testing with the compasses, it will be seen that the dimensions marked on Fig. 76, correspond with the scale measurements on scale C. Thus, **D F** is marked as 1′ in height, and this length corresponds with a measurement of 1′, by scale, on scale C.

Make an *enlarged* copy of Fig. 76, to a scale of $\frac{5}{8}$ths inch to a foot. For this purpose, construct a scale of $\frac{5}{8}$" to a foot. See scale E, Fig. 72. Refer to Fig. 76 for the dimensions. Draw the *centre-line* **H J** (Fig. 77), and by the $\frac{5}{8}$" scale make it = 1′ 6". Draw a horizontal line through **H**, making **H K, H L,** each = 1′ 6". With a set-square, raise perpendiculars from **K** and **L.** Make **K M**=1′. Draw the horizontal line **M N** : then **L N** also equals 1′. Join **J M, J N.** With **H** as centre, and a radius **H O** of 9", describe the semi-circle. Fig. 77 is then an enlarged copy of Fig. 76, to a scale of $\frac{5}{8}$" to a foot. Whenever a diagram has two sides alike, as in Fig. 76, always commence by drawing the *centre-line*, and then mark off the equal dimensions on each side of it, as in Fig. 77.

(A) EQUILATERAL TRIANGLES.

PROBLEM 19.—On a given line, P R, to construct an Equilateral Triangle.

1.—With centre **R**, and radius **R P**, describe the arc **P 1**.
2.—With centre **P**, and the *same* radius, cut the arc **P 1**, at point **2**.
3.—Join **2 P**, **2 R**. Then **P 2 R** is the required EQUILATERAL TRIANGLE.

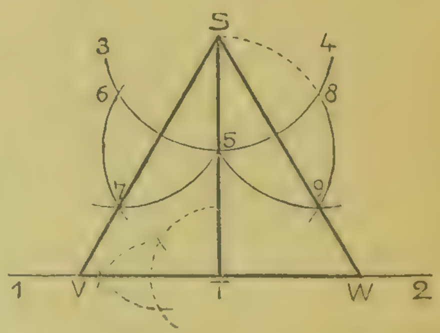


Note (a).—Each angle contains 60°, *i.e.*, one-third of 180°. The three sides are equal, because they are *radii of equal arcs.*

(b).—This triangle can be constructed with a set-square of 60°. Thus, draw lines making 60° with P R, at P and R. The intersection of these lines gives the apex at point 2.

PROBLEM 20.—To construct an Equilateral Triangle, the altitude, S T, being given.

1.—Through **T** (*by Problem* 3), draw the line **1 2**, *perpendicular* to **S T**.
2.—With centre **S**, and the greatest convenient radius, describe arc **3 4**.
3.—With centre **5**, and the *same* radius, describe arc **6 7**; and with centre **6**, and *same* radius, describe arc **5 7**.
4.—Obtain point **9**, in a similar way.
5.—Through points **7** and **9**, draw the lines **S V**, **S W**. Then **S V W** is an EQUILATERAL TRIANGLE, having **S T** for its ALTITUDE.



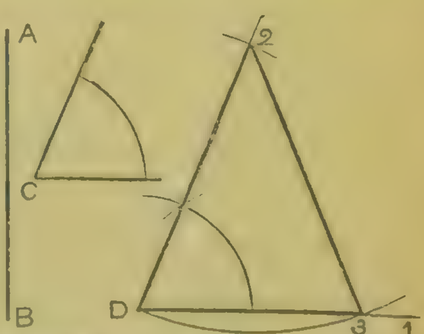
Note (a). An angle of 30° is made on each side of S T, at S (*by Problem* 17, *Fig.* 46); therefore V S T + T S W = 60°. The triangles V S T, T S W, are each similar in shape to a set-square of 60°.

(b).—T V = T W; therefore, to find the altitude of an *equilateral* triangle, bisect the base, and draw a line to the apex.——The above problem can be worked by the use of a set-square of 60°.

(B) ISOSCELES TRIANGLES.

PROBLEM 21. To construct an Isosceles Triangle, when each of the equal sides = A B, and each of the equal angles = C.

1.—Draw **D 1**, of *indefinite* length.
2.—Make angle **1 D 2** equal to angle **C**; and make **D 2** equal to **A B**.
3.—With centre **2**, and radius **2 D**, describe arc **D 3**. Join **2 3**. Then **D 2 3** is the required ISOSCELES TRIANGLE, having two sides equal to the line **A B**, and angles, at **D** and **3**, equal to **C**.



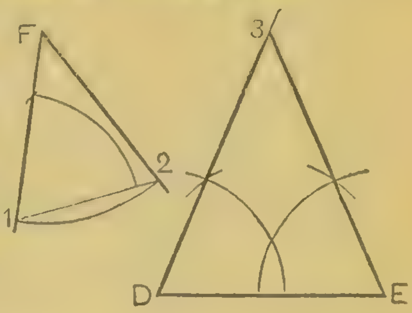
Note (a).—The sides 2 D, and 2 3, must be *equal*, because they are *radii of the same arc.*

(b).—If the *base* D 3, and one of the *equal sides*, A B, are given, with centres D and 3, and radius A B, describe arcs cutting at 2. Join 2 D, 2 3.

PROBLEM 22.—To construct an Isosceles Triangle, the base D E, and F, the size of the vertical angle, being given.

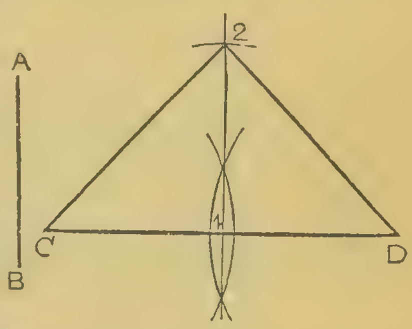
1.—With centre **F**, and any radius, describe arc **1 2**.
2.—Join **1 2**. Make each of the angles, at **D** and **E**, *equal* to the angle **F 1 2**, or **F 2 1** (by Problem 10).
3.—**D 3 E** is the required ISOSCELES TRIANGLE, having an angle at the APEX, equal to angle **F**.

Note.—The triangles F 1 2 and 3 D E are SIMILAR triangles.



PROBLEM 23.—To construct an Isosceles Triangle, when the altitude is equal to A B, and the base, C D, is given.

1.—Bisect **C D** by the *perpendicular* **1 2**, making **1 2** equal to **A B**.
2.—Join **2 C**, **2 D**. Then **C 2 D** is the required ISOSCELES triangle, having an ALTITUDE and BASE equal to two given lines, **A B**, and **C D**, respectively.

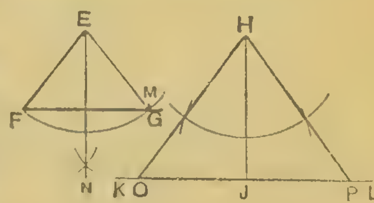


Note (a).—The ALTITUDE of an ISOSCELES triangle BISECTS the BASE. Thus, to find the *altitude* of an *isosceles* triangle, bisect the base, and draw a line to the apex.

(b).—To construct the Isosceles Triangle C 2 D, when one of the equal sides C 2, and the altitude 1 2, are given :—Through point **1**, draw the perpendicular **C D**, of indefinite length. With centre 2, and radius C 2, cut line **C D**, at **C** and **D**. Draw 2 C, 2 D. Then **C 2 D** is the required TRIANGLE.

PROBLEM 24.—Construct an Isosceles Triangle, when each of the base angles = E F G, and the altitude, H J, is given.

1.—Through **J**, draw **K L**, perpendicular to **H J**. (*By Problem* 3.)
2.—With centre **E**, and radius **E F**, describe arc **F M**. Join **E M**. Bisect angle **F E M**, by **E N**.
3.—Make angles **J H O**, **J H P** = angle **F E N**. Then **O H P** is the required ISOSCELES TRIANGLE, having angles at **O** and **P** = **E F G**.



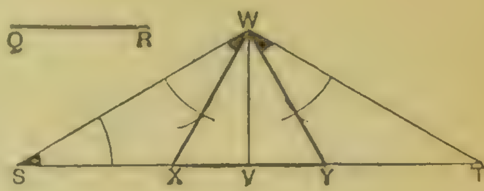
Note (a).—Line E M may be omitted, and E N may be drawn perpendicular to F G ; but the above method is adopted to shew more clearly the construction of the problem. F E M, O H P are *similar* triangles, the angles at H, O, P, are equal to the corresponding angles at E, F, M.

(b).—To construct the Isosceles Triangle, O H P, when the vertical angle, F E M, and the altitude, H J, are given :—Through J, draw K L perpendicular to H J. Bisect F E M by E N. Make angles J H O, J H P = angle F E N or N E M. Then **O H P** is the required TRIANGLE.

PROBLEM 25.—To construct an Isosceles Triangle, when the altitude = Q R, and the perimeter = S T.

1.—Bisect **S T** at **V**, and erect a perpendicular **V W** = **Q R**. Draw **W S, W T.**

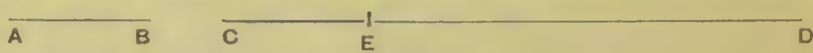
2.—Make angles **S W X, T W Y** = angle **S** or **T**. Then **W X Y** is an ISOSCELES TRIANGLE having an ALTITUDE = **Q R**, and a PERIMETER = **S T.**



Note (a).—S X W, T Y W are equal isosceles triangles, having S X, X W, and T Y, Y W of equal length. Thus, X W + W Y + X Y = S T. We can also get points X, Y, with the bisectors of S W, W T.

(b).—To construct an Isosceles Triangle, the base, **A B**, and **C D**, the perimeter, being given:—Make **C E** = **A B** (Fig. 78); then the length of each of the

FIG. 78.

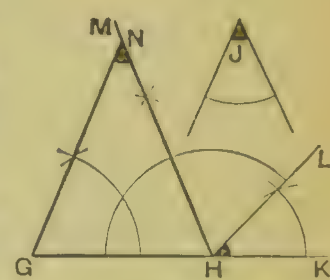


equal sides is equal to half **E D.** In other words, **E D** is equal to the *sum* of the equal sides.

PROBLEM 26.—On a given base, G H, construct an Isosceles Triangle, having a vertical angle equal to J.

1.—Produce **G H**, indefinitely, to **K**. Make angle **K H L** = **J.**

2.—Bisect angle **G H L** by **H M.** Make **H G N** = **G H M.** Then **G H N** is an ISOSCELES TRIANGLE upon the given base **G H**, having a vertical angle **G N H** = **J.**



Note (a).—On **A B** as base, construct an Isosceles Triangle, having its vertical angle half the size of each base angle:—Produce **A B** (Fig. 79). With centre **B**, describe a semi-circle, and divide it into 5 equal parts. Draw **B D** through the **1st.** division from **C.** Through the **2nd.** division from **E** draw **B F.** Make angle **B A F** = **A B F.** Then angle **A F B** is *half* **B A F** or **A B F.** By the construction it is evident that **A B F** = **F B D**, and that **D B C** is *half* **A B F.**

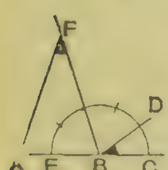


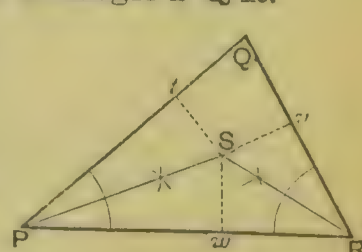
FIG. 79.

(b).—A set-square of 45° is an Isosceles triangle, having the vertical angle (90°) *twice* the magnitude of each base angle (45°).

(C) SCALENE TRIANGLES.

PROBLEM 27.—To find the centre of any Triangle P Q R.

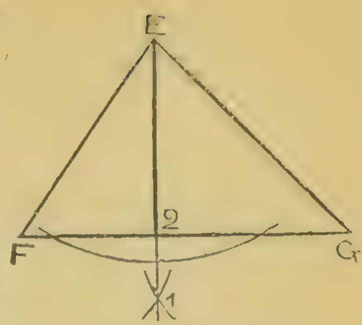
Bisect *any* two of the angles, say **P** and **R.** Then the intersection of the bisectors, at point **S**, is the CENTRE of the TRIANGLE.



Note (a).—If we bisect the angle **Q**, this bisector also passes through **S.** The centre of a triangle is *equidistant from the 3 sides.* Thus, if we draw a perpendicular from **S** to each side, the lines **St, Sv, Sw** are of *equal* length.

(b).—Adopt the same method to find the centre of an equilateral or isosceles triangle. In these triangles the intersection of the *altitude* with the bisector of *one* of the base angles gives the centre.

PROBLEM 28.—To find the altitude of a given Triangle E F G.

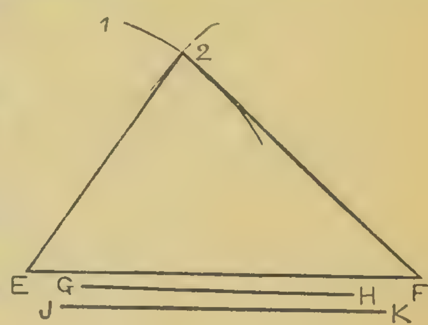


1.—From point **E**, draw **E 1** *perpendicular* to **F G** (*by Problem* 4).

2.—The line **E 2** is the required ALTITUDE.

Note.—If the line E 2 does not fall on the base, the base must be PRODUCED, and we obtain an altitude similar to line M T, in Fig. 64.

PROBLEM 29.—To construct a Triangle, when E F, G H, J K, the lengths of the three sides, are given.



1.—With centre **E**, and radius **G H**, describe arc **1**

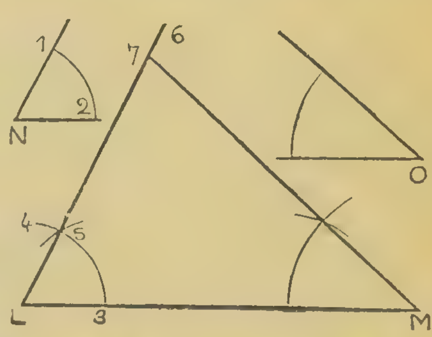
2.—With centre **F**, and radius **J K**, describe an arc cutting arc **1**, at **2**.

3.—Draw lines **2 E**, **2 F**. Then **E 2 F** is the required TRIANGLE.

Note (a).—To connect the side J K at E, instead of at F,—use radius J K, and centre E, for arc 1, and radius G H, and centre F to obtain point 2.

(b).—By this Problem we can construct a triangle *equal and similar* to another triangle.

PROBLEM 30.—To construct a Triangle, when one side equals L M, and the two angles adjacent to this side equal N and O.



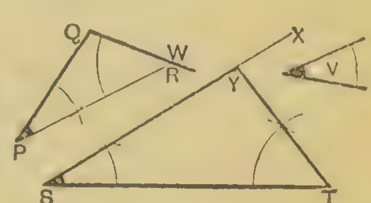
1.—Make the angle **M L 6** = angle **N**, as shewn by the arcs **1 2**, **3 4**, &c.

2.—Make angle **L M 7** = angle **O**. Then the TRIANGLE **L 7 M** is constructed upon **L M**, and the angles at **L** and **M** are equal to **N** and **O**.

Note (a).—The given angles N and O must together be *less* than 180°, because the sum of the *three* angles must = 180°.

(b).—Construct the Triangle, L 7 M, when two sides L 7, L M, and the included angle, equal to N, are given:—Draw two lines of indefinite length, making an angle at L=N. Cut off **L 7**, **L M**=given sides. Join **7 M**.

PROBLEM 31.—To construct a Triangle, when one side equals S T, the angle at S equals angle V, and the angle opposite to S T equals the angle P Q R.



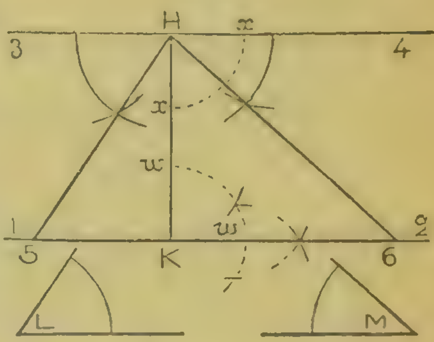
1.—At **P**, make angle **Q P W**=**V**.

2.—Make angle **T S X**=**W P Q**, or **V**.

3.—Make angle **S T Y**=**P W Q**. Then **S Y T** is the required TRIANGLE, constructed upon **S T**, and having an angle **Y**=**P Q R**, and an angle at **S**=**V**.

Note.—The angles P Q R and V must together be less than 180°.

PROBLEM 32.—To construct a Triangle, when the angles at the base equal L and M, and the altitude, H K, is given.



1.—By Problem 3, draw the line 1 2, *perpendicular* to H K.

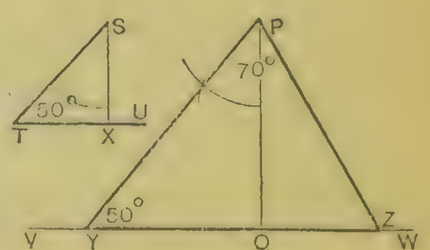
2.—Through H, draw 3 4, *perpendicular* to H K, by making the angle x H x = w K w.

3.—At H make angle 3 H 5 = L. Then angle H 5 2 also equals L. (*See Problem* 11.)

4.—Make angle 4 H 6 = M. Then H 6 5 also equals M. 5 H 6 is the required TRIANGLE.

Note.—Construct the Triangle, 5 H 6, when the base, 5 6, the altitude H K, and an angle L, at the base, are given:—By Problem 6, draw 3 4 parallel to 5 6, at the distance H K from it. At 5, make the angle 6 5 H = L. Join H 6. Then 5 H 6 is the required TRIANGLE.

PROBLEM 33.—Construct a Scalene Triangle, having a given altitude = $1\frac{1}{2}''$, a given vertical angle = 70°, and one base angle = 50°.



1.—Draw P Q = $1\frac{1}{2}''$. Make angle S T U = 50°. (*Use a protractor.*)

2.—Through Q draw V W perpendicular to P Q.

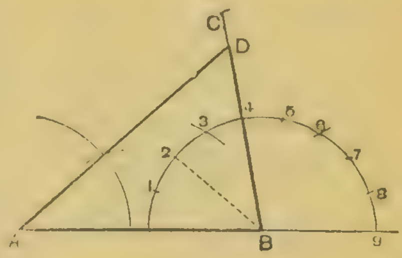
3.—Draw S X perpendicular to T U. Make angle Q P Y = X S T, and angle Y P Z = 70°. Then Y P Z is the required TRIANGLE, having a vertical angle Y P Z = 70°, and a base angle at Y = 50°. (The *diagram* is engraved to "*half-scale*," *i.e.*, a "*scale of* ½ *inch to* 1 *inch*.")

Note a).—Triangle Y P Q is *similar* to triangle T S X, consequently angle Q Y P = angle X T S (50°). If required, the angle at Z, instead of at Y, may be made equal to 50°.

(b).—To construct the Triangle, Y P Z, when a side P Y, base Y Z, and altitude P Q, are given:—Through Q, draw V W perpendicular to P Q. With centre P, and radius equal to given side, describe arc cutting V W at Y. Draw P Y. With centre Y, and radius = base, mark point Z. Draw P Z.

(c).—To construct the Triangle, Y P Z, when the altitude P Q, and the lengths of the two sides, P Y, P Z, are given:—Through Q, draw V W perpendicular to P Q. With centre P, and a radius equal to one of the given sides, cut V W at point Y. Draw P Y. With centre P, and a radius equal to the other given side, cut V W at Z. Draw P Z.

PROBLEM 34.—To construct a Triangle on a given side A B, having angles of a given proportion to one another—say in the proportion of 2, 3, 4.



1.—Produce **A B** indefinitely. With centre *B*, and *any* radius, describe a semi-circle. Then, as 2+3+4=9, divide the arc into 9 equal parts.

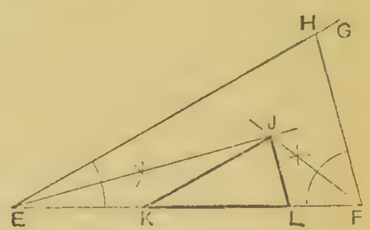
2.—From **B**, draw **B C** through **4**.

3.—Make angle **B A D** = angle **A B 2**. Then **A D B** is the required TRIANGLE, having angles **A, D, B,** respectively, in the proportion of **2, 3, 4.**

Note (a).—To divide the semi-circle into 9 equal parts, set off its *radius* from each extremity of the arc, at 3 and 6; then, by *trial with compasses*, obtain the smaller divisions, at 1, 2, 4, 5, 7, 8.

(b).—If we draw lines from points 1, 2, 3, 4, 5, 6, 7, 8, to B, we obtain 9 equal *sectors*, each containing 20°, total 180°. (*See* § 18.) As B D passes through 4, angle A B D = 4 of these sectors, or 80°. Angle B A D is made = 2 of them, and therefore = 40°. Angle A D B must = 60°, because 80°+40°+60°=180°. (*See* § 76.)

PROBLEM 35.—Construct a Triangle having angles of 30° and 75°, and a perimeter equal to $2\frac{1}{2}$ inches.



1.—Draw **E F** = $2\frac{1}{2}''$. Make angle **F E G** = **30°**, and angle **E F H** = **75°** (*with a protractor*).

2.—Bisect angles **F E H, E F H,** by lines cutting at **J.**

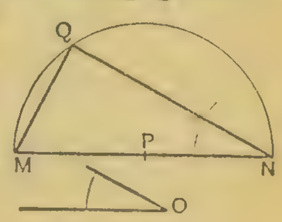
3.—Draw **J K** parallel to **E H**, and **J L** parallel to **F H.** Then **K J L** is the required TRIANGLE, having angles **K** = **30°**, **L** = **75°**, and a perimeter = $2\frac{1}{2}''$. (The *diagram* is engraved to "*half-scale*," *i.e.*, a "scale of $\frac{1}{2}$ inch to 1 inch," or a "scale of 6″ to a foot.")

Note (a).—By this problem a triangle can be constructed *similar* to a given triangle, and with a given *perimeter*. Thus, K L J is similar to E F H, and it has a given perimeter E F.

(b).—Construct the Triangle, K J L, when the angle L, one side J L, and E L, the sum of the other two sides, are given:—Make **J L** = given side. Make **J L E** = given angle, and **E L** = the given sum of the 2 sides. Join **E J.** Make angle **E J K** = **L E J.** Then **K J L** is the required TRIANGLE.

(D) RIGHT-ANGLED TRIANGLES.

PROBLEM 36.—Construct a Right-Angled Triangle, the hypotenuse, M N, and one of the acute angles, O, being given.



1.—Bisect **M N** at **P.** With centre **P**, and radius **P M**, describe a semi-circle.

2.—Make angle **M N Q** = **O.** Draw **M Q.** Then **M Q N** is a RIGHT-ANGLED TRIANGLE.

Note (a).—The angle M Q N is a *right angle*, because the angles in *semi-circles* are always right angles. Thus, if we mark *any* point on the semi-circle, and draw lines from it to M and N, we obtain a *right-angled triangle*. (*See* § 81.)

(b).—**To construct the right-angled Triangle, M Q N, when the two sides M Q, Q N, containing the right angle, are given** :—Draw M Q, Q N, their given lengths, and at right angles. Join points M and N.

(c).—**To construct the right-angled Triangle, M Q N, when the hypotenuse, M N, and one of the other sides, M Q, are given** :—Bisect M N. Describe a semi-circle. With centre M, and radius equal to the given side, obtain point Q. Draw M Q, Q N.

(d).—**Construct the Triangle, the hypotenuse, M N, and the perpendicular distance from the right angle to M N, being given** :—Describe a semi-circle. Draw a line parallel to M N, the given perpendicular distance from M N. This line cuts the arc at point Q. Draw Q M, Q N. The altitude must not be greater than ½ M N. If exactly *half*, point Q bisects the arc, and M Q = Q N.

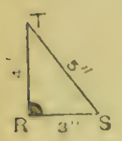


FIG. 80.

(e).—Every triangle having sides in the proportion of 3, 4, 5, is a *right-angled triangle.* Thus, in a triangle R S T, having sides 3″, 4″, and 5″ (Fig. 80), the 3″ and 4″ sides include a *right angle.* Any *similar* proportion may be used, as 6, 8, 10, or 9, 12, 15 ; &c. The lengths may be in feet or yards, &c. By using *any* scale of equal parts, we can draw a line R T, perpendicular to R S. This method is useful in land-surveying—the 100 links on the chain giving us a scale of equal parts.

General Notes about Triangles.

94.—A triangle may be constructed when either of the following particulars are given :—1st. Any 2 sides, and any one angle. 2nd. Any one side, and any two angles. 3rd. The 3 sides. A triangle of a required *size* cannot be constructed if only the 3 angles are given.

95.—The longest side of a triangle is always opposite to, or *subtends*, the greatest angle. The shortest side subtends the smallest angle. (*Euclid* I. 18, 19.)

96.—When one side, N O, of a triangle (Fig. 64) is produced, the *exterior* angle, T N M, is equal to the sum of the two *interior* and *opposite* angles, N M O, M O N. (*Euclid* I. 32.)

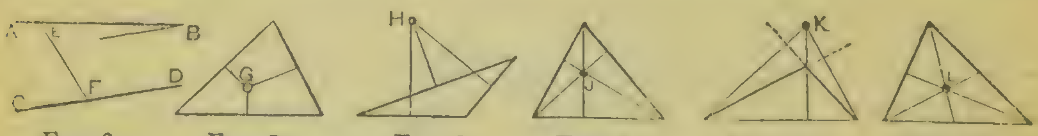


FIG. 81. FIG. 82. FIG. 83. FIG. 84. FIG. 85. FIG. 86.

97.—By knowing that the 3 angles of a triangle = 180°, we can ascertain the magnitude of the angle that would be made by 2 lines, A B, C D (Fig. 81), converging towards an inaccessible point. Draw *any* connecting line E F. Measure angle E F D = 112°, and F E B = 57°. Then the inaccessible angle must = 11°, because 112° + 57° + 11° = 180°. Or, we may ascertain the angle by drawing a line from D or B, parallel to the opposite line ; thus, angle B = 11°.

98.—The bisectors of the 3 *angles* of a triangle intersect at the same point, as in Problem 27. (*Euclid* IV. 4.)

99.—The bisectors of the 3 *sides* of a triangle intersect at the same point, as at G, Fig. 82. The point may come outside the triangle, as at H, Fig. 83. (*Euclid* IV. 5.)

100.—The 3 *altitudes* of a triangle intersect at the same point, either inside the triangle, as at J, Fig. 84 ; or outside, when produced, as at K, Fig. 85.

101.—Lines drawn from each angle to the centre of the opposite side, intersect at one point, as at L, Fig. 86. These 3 lines are called *median* lines, and they divide the triangle into 6 triangles of equal surface, or *area.*

102.—Given, the size, say 25°, of one of the acute angles of a right-angled triangle, find the size of the other acute angle. Thus, 25° + 90° = 115° ; then 180° — 115° = 65°, which is the size of the other acute angle.

103.—In a *right-angled triangle*, the right angle = the sum of the other 2 angles. In an *obtuse-angled triangle*, the obtuse angle is greater than the sum of the other 2 angles. In an *acute-angled triangle*, each angle is less than the sum of the other 2 angles. (*Euclid* III. 31, Cor.)

Work out TEST-PAPER A and the Exercises on TRIANGLES.

CHAPTER IV. QUADRILATERALS.

CLASS-SHEETS 4 AND 5.

DEFINITIONS:—104.—A Quadrilateral figure, or **Quadrangle,** is a plane figure bounded by four straight lines. The side upon which it stands is called the BASE. Any side may be given as the base. (*Euclid* I. *Def.* 22.)

105.—A Diagonal of a quadrangle is a straight line which connects any two opposite angles, as D B (Fig. 87), K H, G J (Fig. 88), and O Q (Figs. 89, 94). Every quadrangle has two diagonals, each of which divides the figure into two triangles. (*Euclid* I. *Def.* A.)

106.—A Parallelogram is a quadrangle which has its opposite sides equal and parallel, and its opposite angles equal. The SQUARE, RECTANGLE, RHOMBUS and RHOMBOID are parallelograms. Each diagonal divides a *parallelogram* into two *equal* triangles. The point of intersection of the diagonals is the *centre* of the figure, as at N (Fig. 88), and Z (Fig. 89). (*Euclid* I. 34.)

107.—A Square, A B C D (Fig. 87), has four equal sides, and four right angles. A square is said to be constructed *upon* a given line, as upon A B. (*Euclid* I. *Def.* 30.)

108.—A Rectangle, or **Oblong,** G H J K (Fig. 88), has its opposite sides equal, and has four right angles. We say that a rectangle is *contained* by any pair of its adjacent sides. (*Euclid* I. *Def.* 31; II. *Def.* 1.)

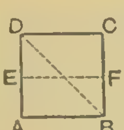


FIG. 87.

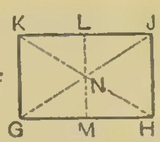


FIG. 88.

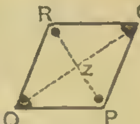


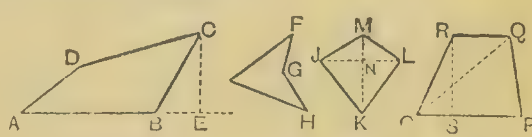
FIG. 89.

FIG. 90.

109.—A Rhombus, or **Lozenge,** O P Q R (Fig. 89), has *four* equal sides, It has a pair of equal obtuse angles, opposite to each other; and a pair of equal acute angles, opposite to each other. (*Euclid* I. *Def.* 32.)

110.—A Rhomboid, S T V W (Fig. 90), has its *opposite* sides equal. It has a pair of equal obtuse angles, opposite to each other; and a pair of equal acute angles, opposite to each other. (*Euclid* I. *Def.* 33.)

111.—A Trapezium, A B C D (Fig. 91), is a quadrangle, which has none of its sides parallel. Some of the sides or angles may, or may not, be equal. Fig. 92 is also a trapezium; and the angle bent inwards, as at F G H, is called a *re-entrant* angle. When there are two pairs of equal sides, as M J, M L, and K J, K L (Fig. 93), the figure is called a **Trapezion,** or **Kite.** (*Euclid* I. *Def.* 34.)



FIGS. 91. 92. 93. 94.

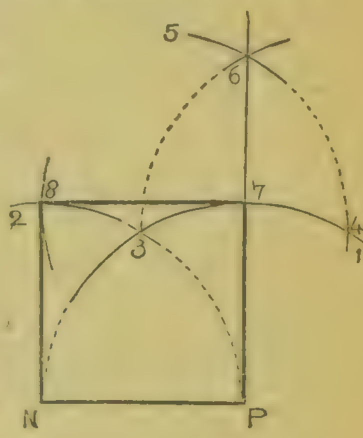
112.—A Trapezoid, O P Q R (Fig. 94), is a quadrangle, which has two sides parallel. Some of the sides or angles may, or may not, be equal. This figure is sometimes called a *trapezium.*

113.—The **Altitude** of *any* rectilineal figure is the perpendicular distance from the highest angle, or vertex, to the base, or base produced, as V Y, W X (Fig. 90), C E (Fig. 91), or R S (Fig. 94). *Any* side of a *rectangle* is its altitude, according to which side is given as the base, as K G; or G H, if K G be the base (Fig. 88). The altitude is sometimes called the *perpendicular height* or *distance.* (*Euclid* VI *Def.* 4.)

114.—A Diameter of a *parallelogram* is a straight line which passes through the *centre* of the figure, and is terminated by the opposite sides, as E F (Fig. 87), and L M (Fig. 88). A diameter divides the figure into two equal parts.

115. In naming a parallelogram we use the letters at the four angles: we say the parallelogram S T V W (Fig. 90): or we mention the letters at the opposite angles, and say the parallelogram S V, or W T.

PROBLEM 37.—To construct a Square upon a given line N P.



1.—With **N P** as radius, strike the arcs **N 1**, **P 2**, **4 5** **3 6**, and draw **P 6**, *perpendicular* to **N P**, as in Problem 3. Then **P 7** is one side of the square.

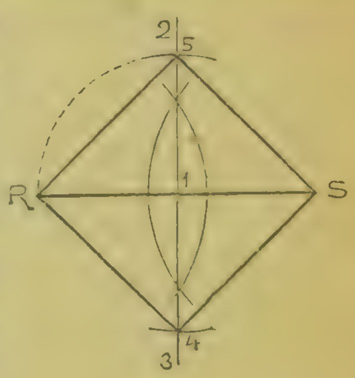
2.—With centre **7**, and radius **N P**, cut the arc **P 2** at point **8**.

3.—Join **8 N** and **8 7**. Then **N P 7 8** is the required SQUARE.

Note (a).—If N P is too long to use conveniently as a radius, then raise the perpendicular at P, by using a *smaller* radius. Make P 7 = N P. With centres N and 7, and radius N P, strike arcs cutting at 8. Join 8 N, 8 7.

(b).—To test the accuracy of the work, see that the *diagonals* N 7, P 8, are of *equal length*.

PROBLEM 38.—To construct a Square, a diagonal, R S, being given.



1.—Bisect **R S**, by a perpendicular **2 3**. Cut off **1 4**, and **1 5**, equal to **1 R**, or **1 S**.

2.—Join **R 5**, **5 S**, **S 4**, **4 R**. Then **R 5 S 4** is the SQUARE required, having a given DIAGONAL **R S**.

Note (a).—The DIAGONALS of a SQUARE are *equal* to each other, and at *right angles*; and they *bisect* each other. Point 1 is the *centre* of the square.

(b).—The two diagonals divide the square into four equal and similar triangles (*See* § 80). The diagonals bisect the angles.

(c).—A square, or rectangle, has all its angles in the circumference of a surrounding circle. Thus, if we complete the circle of which R 5 is an arc, the circumference passes through points S and 4. (*See also Problem* 40.)

PROBLEM 39.—Construct an Oblong, or Rectangle, the lengths of two of the adjacent sides, T V and W X, being given.

1.—At **V**, erect **V 1**, *perpendicular* to **T V**, and *equal* to **W X**.

2.—With centre **T**, and radius **W X**, describe arc **2**.

3.—With centre **1**, and radius **T V**, cut arc **2**, at point **3**.

4.—Join **3 1**, and **3 T**. Then **T 3 1 V** is the required RECTANGLE, having adjacent sides equal to **T V** and **W X**.

Note (a).—To test the accuracy of the work, see that the diagonal 3 V = T 1.

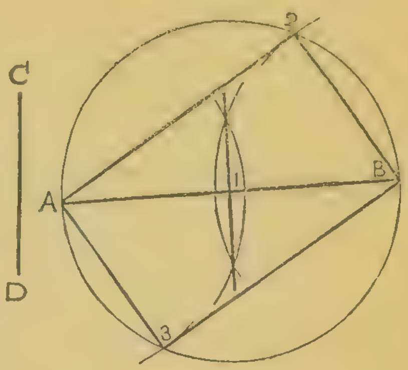
(b).—In all practical work, the figures constructed in Problems 37, 38, and 39, can be obtained with a set-square and straight-edge.

PROBLEM 40.—To construct a Rectangle, when each of the diagonals is equal to A B, and each of one pair of opposite sides is equal to C D.

1.—Bisect A B at 1, and with centre 1, and radius 1 A, describe a circle.

2.—With centres A and B, and radius C D, obtain points 2 and 3.

3.—Join A 2, 2 B, B 3, 3 A. Then A 2 B 3 is the required RECTANGLE.



Note (a).—If the *longer* side A 2 is given, instead of the *shorter* side, then get arcs at 2 and 3, with the *longer* side as *radius*, &c.

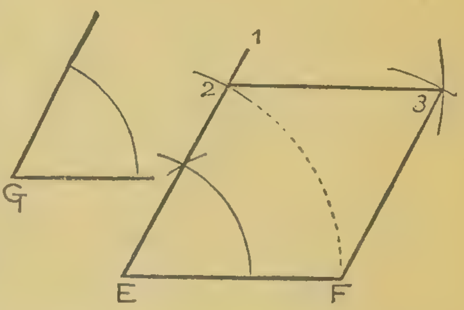
(b).—The other diagonal, 2 3, passes through point 1. The diagonals of a rectangle are equal; and they bisect each other, but not at right angles. — The angles A 2 B, A 3 B, are right angles, because they are angles in semi-circles. (*See* § 81.) If we join 2 3, the angles at A and B are also in semi-circles, and are right angles. *Any* parallelogram with its 4 angles in the circumference of a circle, must be *rectangular*.

PROBLEM 41.—To construct a Rhombus, when each of one pair of opposite angles is equal to G, and a side, E F, is given.

1.—Make angle F E 1 equal G. Cut off E 2, equal to E F.

2.—With centres 2 and F, and radius E F, describe arcs intersecting at 3.

3.—Join 2 3, and 3 F. Then E 2 3 F is the required RHOMBUS, having angles at E and 3, equal to G.

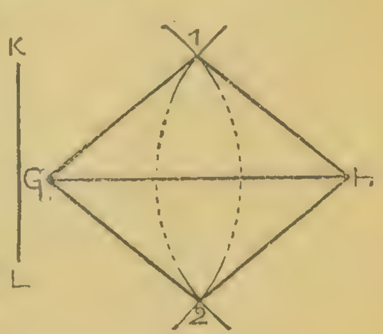


Note.—Construct a Rhombus, when one side E F (1½ in. long), and the length of the altitude (1 in.), are given:—Draw E F = 1½ in. Draw an *indefinite* line 2 3, parallel to E F, and 1 in. from it (*by Prob.* 6). With centre E, and radius E F, strike arc F 2, cutting the indefinite line at 2. Make 2 3 = E F. Join 3 F.

PROBLEM 42. Construct a Rhombus, or Lozenge, a diagonal, G H, and K L, the length of each side, being given.

1.—With centres G and H, and radius K L, describe arcs cutting at 1 and 2.

2.—Join G 1, 1 H, H 2, and 2 G. Then G 1 H 2 is the required RHOMBUS.



Note (a).—The diagonals of a rhombus bisect each other at right angles, and they bisect the angles of the figure. Every quadrilateral, having equal sides, must be either a square or a rhombus.

(b).—To construct a Rhombus when the two diagonals are given:—Draw them at right angles, and bisecting each other, then join the extremities of the diagonals.

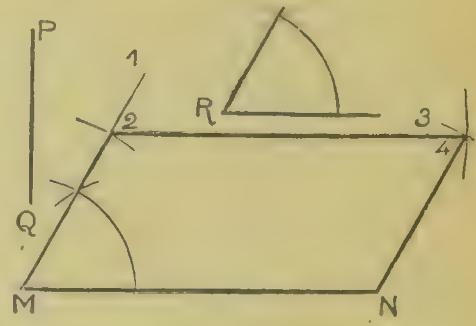
PROBLEM 43.—Lines **M N** and **P Q** are the lengths of two adjacent sides of a Rhomboid, and **R** gives the magnitude of each of one pair of opposite angles. Construct the Rhomboid.

1.—Make the angle **N M 1** equal to **R.** Cut off **M 2** equal to **P Q.**

2.—With centre **N**, and radius **P Q**, describe arc **3.**

3.—With centre **2**, and radius **M N**, cut arc **3**, at **4.**

4.—Join **2 4**, **4 N.** Then **M 2 4 N** is the required RHOMBOID.



Note (a).—If the angle at M or N is given in *degrees*, say 75° or 125°, with a protractor, draw such angles at M or N. Make M 2 and N 4 = P Q. Join 2 4.

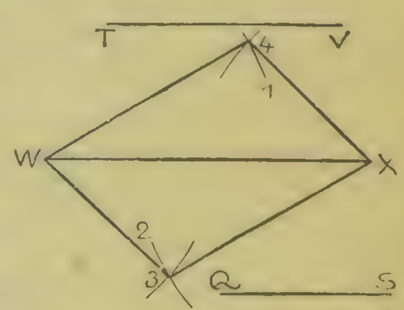
(b).—Construct a Rhomboid, when the base **M N** (2 inches long), an angle **R**, and the length of the altitude (1 inch) of the Rhomboid, are given :—Draw M N = 2 in. Draw a line **2 4**, of *indefinite* length, parallel to **M N**, and 1 inch from it (*by Problem* 6). Make angle **M = R.** We get point **2** where **M 1** cuts the indefinite line. Make **2 4 = M N.** Draw **4 N.**

PROBLEM 44.—Lines **Q S** and **T V** are the lengths of two adjacent sides of a Rhomboid, and **W X** is one of the diagonals. Construct the Rhomboid.

1.—With centres **W** and **X**, and radius **T V**, describe arcs **1** and **2.**

2.—With the same centres, and radius **Q S**, cut arcs **1** and **2**, at **3** and **4.**

3.—Join **W 4**, **4 X**, **X 3**, and **3 W.** Then **W 4 X 3** is the required RHOMBOID.



Note (a).—A diagonal of a parallelogram makes angle 4 W X = W X 3, and angle 3 W X = W X 4.

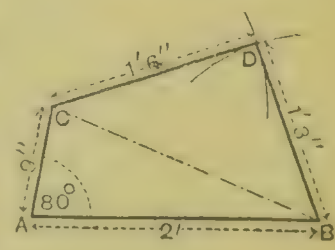
(b).—If the lengths of the 2 diagonals, and one of the angles made at their intersection be given, draw the 2 diagonals bisecting each other, and at the given angle. Join the extremities of the diagonals.

PROBLEM 45.—Construct the Trapezium, **A B D C**, from given dimensions. **A B** = 2 feet; **A C** = 9 inches, and makes an angle of 80° at **A**; **C D** = 1 foot 6 inches; and **B D** = 1 foot 3 inches. (SCALE 1 inch = 1 foot.)

1.—Draw the base **A B** = 2 feet *by scale of* 1 *inch* = 1 *foot.*

2.—Make angle **B A C** = 80°, and **A C** = 9 inches.

3.—With centre **C**, and radius of 1 foot 6 inches, describe an arc. With centre **B**, and radius of 1 foot 3 inches, cut this arc at point **D.** Join **C D**, **B D.** Then **A B D C** is the required TRAPEZIUM. (The above *diagram* is drawn to a *smaller* scale.)

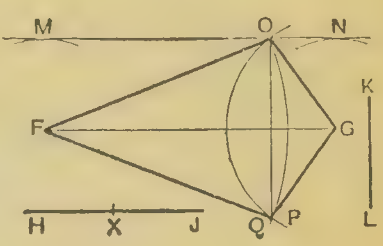


Note (a).—If the lengths of the sides are given as above, and also the length of the diagonal C B, which equals 2 feet, draw C B = 2 feet, by a scale of 1 inch = 1 foot. Obtain sides C D, D B, as in the above problem. With centre C, and radius of 9 inches, and centre B, and radius of 2 feet, describe arcs cutting at A. Join A C, A B.

FIG. 95.

(b).—The diagram, H J K L (Fig. 95), is roughly and inaccurately drawn. Make a *correct* drawing, according to the following dimensions :—H J = 4′; J K = 2′; K L = 5′; L H = 3′; angle H J L = 40°. Scale ½″ = 1′.

PROBLEM 46.—Construct a Trapezion, the lengths of the two diagonals, F G, H J, and K L, the length of each of one pair of equal sides, being given. The short diagonal to be bisected by the long diagonal.



1.—Bisect **H J** at **X.** Draw **M N,** parallel to **F G,** and the distance, **H X,** from it.

2.—With centre **G,** and radius **K L,** describe arc **O P.** With centre **F,** and radius **F O,** cut arc **O P** at **Q.**

3.—Join **F O, O G, G Q, Q F.** Then **F Q G O** is the required TRAPEZION.

Note (a).—Proceed in the same way if the short diagonal is to bisect the long one.

(b).—The diagonal O Q divides the trapezion into 2 unequal isosceles triangles. A trapezion has one pair of equal angles, as at O and Q. The diagonal which joins the unequal angles bisects them, and it is called the *symmetrical axis.*

(c).—**Construct the Trapezion, the diagonal F G, one side F O, and the angle Q F O, being given** :—Draw **Q F, F O,** each equal to the given side, and making the given angle at **F.** Bisect angle **Q F O** by **F G,** and make **F G** equal to the given diagonal. Join **O G, G Q.**

General Notes about Quadrilaterals.

116.—Any 3 sides of a quadrilateral are together greater than the remaining side.

117.—Straight lines which join the ends of equal parallel lines are also equal and parallel. (*Euclid* I. 33.)

118.—In ALL quadrilaterals the 4 interior angles together equal 4 right angles, or 360°. The *square* and *rectangle* are constructed with 4 right angles. In a *rhombus* (Fig. 89), if angle O = 70°, Q also = 70°, then R and P each = 110°, because 70° + 70° + 110° + 110° = 360°. In a *rhomboid* (Fig. 90), if angle S = 124°, V also = 124°, then W and T each = 56°, because these 4 angles together = 360°. Thus, in a *parallelogram,* if one angle be given, we know the sizes of the other angles. In a *trapezium* (Fig. 91), if angle A = 40°, B = 120°, and D = 155°, then C must = 45°, to make the total = 360°. Thus, if we know the sizes of any 3 angles of a trapezium, we also know, by calculation, the size of the 4th angle. A diagonal divides *any* quadrilateral into 2 triangles, and since a triangle contains 180°, *every* quadrilateral must contain twice 180°, *i.e.*, 360°.

FIG. 96.

FIG. 97.

119.—When 2 angles of a *trapezoid* are right angles, E, F (Fig. 96), the figure is sometimes called a *right-angled trapezoid.* When the sides connecting the parallel sides of a *trapezoid* are equal, G, H (Fig. 97), the figure is called an *isosceles* or *symmetrical trapezoid,* and the central line, J, is called the *median line,* or *axis of symmetry.*

Work out the Exercises on QUADRILATERALS.

CHAPTER V. POLYGONS.

CLASS-SHEETS 5 AND 6.

DEFINITIONS :—120.—A Polygon is a plane rectilineal figure, having more than four sides, or angles. Polygons are also called MULTILATERAL, or MULTANGULAR figures (*i.e.*, *many-sided*, or *many-angled*). A polygon has the same number of angles as it has sides. Some geometricians consider triangles and quadrilaterals to be polygons. The word "polygon" in this book will mean a figure having *more* than four sides. (*Euclid* I. *Def.* 23.)

FIG. 98. FIG. 99. FIG. 100. FIG. 101. FIG. 102.

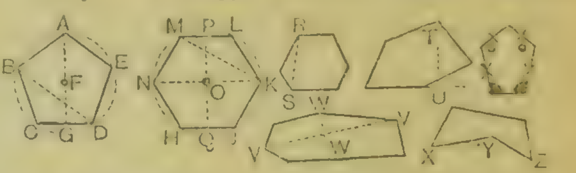


FIG. 103. FIG. 104.

121.—A Regular Polygon has all its sides and all its angles equal (Figs. 98, 99). A *regular polygon* has *all* its angles upon the circumference of a circle, and *the centre of the circle is also the centre of the polygon*, as at F and O. The centre of a regular polygon is *equidistant* from each *angle*, and it is also *equidistant* from each *side* of the figure. In all regular polygons—having an even number of sides—the *opposite* sides are parallel to each other, as in Fig. 99.

122.—A *regular* polygon is said to be **inscribed** in a circle, because all the angles are upon the circumference: and the circle is said to be **circumscribed** about, or *described about*, the polygon. (*Euclid* IV. *Defs.* 3, 6.) Throughout this book, when the word "polygon" is used, a *regular* polygon is meant, except when otherwise stated.

123.—An Irregular Polygon may have unequal sides and equal angles (Fig. 100); or equal sides and unequal angles (Fig. 101); or some of the sides, and some of the angles, may be equal (Fig. 102); or none of the sides nor angles may be equal (Fig. 103). Figure 104 is an irregular polygon, having a re-entrant angle at X Y Z.

124. REGULAR and IRREGULAR POLYGONS are named according to the number of sides or angles they have. Thus, a polygon having

5	sides is	a	**Pentagon**;	13	sides is	a	**Tridecagon**;
6	,,	a	**Hexagon**;	14	,,	a	**Tetradecagon**;
7	,,	a	**Heptagon**;	15	,,	a	**Pentadecagon**, or **Quindecagon**;
8	,,	an	**Octagon**;				
9	,,	a	**Nonagon**;	16	,,	a	**Hexadecagon**;
10	,,	a	**Decagon**;	17	,,	a	**Heptadecagon**;
11	,,	an	**Undecagon**;	18	,,	an	**Octadecagon**;
12	,,	a	**Dodecagon**, or **Duodecagon**;	19	,,	a	**Nonadecagon**;
				20	,,	a	**Bisdecagon.**

125.—A Diagonal of a POLYGON is a line drawn across the figure, joining any two of its angles, as B D (Fig. 98), M K, N K (Fig. 99), R S (Fig. 100), and V V (Fig. 103). Thus, a pentagon has 5 diagonals, a hexagon 9, a heptagon 14, an octagon 20, &c.

126.—A Diameter of a REGULAR POLYGON, that has an *even* number of sides, is a line drawn through the centre, and terminated by the opposite sides, as P Q (Fig. 99). When the sides are *uneven* in number, a line drawn from an angle

through the centre, to the middle point of the opposite side, is called a *diameter*, as A G (Fig. 98). A diameter divides a regular polygon into 2 equal parts.

127.—An Altitude of a POLYGON. Lines A G (Fig. 98), T U (Fig. 101), and W W (Fig. 103), are altitudes. (*See* § 113.)

128.—A Chord is a straight line, A B (Fig. 105), drawn across a CIRCLE, and terminated at each end by the circumference. The longest chord in a circle is a diameter.

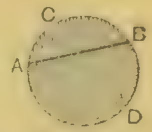
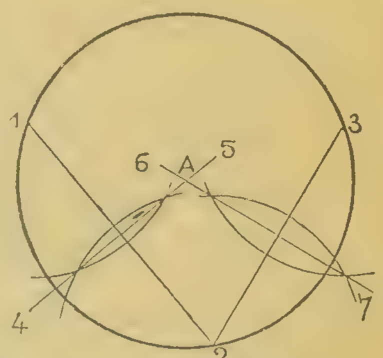


FIG. 105.

129. A Segment of a CIRCLE is a figure contained by a chord, and that portion of the circumference which it cuts off, as A B C A, or A B D A (Fig. 105). (*Euclid* III. *Def.* 6.)

PROBLEM 47.—To find the centre of a given Circle, or Arc of a Circle.



1.—Draw *any* two *chords*, **1 2** and **2 3**.

2.—Bisect these chords, by perpendiculars **4 5**, and **6 7**, intersecting at **A**. Point **A** is the CENTRE of the CIRCLE, or ARC.

Note (a).—The *chords* are not obliged to meet at 2. They may be drawn *anywhere* in the circle or arc, but it is better, when possible, to let them be at about *right angles* to each other. The chords may intersect. They should not be made *too short*.

(b).—Two or more arcs of the *same* circle have the *same* centre, H (Fig. 106).

(c).—To describe a Circle, the circumference to pass through three given points, 1, 2, 3 :—Join **1 2**, **2 3**. Draw the bisectors **4 5**, **6 7**, intersecting at **A**. With centre **A**, and radius **A 1**, **A 2**, or **A 3**, describe the required CIRCLE. The circumference passes through points **1**, **2**, and **3**. We may join **1 3**, and either **1 2** or **2 3**, and proceed as above, because the bisectors of *all* the chords of a circle pass through the centre.

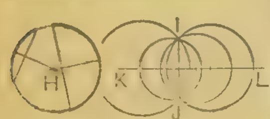
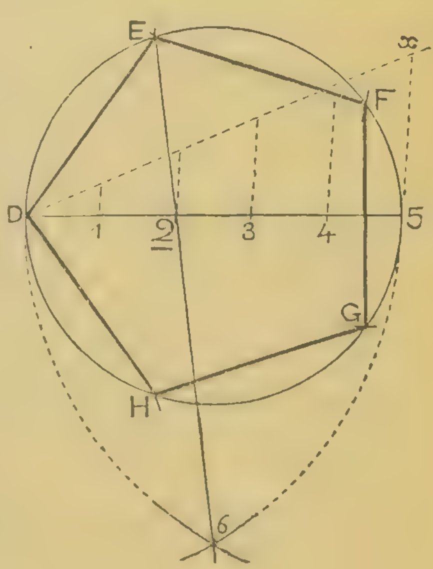


FIG. 106. FIG. 107.

(d).—Only *one* circle can be described to pass through 3 given points; but an *infinite* number can be made to pass through 2 given points I, J (Fig. 107). Line I J is a chord, common to all these circles, and the centres are all in K L, the bisector of the chord.

PROBLEM 48.—In a given circle, to inscribe ANY Regular Polygon—say a Pentagon.



1.—Draw a diameter **D 5**. Divide the DIAMETER into as many EQUAL PARTS, *as the polygon is to have sides*, in this case, FIVE equal parts (*by Problem* 16).

2.—With points **D** and **5**, as centres, and the *diameter* **D 5** as *radius*, describe arcs intersecting at **6**.

3.—From **6**, draw a line through **Point 2**, to **E**. Join **D E**, which is ONE SIDE OF THE REQUIRED POLYGON.

4.—Make **E F**, **F G**, **G H**, each equal **D E**.

5.—Join **E F**, **F G**, **G H**, **H D**. Then **D E F G H** is the required POLYGON.

Note (a).—The above method is only *approximately* correct. It is, however, sufficiently accurate for all practical work. On the same principle, we can (approximately) divide an *arc* into any number of equal parts, or a circle into equal *sectors*.

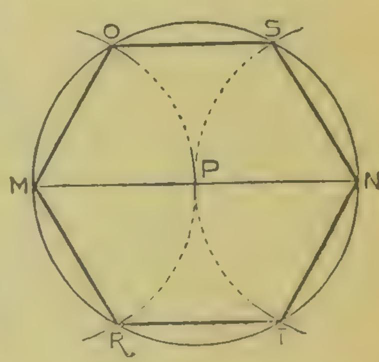
(b).—By this method, a REGULAR POLYGON, having ANY number of sides, can be inscribed (approximately), within a given circle. If a NONAGON is to be inscribed, divide the *diameter* into *nine* equal parts, and then proceed as above. To get the first side of the polygon, ALWAYS draw a line from point 6, through the 2ND DIVISION on the diameter, no matter how many sides the polygon is to have.

(c).—In a polygon, that has an *even* number of sides, a line drawn from one angle to the *opposite* angle (*a diagonal*), passes through the *centre*. When there is an *odd* number of sides, a line drawn from an angle, through the *centre*, *bisects* the opposite *side*. Note these facts, as tests for accuracy in the work.

(d).—The sides of a regular polygon are equal chords, which cut off equal segments from the circumscribed circle.

(e).—If we bisect the arcs of the circumscribed circle, that are between the angles of a regular polygon, we obtain the angular points for a regular polygon of *twice* the number of sides.

PROBLEM 49.—To inscribe a Regular Hexagon within a given circle. (SPECIAL METHOD.)



1.—Draw a diameter **M N.** With centres **M** and **N**, and **P M**, *the radius of the circle*, as radius, strike arcs at **O, R,** and **S, T.**

2.—Join points **M O, O S, S N,** &c., and we obtain the required REGULAR HEXAGON.

Note (a).—When inscribing a regular *hexagon* in a circle, the *special* method is preferable to that shewn in Problem 48. (*See note* a, *Problem* 17.) The three long diagonals divide a regular hexagon into six equal *equilateral triangles.* The regular hexagon is the only polygonal figure, which, by repetition, will cover a surface, without leaving any blank spaces, as in Fig. 108.

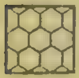
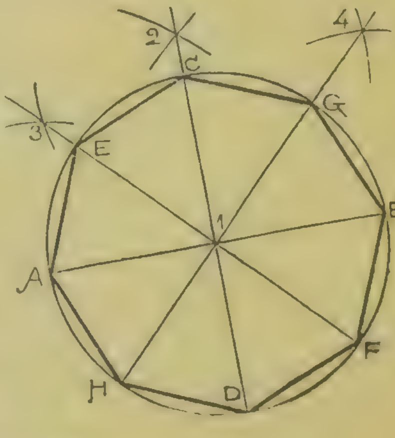


FIG. 108.

(b).—To construct a REGULAR HEXAGON, when one of the *long diagonals*, M N, is given :—Bisect M N at P. Describe a circle and proceed as above.

(c).—If we join the *alternate* angles of a regular hexagon, we obtain an EQUILATERAL TRIANGLE. If we bisect the arcs M O, O S, &c., we obtain, with points M, O, S, &c., the positions of the angular points of a REGULAR DODECAGON.

PROBLEM 50.—Inscribe a Regular Octagon within a given circle. (SPECIAL METHOD.)



1.—Let one of the angles touch point **A.**

2.—From **A**, draw the diameter **A B.**

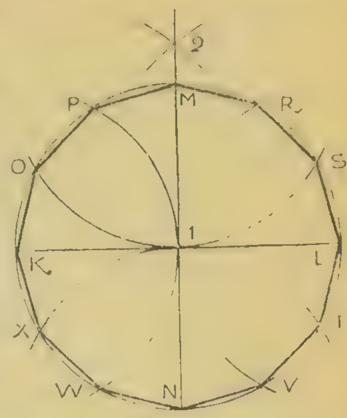
3.—By using **A** and **B** as centres, and describing arcs at **2**, and drawing from **2** through **1**, we obtain the diameter **C D**, at *right angles* to **A B.**

4.—Bisect angle **A 1 C**, by **3 F**; and draw **4 H**, bisecting the angle **C 1 B.**

5.—Join **A E, E C, C G,** &c., and we obtain the required OCTAGON.

Note.—Lines drawn from the centre to the angles of any *regular polygon*, cut the circumscribed circle into as many equal *sectors* as the polygon has sides. By joining the alternate angles of an *octagon* we obtain a SQUARE.

PROBLEM 51.—To inscribe a Regular Dodecagon within a given circle. (SPECIAL METHOD.)

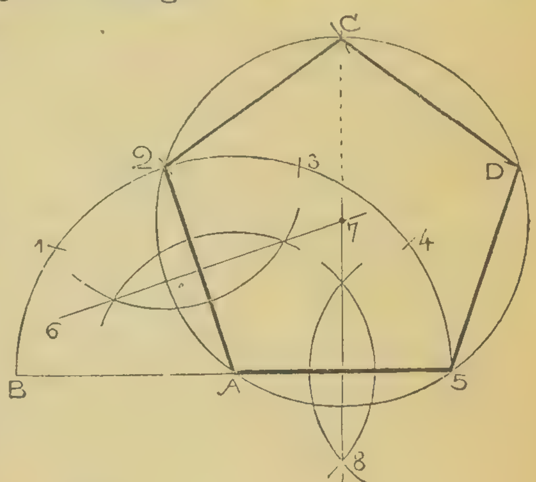


1.—Draw any two diameters, **K L** and **M N**, at *right angles* (*as in last problem*).
2.—With centres **K, L, M, N**, and radius **1 K**, strike arcs at **P, W; R, V; O, S; X, T.**
3.—Join **K O, O P, P M, M R**, &c., and we obtain the required DODECAGON.

Note (a).—Each right angle, or quadrant, is *trisected.* Points R, S, &c., can be obtained with a set-square of 60°. (*See note* c. *Problem* 18.) The *longest* diagonals of *any* regular polygon, of an even number of sides, bisect the angle at each extremity. Thus, K L bisects angles O K X, T L S.

(b).—Lines joining every 2nd angle of a dodecagon make a HEXAGON; every 3rd angle, a SQUARE; every 4th angle, an EQUILATERAL TRIANGLE.

PROBLEM 52.—On a given straight line, A 5, to construct ANY Regular Polygon—say a Pentagon.



1.—Produce the given side **A 5**—say towards the *left.* With centre **A**, and radius **A 5**, describe a *semi-circle.*

2.—Divide the semi-circle into *as many* EQUAL *parts*, as the polygon is to have *sides*; in this case, 5 *equal parts*, by trial with compasses.

3.—From **A**, draw **A 2**, which gives us another *side* of the polygon; and NO MATTER HOW MANY SIDES the polygon is to have, always draw from **A**, to the SECOND division on the semicircle.

4.—*Bisect* the sides **2 A, A 5**, by lines **6 7** and **8 7**, intersecting at point **7**, which is the *centre of the polygon.* With centre **7**, and radius **7 A**, draw the circle.

5.—Mark off, on the circumference, the divisions **2 C, C D**, *equal* to **A 5.** Join **2 C, C D, D 5.** Then **A 2 C D 5** is the required REGULAR POLYGON.

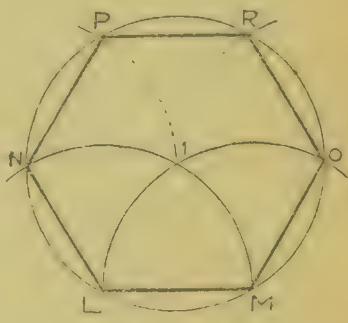
Note (a).—To divide a *semi-circle* into 3 equal parts, mark the radius 3 times round the arc. To get 6 equal divisions, bisect each third part; to get 12, bisect each sixth part; to get 9, divide each third part into 3. To get 4 equal divisions, bisect each half of the arc; to get 8, bisect each fourth part; &c. If the semi-circle to be divided is very small, describe a larger one, having the same centre, and with the diameter covering that of the smaller one. Then divide the *larger* semi-circle into the required number of equal parts, and draw lines to the centre. These lines divide the *smaller* semi-circle as required. Adopt a similar method for dividing a small *circle* into a given number of equal parts.

(b).—If we draw the diagonals, A C, A D, they pass through 3 and 4. In ALL regular polygons, each diagonal drawn from A—the angle at the centre of the semi-circle—passes through one of the equal divisions on the arc. We should note these facts, as tests for accuracy.

(c).—To construct a Regular Polygon, when 2 of the adjacent sides, A 2, A 5, and the angle contained by them, are given:—Draw the bisectors **6 7, 8 7.** With centre **7**, and radius **7 2, 7 A**, or **7 5**, describe the circle. Cut off **2 C, 5 D = A 2** or **A 5**,

and join the points obtained. The given sides need not be *adjacent*. Thus, if A B and C D are given in their proper positions, draw their bisectors, and proceed as above. In ANY *regular polygon*, the bisectors of the *sides*, *angles*, and *diagonals*, intersect at the CENTRE of the polygon.

PROBLEM 53. On a given straight line, L M, to construct a Regular Hexagon. (SPECIAL METHOD.)



1.—With centres **L** and **M**, and *radius* **L M**, describe arcs intersecting at **1**.

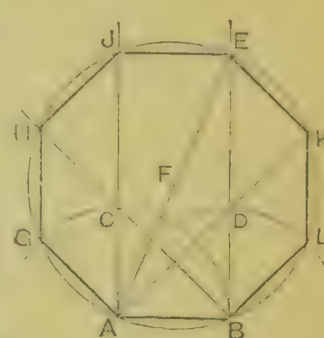
2.—With centre **1**, and the *same* radius, describe a circle.

3.—With centres **N** and **O**, and the *same* radius, mark points **P** and **R**. Join **L N, N P**, &c. Then **L N P R O M** is the required HEXAGON.

Note (a).—For a *regular hexagon*, the *special* method is preferable to that shewn in Problem 52. The *side* of a regular hexagon is equal to the *radius* of the *circumscribed circle*. (*See note* a, *Problem* 17.)

(b).—We can construct a regular hexagon, on a given *line*, L M, with a *set-square* of 60°. Thus, place a straight-edge, to coincide with L M. Draw L N, M O, each making 60° with the straight-edge, and each equal to L M. Raise perpendiculars at L and M. From L and M, draw lines at 60° with L M, cutting the perpendiculars at R and P. Join N P, P R, R O.

PROBLEM 54.—On a given straight line, A B, construct a Regular Octagon. (SPECIAL METHOD.)



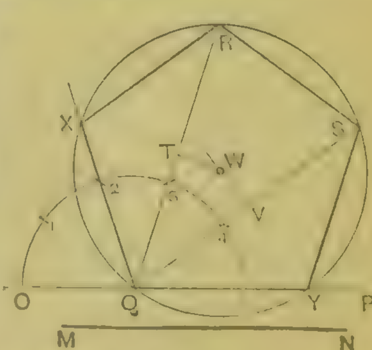
1.—At **A** and **B**, erect perpendiculars of indefinite length, with a set-square.

2.—With centres **A** and **B**, and radius **A B**, describe the arcs. From **A** and **B**, draw lines of indefinite length through **D** and **C**.

3.—Make **D E** = **D A**. Join **A E**, and bisect it at **F**. With centre **F**, and radius **F A** or **F E**, describe the circle, which gives points **G, H, J, K, L**. Join **A G, G H**, &c. Then **A G H J E K L B** is the required OCTAGON.

Note.—If A B is produced both ways, each of the *exterior* angles, at A and B, equals 45°. Thus a *regular octagon* can be constructed on a given line, with a *set-square* of 45°. (*See note* b, *Problem* 53.)

PROBLEM 55.—The length of ANY diagonal of ANY Regular Polygon being given, to construct the polygon.



1.—Suppose **M N** to be the length of one of the diagonals of a *regular pentagon*. Draw **O P**, of indefinite length. With any point **Q** as centre, describe a semi-circle, and divide it into as many equal parts, as the polygon is to have sides—*i.e.*, 5 equal parts.

2.—Through **2**, draw a line of indefinite length, a portion of which will be one side of the polygon. (*See Problem* 52.) Through **3** and **4**, draw **Q R, Q S**, = **M N**.

3.—Bisect **Q R, Q S**, by perpendiculars **T W, V W**, cutting at **W**. With centre **W**, and radius **W Q, W R**, or **W S**,

describe the circle, which gives points X and Y. Join X R, R S, S Y. Then Q X R S Y is the required PENTAGON. (*See note* b, *Problem* 52.)

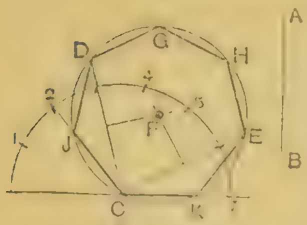
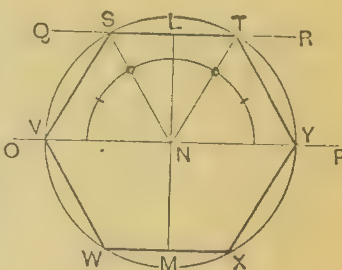


Fig. 109.

Note (a).—Proceed in the same way for ANY regular polygon, no matter which diagonal is given. Thus, suppose A B (Fig. 109) to be the length of a *short* diagonal of a regular heptagon. Get the pair of diagonals at C D, C E, and their bisectors cutting at F; &c. If a *long* diagonal, C G, is given, draw the pair of equal diagonals through 4 and 5; &c.

(b).—If a *central* diagonal of a polygon that has an *even* number of sides, is given, divide it into the same number of equal parts as the polygon has sides, and proceed as in Problem 48.

PROBLEM 56.—Given a diameter of ANY Regular Polygon that has an even number of sides. Construct the Polygon.



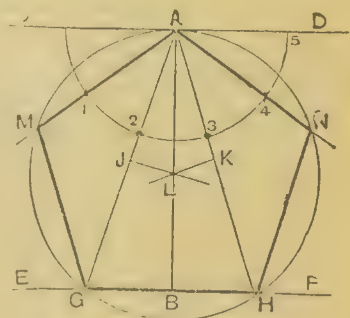
1.—Let L M be a diameter of a *regular hexagon*. Bisect L M, at N, which is the centre of the polygon. Through N, draw O P perpendicular to L M. Through L, draw Q R perpendicular to L M.

2.—With centre N, and *any* radius, describe a semi-circle, and divide it into as many equal parts as the polygon is to have sides, *i.e.*, 6. From N, draw N S, N T, through the 1st division on the semi-circle, on each side of L M. Then S T is one side of the polygon.

3.—With centre N, and radius N S, or N T, describe a circle, which gives points V and Y. Make V W, Y X, equal S T. Join S V, V W, &c. Then W X Y T S V is the required HEXAGON.

Note.—Proceed in the same way for ALL such polygons as mentioned, and *always* draw lines from N, through the 1st division on the arc, on each side of L M.

PROBLEM 57.—Line A B is the distance from an angle of a Regular Pentagon, to the middle point of the opposite side. Construct the Pentagon. (GENERAL METHOD *for* ALL *regular polygons with an odd number of sides.*)



1.—Through A and B, draw C D, E F, perpendicular to A B. With centre A, and *any* radius, describe a semi-circle, and divide it into as many equal parts as the polygon is to have sides, *i.e.*, 5.

2.—From A, draw A G, A H, through the first division of the semi-circle, on each side of A B. Then G H is one side of the polygon. Bisect A G, A H, at J and K. From J and K, draw perpendiculars cutting at L, which is the centre of the pentagon. From A, through 1 and 4, draw lines of indefinite length.

3.—With centre L, and radius L A, describe a circle, which gives points M and N. Join M G, N H. Then G H N A M is the required PENTAGON.

Note.—Proceed in the same manner for ALL regular polygons, with an odd number of sides. *Always* draw the lines A G, A H, through the 1st divisions on the semi-circle, on each side of A B. Where the circle cuts the radials, drawn from A, we get the positions of the angles.

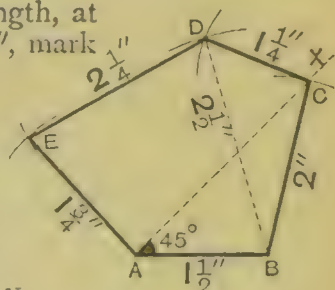
PROBLEM 58.—On a line A B, $1\frac{1}{2}$ inches long, construct an Irregular Polygon of 5 sides. Let B C be 2 in., C D $1\frac{1}{4}$ in., D E $2\frac{1}{4}$ in., and E A $1\frac{3}{4}$ in. One diagonal A C makes an angle of 45° with A B. Another diagonal B D is $2\frac{1}{2}$ in. (SCALE, FULL-SIZE.)

N.B.—*When required to construct an irregular figure, from given dimensions,* MAKE A ROUGH SKETCH FIRST, *so as to understand the order in which the lines are to be drawn.*

1.—Draw **A B** = $1\frac{1}{2}''$. Draw **A X,** of indefinite length, at 45° with **A B.** With centre **B,** and a radius of $2''$, mark point **C.** Join **B C.**

2.—With centre **B,** and a radius of $2\frac{1}{2}''$; and with centre **C,** and a radius of $1\frac{1}{4}''$, describe arcs intersecting at **D.** Join **C D.**

3.—With centre **D,** and a radius of $2\frac{1}{4}''$; and with centre **A,** and a radius of $1\frac{3}{4}''$, describe arcs intersecting at **E.** Join **D E** and **E A.** Then **A B C D E** is the required IRREGULAR POLYGON.



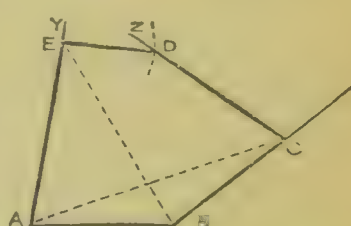
Note.—Observe that we commence by constructing the *triangles* A B C, B C D.

PROBLEM 59.—Construct an Irregular Pentagon A B C D E, on a base A B, 2″ long. B C is inclined to A B at an angle of 140°, and the diagonal A C makes an angle of 20° with A B. Line A E is at 80° with A B, and diagonal B E is at 60° with A B. Line C D makes an angle of 75° with B C. Line E D is $1\frac{1}{4}''$ long. (SCALE, FULL-SIZE.)

1.—Draw **A B** = 2″. **B C** is inclined to **A B** at 140°, so make angle **A B C** = 140°. **A C** is inclined to **A B** at 20°, so make angle **B A C** = 20°.

2.—**A E** is inclined to **A B** at 80°, so make angle **B A Y** = 80°. **B E** is inclined to **A B** at 60°, so make angle **A B E** = 60°.

3.—**C D** is inclined to **B C** at 75°, so make angle **B C Z** = 75°. **E D** = $1\frac{1}{4}''$, so with centre **E,** and a radius of $1\frac{1}{4}''$, mark point **D,** on **C Z.** Join **E D.** Then **A B C D E** is the required IRREGULAR PENTAGON.



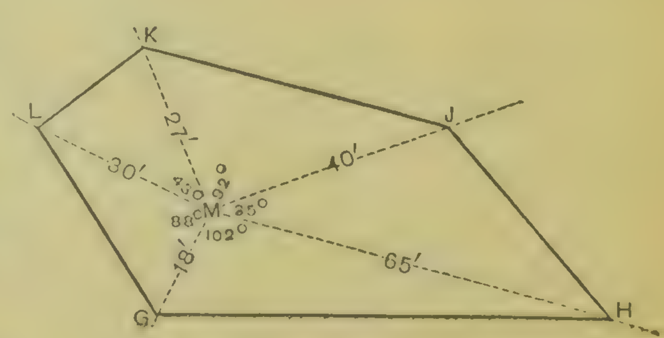
Note.—Observe that we commence by constructing the *triangles* A B C, A B E.

PROBLEM 60.—Construct the Irregular Polygon, G H J K L, from the dimensions given on the accompanying diagram. (SCALE 1 INCH TO 30 FEET.)

1.—Construct a scale of 1 inch to 30 feet. (*See scale* O, *Fig.* 73.)

2.—Draw the line **M K** = 27′, by *scale.*

3.—From **M,** draw the radiating lines of *indefinite* length, and making angles at **M,** of 92°, 35°, 102°, 88°, and 43°.



4.—From **M,** set off, on the

radiating lines, **M L** = 30′, **M G** = 18′, **M H** = 65′, and **M J** = 40′. Join the points obtained. Then **G H J K L** is the required IRREGULAR POLYGON. *Mark on the drawing the length of each* SIDE *of the polygon.*

FIG. 110.

FIG. 111.

FIG. 112.

Note (a).—The angles at M, added together, equal 4 right angles, or 360°. (*See* § 49.)

(b).—The problem can be stated under different conditions. Thus, we can give the lengths of *all* the lines radiating from an *angle* N (Fig. 110); or from a point O (Fig. 111), in a *side;* or from a point P (Fig. 112), *outside* the figure, the angles made at the point, in each case, being given.

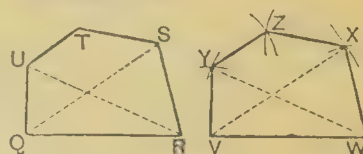
PROBLEM 61.—**Construct a figure equal and similar to the Irregular Polygon Q R S T U.** (TWO SHAPES, A and B, ARE GIVEN.)

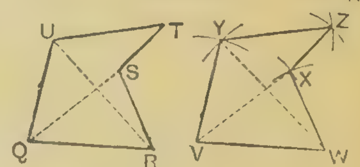
1.—Draw **V W** = **Q R.** With centres **V** and **W,** and distances **Q S** and **R S,** describe arcs cutting at **X.** Join **W X.**

2.—With centres **V** and **W,** and distances **Q U** and **R U,** describe arcs cutting at **Y.** Join **V Y.**

3.—With centres **Y** and **X,** and distances **U T, S T,** describe arcs cutting at **Z.** Join **Y Z, Z X.** Then **V W X Z Y** is the required IRREGULAR POLYGON. (*The foregoing explanation applies to each shape, A and B.*)

SHAPE A.





SHAPE B.

Note (a).—We construct upon V W, as a base, a series of *triangles, equal and similar* to those on Q R. Adopt a similar method for ANY arrangement of *unconnected*, or *intersecting* lines. When regular or irregular figures, of ANY number of sides, *are exactly alike in shape and size,* they are said to be EQUAL AND SIMILAR.


FIG. 113.

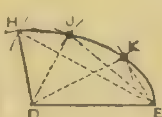


FIG. 114.

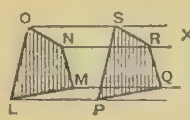


FIG. 115.

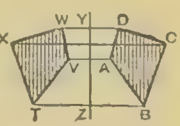


FIG. 116.

(b).—To copy a figure having a *curve*, as A B C (Fig. 113):—Mark *any* number of points F, G, on the curve. Make D E = A B. On D E (Fig. 114) as base, construct triangles, *equal and similar* to those on A B.——Fig. 115 shews how a diagram, L M N O, can be copied, the same size, by another method. The *parallels*, O S, N R, M Q, L P, are equal in length.——Fig. 116 shews how a figure T V W X. may be copied, *reversed* in position. Draw Y Z. Make the *perpendicular* distances from Y Z to A, B, C, D = those from Y Z to V, T, X, W.

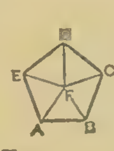
General Notes about Polygons.

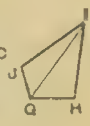
130.—The *special* methods for constructing regular polygons on lines, or in circles, should not be adopted, except when they are simpler in construction than the *general* methods.

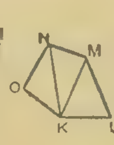
131.—In any regular polygon, each of the longest diagonals divides the figure into 2 equal and similar portions, as line M N (Problem 49); A B, E F, &c. (Problem 50); and K L, M N (Problem 51).

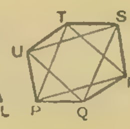
132.—The *vertical angle*, of a polygon having an odd number of sides, is the one most distant from the base, as angle D (Fig. 117).

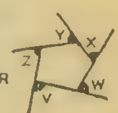
133.—If we draw lines from the centre to the angles of *any* REGULAR POLYGON, *the angles at the* CENTRE *are all equal,* and we divide the polygon into as many *equal isosceles triangles,* as there are sides to the polygon. Thus, in Fig. 117, the











FIGS. 117. 118. 119. 120. 121.

pentagon is divided into five equal isosceles triangles. All the angles at the centre are together equal to 360° (*see* § 19). To find the magnitude of such an angle, divide 360° by the same number as the polygon has sides, which in the *pentagon* gives 72° (360÷5=72). In a *hexagon* the angle at the centre = 60° ; in a *heptagon*, $51\frac{3}{7}$° ; in an *octagon*, 45° ; &c. Each base angle of each isosceles triangle is half the angle of the polygon. Thus, F A B is half E A B.

134. The *interior* angles of *any* rectilineal figure—REGULAR OR IRREGULAR—are together equal to twice as many right angles as there are triangles, made by diagonals drawn from any *one* of the angles of the figure. Thus, in the *quadrilateral*, G H I J (Fig. 118), the diagonal G I divides it into 2 triangles, and as each triangle contains 2 right angles (*i.e.*, 180°). G H I J must contain 4 right angles (*i.e.*, 360°). The *irregular pentagon*, K L M N O (Fig. 119), is divided by diagonals, drawn from *any one* angle, into 3 triangles, and as each triangle contains 2 right angles, the pentagon must contain 6 right angles (*i.e.*, 540°). For every *side* added to a rectilineal figure, we have an increase of TWO RIGHT ANGLES (180°). Thus, a TRIANGLE contains 2 right angles; a QUADRILATERAL, 4; a PENTAGON, 6; a HEXAGON, 8; &c. (*See Euclid* I. 32, *Cor.* 1.)

135.—To ascertain the *number of degrees* contained in an angle of a REGULAR POLYGON—say, a PENTAGON :—The preceding section shews that in a pentagon (*regular or irregular*) the interior angles together equal 6 right angles. Reduce the 6 right angles to degrees. Thus, 90° × 6 = 540°, which, divided by 5 (the *number of sides*), gives 108°, the size of each angle of a pentagon. A HEXAGON contains 8 right angles ; and 90° × 8 = 720°, which, divided by 6 (the number of sides), gives 120°, the size of each angle. An angle of a HEPTAGON = $128\frac{4}{7}$° ;—an OCTAGON = 135° ;—a NONAGON = 140° ;—a DECAGON = 144°; &c.

136.—*Any* rectilineal figure, having an *even* number of *equal* sides, has its opposite sides parallel, its opposite angles equal, and its opposite diagonals equal and parallel (Fig. 120).

137.—The *exterior* angles of *any* rectilineal figure are together equal to 4 right angles. Thus, in Fig. 121, the angles V, W, X, Y, Z, together equal 360°. (*Euclid* I. 32, *Cor.* 2.)

138.—A knowledge of the construction of polygons is of great service to Designers, Architects, Engineers, &c. Many ornamental and constructive forms are based upon polygonal figures, as in Gothic tracery, the plans of columns, capitals, &c. In nature, we find the shapes of many flowers are based upon polygonal forms—the dog-rose and convolvulus, upon the pentagon—the wood anemone and the narcissus, upon the hexagon.

Work out TEST-PAPER B, and the Exercises on POLYGONS.

CHAPTER VI. CIRCLES AND TANGENTS.

CLASS-SHEETS 7, 8, AND 9.

DEFINITIONS, etc :—139.—See the definitions, &c., on the **circle**, given in sections **38, 39, 40, 41, 42, 48, 49, 50, 51, 81, 128, 129.** See also PROBLEM **47.**

140.—Strictly speaking, the word CIRCLE means the *entire surface* of the figure, and the *boundary line* is the *circumference*. The word circle, however, is frequently intended to mean the circumference. A circle may be indicated by the letter at the centre. Thus, we say the " circle D " (Fig. 122).

141.—A Tangent to a CIRCLE is a straight line, A B, of *indefinite* length (Fig. 122), drawn outside the circle, and touching the circumference in only *one* point, C, which is called the POINT OF CONTACT. The radius D C, drawn to the point of

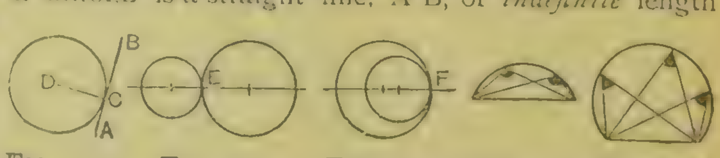


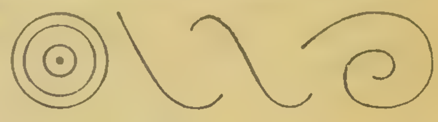
FIG. 122. FIG. 123. FIG. 124. FIG. 125. FIG. 126.

contact, is *perpendicular* to the tangent. Whenever the word "tangent" is used by itself in this book, a "*straight*" or "*right tangent*" is intended. (*Euclid* III. *Def.* 2.)

142.—Circles are said to *touch* each other, when they meet in *one* point only, as at E, or F (Figs. 123, 124), and **the straight line drawn through their centres passes through the point of contact,** as at E or F. When arcs or circles *touch*, they are also said to be *tangential* to each other. (*Euclid* III. 11, 12.)

143.—All the angles in the same SEGMENT of a circle are *equal* to one another. In § 81, we see that the angles in a *semi-circle* are *right angles*. In a segment *less* than a semi-circle, the angles are *equal obtuse angles* (Fig. 125); in one *greater* than a semi-circle (Fig. 126), the angles are *equal acute angles*. (*Euclid* III. 21.)

144.—Two or more circles, having the *same* centre, are called CONCENTRIC circles. (Fig. 127.)



FIGS. 127. 128. 129. 130.

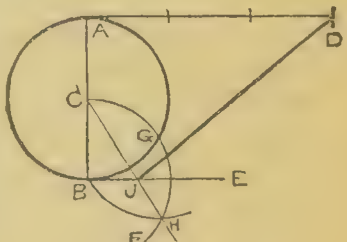
145.—A Curved Line is one that is bent in a regular manner, and no portion of it is straight. In Fig. 128, we have a SIMPLE CURVE; in Fig. 129, a DOUBLE, or COMPOUND CURVE, also called an OGEE; in Fig. 130, a SPIRAL, or VOLUTE. Curves may, or may not, be formed by combinations of arcs of circles.

As a general rule, curved lines—simple, compound, or spiral—used in *ornamental design*, should not be composed of arcs of circles. Such lines are not so beautiful in form as those having a variety in their bend or flexure. The curves shewn in Figs. 128, 129, 130, could not be produced with arcs of circles, and they are far more ornamental in character than those constructed with arcs in Problems 76, 83, 84: the latter lines have too much *sameness* in the quality of the curve. The grace and refinement seen in the decorative work of ancient Greece, are greatly due to the fact that the curves used for the ornament, and the contours employed for mouldings, vases, &c., are curves produced by means of ellipses and other conic sections, which cannot be obtained by any combination of circular arcs.

146.—Curvilineal or **Curvilinear Figures** are those bounded by curved lines.

(A) THE CIRCLE.

PROBLEM 62.—To ascertain, approximately, the length of the circumference of a given circle.



1.—Draw a diameter **A B**. Find centre **C**. Draw **A D** perpendicular to **A B**, and 3 times the length of the radius. Draw **B E** perpendicular to **A B**.

2.—With a set-square, or by Problem 17, make angle **B C H** = 30°. Mark point **J**, on **B E**.

3.—Join **J D**. Then line **J D** is (*approximately*) equal to *half* the circumference; and twice **J D** = the whole circumference.

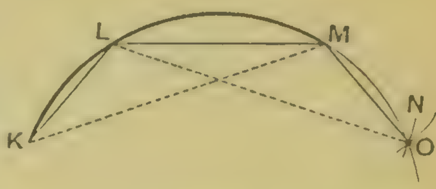
Note (a).—The above method is sufficiently accurate for all practical work, because the result is wrong—only by about the $\frac{1}{100000}$ part. This problem enables us to ascertain, approximately, the length of certain portions of the circumference. Thus $\frac{1}{3}$ of J D = $\frac{1}{6}$ of the circumference. Archimedes demonstrated that the diameter is to the circumference—within a minute fraction—as 7 is to 22, or 1 to $3\frac{1}{7}$. Thus, for all practical purposes, we may say that if the *diameter* = 1″, the *circumference* = $3\frac{1}{7}$″.

(b).—To describe a circle, having a circumference equal to the circumferences of any number of given equal or unequal circles:—Draw a line equal to the sum of the diameters of the given circles. This line is the diameter of the required circle.

PROBLEM 63. K L M is a given arc. Find one or more points, situated in the continuation of the arc, when the centre is inaccessible.

1.—Draw *any* two adjacent *chords*, **K L**, **L M**.

2.—With centre **L**, and radius **K M**, strike arc **N**. With centre **M**, and radius **K L**, cut arc **N** at **O**. Then **O** is a POINT in the continuation of the arc.



Note (a).—In the same way, *any* number of such points may be obtained. K L M, L M O, are *equal and similar triangles*, but reversed in position. This problem is of use in setting out large arcs on land, and is based upon § 143. If we join K O, we have K L O, K M O, *equal angles in the segment* K L M O.

(b).—The problem might be stated thus:—K, L, M, are 3 points in the circumference of a circle. Find other points in the circumference, without using the centre.

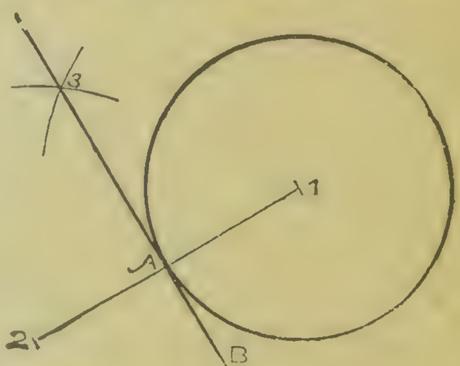
(B) CIRCLES AND TANGENTS.

PROBLEM 64.—Through a given point A, in the circumference, or arc, of a given circle, to draw a Tangent.

1.—From the centre **1**, through **A**, draw **1 2**, making **A 2 = A 1**.

2.—With centres **1** and **2**, and *any* radius, describe arcs cutting at **3**.

3.—Through **A**, draw **3 B**, which is the required TANGENT, touching **A**, the POINT OF CONTACT.



Note (a).—3 B is *perpendicular* to A 1. Only *one* tangent can be drawn through a given point in the circumference.

(b).—A tangent 3 B being given, to find the point of contact:—Draw 1 A perpendicular to 3 B. The position of the centre 1 being given, describe a circle to touch a given line 3 B:—Draw 1 A perpendicular to 3 B; &c.

PROBLEM 65.—Through any given point B, in the arc of a circle, A B C, to draw a Tangent, when the centre of the arc cannot be obtained.

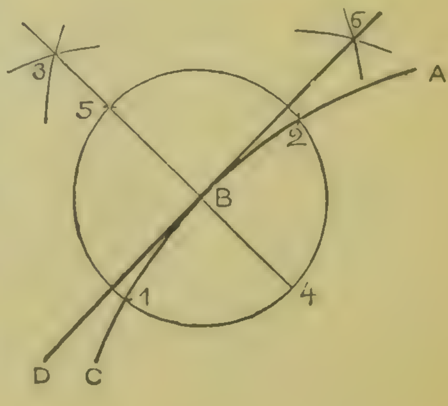
1.—With centre **B**, and *any* radius, describe a circle, cutting the arc at **1** and **2**.

2.—With centres **1** and **2**, and *any* radius, describe arcs intersecting at **3**.

3.—From **3**, through **B**, draw the line **3 4**, cutting the small circle at **4** and **5**.

4.—With centres **4** and **5**, and *any* radius, describe arcs cutting at **6**.

5.—From **6**, through **B**, draw the line **6 D**, which is the required TANGENT.



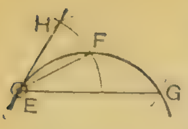
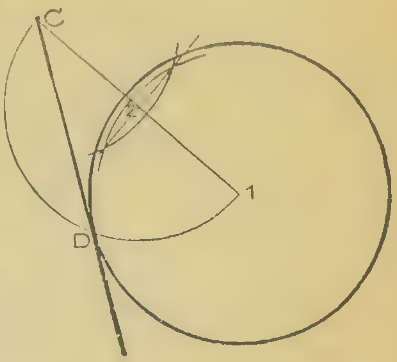


FIG. 131.

Note (a).—B 4 is a portion of the radius of the arc. The problem may be worked by joining 1 2, and drawing D 6, through B, *parallel* to the *chord* 1 2.

(b).—When the given point, E (Fig. 131), **is at, or near the end of the arc,** mark any two equal distances, E F, F G. Join E F, E G. Make angle F E H = angle F E G. Then E H is the required TANGENT.

PROBLEM 66.—To draw a Tangent to a given circle from any given point C, outside the circle.



1.—Point **1** is the centre of the given circle. Join **C 1.**

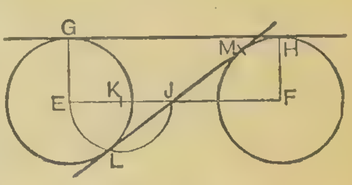
2.—Bisect **C 1** at **2**; and with centre **2,** and radius **2 C** or **2 1,** describe a semi-circle, cutting the circle at **D.** Point **D** is the *point of contact.*

3.—Through **D,** draw **C D,** which is the required TANGENT.

Note (a).—If we join D 1, the angle C D 1 is a *right angle.* (*See* § 141.)

(b).—Another tangent can be drawn from C, on the other side of the circle. The distance from C to the point of contact of each tangent is the same.

PROBLEM 67.—To draw, 1st, an Exterior Tangent; and 2nd, an Interior Tangent, to two given equal circles, E and F.

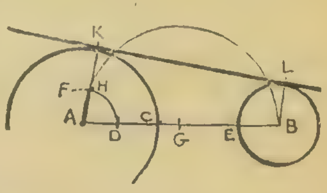


1.—Join the centres, **E, F.** Draw **E G, F H,** perpendicular to **E F.** Through the *points of contact,* **G, H,** draw the EXTERIOR TANGENT **G H.**

2.—Bisect **E F** at **J,** and **E J** at **K.** With centre **K,** and radius **K E,** or **K J,** describe a semi-circle. With centre **J,** and radius **J L,** mark point **M,** on the circle **F.** Through the *points of contact,* **L** and **M,** draw the INTERIOR TANGENT **L M.**

Note.—From J, by Problem 66, we have drawn a tangent to circle E. The tangent through the middle point J, is a tangent common to both circles. An EXTERIOR tangent to *any* 2 equal or unequal circles, is one drawn wholly on one side of the line joining the centres; an INTERIOR tangent is one that *crosses* that line.

PROBLEM 68.—To draw an Exterior Tangent to two unequal circles, A and B.



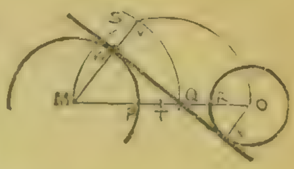
1.—Join the centres **A** and **B.** Bisect **A B** at **G.** With centre **G,** and radius **G A,** describe a semi-circle.

2.—From **C,** on the *larger* circle, mark off **C D** = **B E,** the radius of the *smaller* circle.

3.—With centre **A,** and radius **A D,** describe arc **D F,** cutting the semi-circle at **H.** Through **H** draw **A K.** Draw **B L** parallel to **A K.** Through the *points of contact,* **K, L,** draw the EXTERIOR TANGENT **K L.**

Note.—The unequal circles may *intersect,* or *touch* each other. In the *latter* case, the semi-circle on A B *touches* the tangent.

PROBLEM 69. To draw an Interior Tangent to two unequal circles, M and O.



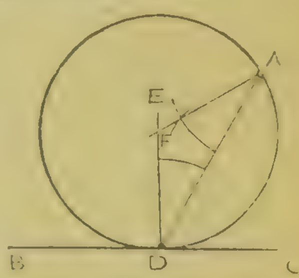
1.—Join the centres **M, O.** Bisect **M O** at **T**, and describe a semi-circle on **M O.**

2.—From **P**, on the *larger* circle, mark off **P Q = O R**, the radius of the *smaller* circle.

3.—With centre **M**, and radius **M Q**, describe arc **Q S**, cutting the semi-circle at **V.** Join **M V**, and mark point **W.** Draw **O X** parallel to **M V.** Through the *points of contact*, **W**, and **X**, draw the INTERIOR TANGENT **W X.**

Note.—For *exterior* tangents, the radius of arc D F (Problem 68) = the *difference* of the radii of the circles. For *interior* tangents, the radius of arc Q S = the *sum* of the radii. ——If two *equal* or *unequal* circles are *apart* from each other, *two* exterior and *two* interior tangents can be drawn. If they *intersect*, an *interior* tangent cannot be obtained. If they *touch*, there can only be *one interior tangent*, which passes through the point of contact of the circles, and is perpendicular to the line joining the centres.

PROBLEM 70.—Describe a Circle to pass through a given point A, and to touch a line B C, at a given point D.



1.—At **D**, draw **D E** perpendicular to **B C.**

2.—Join **A D.** Make the angle **D A F** equal to the angle **A D E.** Mark point **F.**

3.—With centre **F**, and radius **F D** or **F A**, describe the required CIRCLE, which passes through point **A**, and touches **B C** at point **D.**

Note (a).—F A = F D, because D F A is an *isosceles* triangle. When a circle touches a straight line at a given point, the centre is in a perpendicular drawn from the given point. The *bisector* of A D gives point F on line D E.

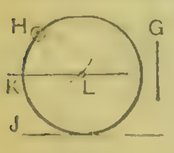
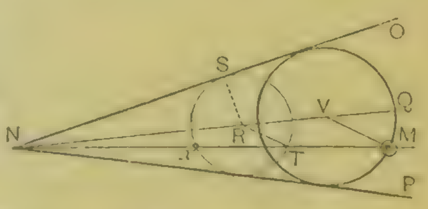


FIG. 132.

(b).—Describe a circle of a given radius, **G** (Fig. 132), to pass through point **H**, and touch line **J** : Draw **K** parallel to **J**, at the distance **G** from it. With centre **H**, and radius **G**, obtain point **L.** With centre **L**, and radius **G**, or **L H**, draw the required CIRCLE.

PROBLEM 71. Describe a Circle to touch two given converging lines, N O, N P, and to pass through a given point M, between them.



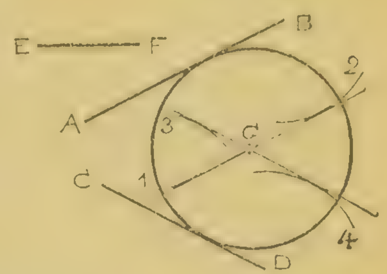
1.—Draw **N Q**, the bisector of the angle **O N P.** From *any* convenient point **R**, in **N Q**, draw **R S**, perpendicular to **N O**, or **N P.** With centre **R**, and radius **R S**, describe a circle, which touches **N O** and **N P.**

2.—Draw **N M**, cutting circle **R** at **T.** Join **R T.** Draw **M V** *parallel* to **T R.** With centre **V**, and radius **V M**, describe the required CIRCLE.

Note (a).—Another circle can be drawn to pass through M, by joining x R, and drawing from M, a line parallel to x R, which gives the centre of the required circle, on N Q.

(b).—Describe an arc, to touch 2 given converging lines, **O N, P N**, and one of them at a given point **S** :—Draw the bisector **N Q.** Draw **S R** perpendicular to **O N.** With centre **R**, and radius **R S**, describe the ARC, which touches **P N** at a point, the same distance from **N**, as **S.**

PROBLEM 72.—To describe a Circle, or Arc, of a given radius, E F, which shall touch any two given converging lines, A B, C D.

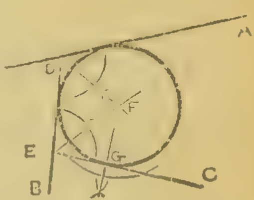


1.—With *any* two points on **A B**, as centres, and with radius **E F**, describe arcs **1** and **2**. With *any* two points on **C D**, as centres, and *same* radius, obtain arcs **3** and **4**.

2.—Draw lines *tangential* to each *pair* of arcs, intersecting at **G**. With centre **G**, and radius **E F**, describe the required CIRCLE or ARC.

Note.—If A B, C D, *meet*, bisect the angle, and draw only *one* of the lines 1 2, or 3 4, to cut the bisector at a point, which is the centre of the circle.

PROBLEM 73.—To describe a Circle to touch three given lines, A, B, and C.



1.—Produce lines **B** and **C**, so as to form angles at **D** and **E**. Bisect angles **A D B, C E D**, by lines cutting at **F**.

2.—From **F**, draw a perpendicular to *either* of the given lines, as at **F G**. With centre **F**, and radius **F G**, describe the CIRCLE.

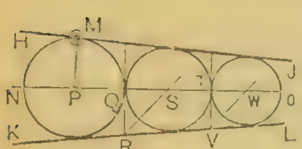


FIG. 133.

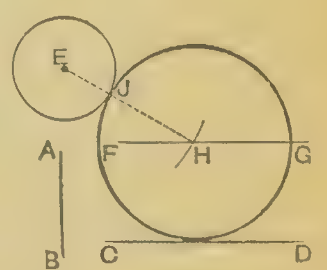
Note (a).—Lines A, B, C, are *tangents* to the circle. If A and C are produced to the left, another circle can be described to touch the 3 lines, on the left of D. If B makes *equal angles* with A and C, the 2 circles will *touch* each other.

(b).—To draw a series of Circles, touching each other, successively, and 2 given converging lines, H J, K L (Fig. 133), one of them at a given point M:—Draw the bisector **N O**, and **M P** perpendicular to it. With centre **P**, and radius **P M**, describe the circle. At **Q**, draw **Q R** perpendicular to **N O**. Bisect angle **Q R L** by **R S**. With centre **S**, and radius **S Q**, describe the circle. In the same way, get **T V** and **W**; &c. If **H J, K L**, are produced both ways, an *infinite* number of circles can be drawn on each side of circle **P**.

(C) CIRCLES TOUCHING CIRCLES AND STRAIGHT LINES.

PROBLEM 74. To describe a Circle, with a given convenient radius A B, to touch a given line C D, and a given circle E.

1.—Draw **F G** parallel to **C D**, and the distance **A B** from it.



2.—With centre **E**, and a distance equal to the *sum* of the radii of both circles, cut **F G** at **H**. With centre **H**, and radius **A B**, draw the required CIRCLE.

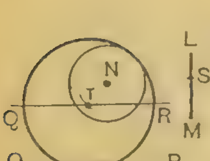


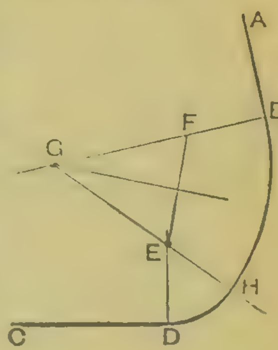
FIG. 134.

Note (a).—J is a point of contact. A perpendicular to C D, from H, gives the other point of contact. Line C D may *touch*, or *cut*, circle E. (*See* § 142.)

(b). When the required Circle, of a given convenient radius, L M, is to touch and enclose the given circle N, and touch O P draw **Q R** (Fig. 134) parallel to **O P**, the distance **L M** from it. Make **M S** = radius of **N**. With centre **N**, and radius equal to **L S**—the

difference between the 2 radii—cut Q R at T. With centre T, and radius L M, describe the CIRCLE. If O P *touches* circle N, then centres N and T are in the perpendicular drawn from the point of contact.

PROBLEM 75. Join the ends of two given lines, A B, C D, with a Simple Curve, made by two arcs touching each other and the given lines—say connect points B and D.



1.—At **B** and **D**, draw perpendiculars of indefinite length. Mark off two *equal* distances, **B F**, **D E**, of *any* CONVENIENT *length*.

2.—Join **E F**. Bisect **E F** by a perpendicular, meeting the perpendicular drawn from **B**, at **G**.

3.—From **G**, through **E**, draw a line of indefinite length.

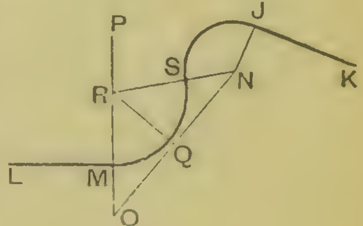
4.—With centre **G**, and radius **G B**, strike arc **B H**. With centre **E**, and radius **E D** or **E H**, strike arc **D H**. Then **D H B** is the required SIMPLE CURVE. (*See* § 142.)

Note (a).—Another pair of such arcs can be described to connect points A and C. If the *equal distances* D E, B F, are not made of *convenient* length, the problem is unworkable.

(b).—The given lines may CONVERGE, or DIVERGE, at *any* inclination, or they may be PARALLEL. According to the position of the lines, and the length of the equal distances, B F, D E, point G may come sometimes on the perpendicular drawn from B, and sometimes on that drawn from D: it may also fall on either side of line E F. The triangle E F G is always *isosceles*. If a line, joining the ends of the given lines, makes *equal angles* with them, *two* such arcs cannot be obtained—only one connecting arc can be described. (*See note* b, *Problem* 71.)

(c).—*It is* MOST IMPORTANT *to observe that when an arc* TOUCHES *a straight line*, **the centre of the arc is in the perpendicular which is drawn to that straight line, at the point of contact.** *Thus, points E and G are in perpendiculars drawn from D and B.*

PROBLEM 76.—To join the extremities, J and M, of two given lines, J K, L M, with a Compound Curve, or Ogee, made by two arcs touching each other.



1.—At **J** and **M**, draw *equal* perpendiculars, **J N**, **M O**, of *any* CONVENIENT *length*. Produce **O M**, indefinitely, to **P**.

2.—Join **O N**. Bisect **O N**, by a perpendicular meeting **O P** at **R**. Join **R N**.

3.—With centre **N**, and radius **N J**, strike arc **J S**. With centre **R**, and radius **R S** or **R M**, strike arc **M S**. Then **M S J** is the required COMPOUND CURVE.

FIG. 135.

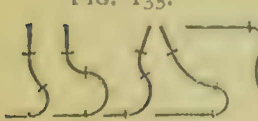
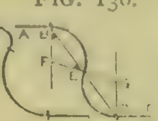


FIG. 136.



Note (a).—O N R is an *isosceles* triangle. The given lines may be inclined at *any* angle, or they may be *parallel*. For practice, connect the ends of such pairs of lines as are shewn at H, I, J, K, L (Fig. 135).

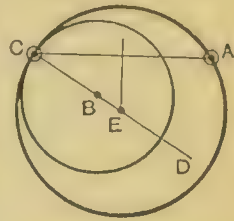
(b).—To connect the ends of 2 *parallel* lines, A B C D, by a pair of *equal* arcs (Fig. 136):—At B and C, draw perpendiculars of indefinite length. Join B C. Bisect B C at E. Make angle B E F=E B F. Produce F E to G. With centres F and G, and radius F B or G C, describe arcs *touching* at E.

(D) CIRCLES IN CONTACT.

PROBLEM 77.—To describe a Circle, passing through a given point A, enclosing a given circle B, and touching it at a given point C.

1.—From **C**, through **B**, draw **C D** of indefinite length.

2.—Join **A C**. Bisect **A C** by a perpendicular meeting **C D** at **E**. With centre **E**, and radius **E C** or **E A**, describe the required CIRCLE.



Note (a).—If we join E A, then C E A is an *isosceles* triangle.

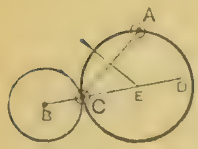


FIG. 137.

FIG. 138.

(b).—Fig. 137 shews how to proceed, when the required circle E is to be *outside* the given circle B. Fig. 138 shews the working lines, when the given pt. A falls *inside* the given circle B. The explanation given to the above problem is sufficient for Figs. 137, 138.

(c).—If the given pts. are *in one straight line with the centre of the given circle*, the bisection of the distance between the given pts. is the centre of the required circle.

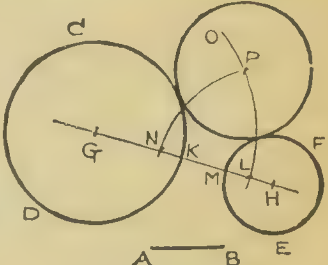
PROBLEM 78.—Describe a Circle of a given convenient radius A B, to touch two given circles, C D, E F, externally.

1.—Draw a line of *indefinite* length through the centres **G** and **H**.

2.—Make **K L** and **M N** each equal to **A B**.

3.—With centre **G**, and radius **G L**, describe the arc **L O**.

4.—With centre **H**, and radius **H N**, describe arc **N P**. With centre **P**, and radius **A B**, describe the required CIRCLE.



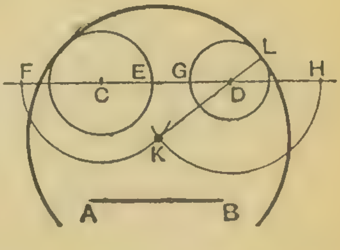
Note.—If we join G P, H P, we get the pts. of contact. The circle P may, if required, be drawn on the other side of the given circles. The given circles may be *equal* or *unequal*, and they may *touch* or *intersect*. If the diameter of the required circle equals the space between circles G and H, join G H, and the middle point of K M is the cent. of the required circle, which is the *smallest possible* circle that can be drawn to touch circles G and H.

PROBLEM 79.—Describe a Circle of a given convenient radius, A B, to enclose and touch two given circles, C and D.

1.—Through **C** and **D**, draw a line of indefinite length.

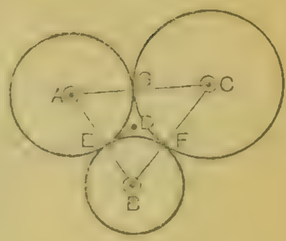
2.—Mark off **E F** and **G H**, each equal to **A B**.

3.—With centre **C**, and radius **C F**; and with centre **D**, and radius **D H**, describe arcs intersecting at **K**. Through **C** or **D**, draw **K L**. With centre **K**, and radius **K L** or **A B**, describe the required CIRCLE.



Note.—The large circle is said to touch the smaller ones *internally*; and the smaller circles touch the large one *externally*. The given circles may be *equal* or *unequal*, and they may *touch* or *intersect*. The principle of construction is the same for Probs. 78-79.

PROBLEM 80. To describe three Circles, touching one another, the positions of their centres, A, B, C, being given.



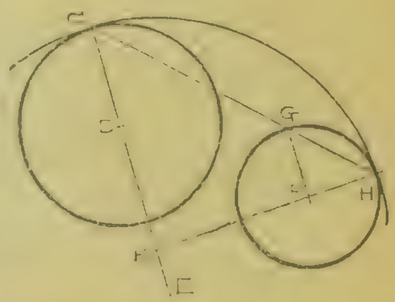
1.—Join **A B**, **B C**, **C A**. By Prob. 27, find **D**, the *centre* of the triangle **A B C**.

2.—From **D**, draw a perpendicular to *either* side of the triangle, as **D E**.

3.—With centre **A**, and radius **A E**; and with centre **B**, and radius **B E**, describe CIRCLES. With centre **C**, and radius **C F** or **C G**, describe the remaining CIRCLE.

Note.—If we join D F, D G, the triangle is divided into 3 *trapezions*, for A G = A E, B F = B E, C G = C F. If the cents. of the circles are given in a *straight line*, the problem is unworkable. Probs. 78, 79, 80, are of use when drawing wheels in gear.

PROBLEM 81. To describe an Arc, or Circle, to touch two given circles, D and F, internally, and one of them in a given convenient point C.



1.—From **C**, through centre **D**, draw **C E**, of indefinite length.

2.—From centre **F**, draw **F G**, *parallel* to **C E**. (Use a *set-square.*)

3.—From **C**, through **G**, draw **C H**; and from **H**, through **F**, draw **H K**.

4.—With centre **K**, and radius **K C**, describe the required ARC, or CIRCLE. (*See* § 142.)

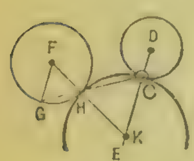


FIG. 139.

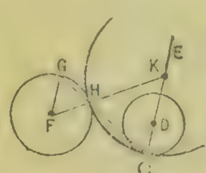
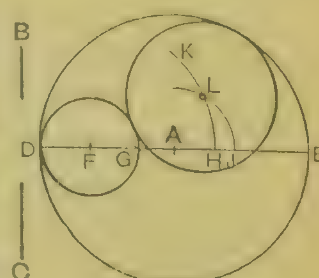


FIG. 140.

Note.— Fig. 139 shews the construction, when the arc is to touch the given circles *externally*; and Fig. 140, when the arc is to pass *between* the given circles. C and H are the points of contact.

The explanation given in Problem 81, is sufficient for Figs. 139, 140. Proceed on the same principle when one of the given circles is *within* the other. In some cases the given circles may *touch* or *intersect*. C H K, G H F, are *similar isosceles triangles*.

PROBLEM 82. Within a given circle A, to inscribe two Circles, having given convenient radii, B and C. The three circles are to touch one another.

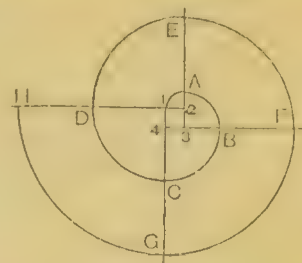


1.—Draw a diameter **D E**. Make **D F** equal **B** or **C**, say = **B**. With centre **F**, and radius **F D**, describe the CIRCLE.

2.—Make **G H** and **E J** = **C**. With centre **F**, and radius **F H**, strike arc **H K**. With centre **A**, and radius **A J**, strike arc **J L**. With centre **L**, and radius **C**, describe the CIRCLE.

Note.—If we join A L, and produce the line; and if we join F L, we obtain *points of contact*. The centre L may fall on the other side of D E. The inscribed circles may be *equal* or *unequal*. If the diameters of the 2 inscribed circles together equal D E, then obtain circle F as above, and the *middle point* of G E will be the centre of the other circle.

PROBLEM 83. To construct a Spiral, or Volute, by means of tangential arcs of circles.



1.—Construct a square **1 2 3 4**, and produce the sides, as shewn.

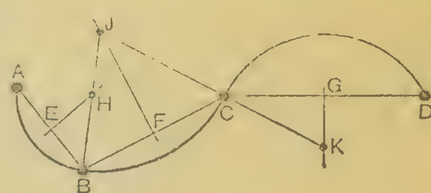
2.—With centre **2**, and radius **2 1**, strike arc **1 A**;—centre **3**, and radius **3 A**, strike arc **A B**;—centre **4**, and radius **4 B**, strike arc **B C**;—centre **1**, and radius **1 C**, get **C D**;—centre **2**, and radius **2 D**, get **D E**. In the same way get *any* no. of arcs, **E F, F G, G H,** &c. The curve obtained is a SPIRAL or VOLUTE.


FIG. 141.

Note (a).—1 2 3 4 is the *eye* of the spiral. The eye can be formed by *any* regular or irregular rectilineal figure, not having a re-entrant angle. In every case, proceed as above.

(b).—Fig. 141 shews a simple form of volute, in which the points of contact of semi-circles are given at equal distances, on one straight line. There are many intricate methods for constructing volutes, which cannot be given in this course.

PROBLEM 84. Through three or more conveniently arranged points, A, B, C, D, to draw a continuous Curve of tangential arcs of circles.



1.—Draw **A B, B C, C D.** *Bisect* these lines at **E, F, G.**

2.—Draw a perpendicular to **A B** from **E**, and on it mark *any convenient* point **H.** With centre **H**, and radius **H A**, strike arc **A B.**

3.—At **F**, draw a perpendicular to **B C.** Through **H**, draw **B J.** With centre **J**, and radius **J B**, strike arc **B C.**

4.—At **G**, draw a perpendicular to **C D.** Through **C**, draw **J K.** With centre **K**, and radius **K C**, strike arc **C D.** Then **A B C D** is the required CURVE.

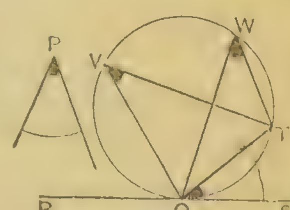

FIG. 142.


FIG. 143.

Note.—*Either* of the arcs may have *any convenient* centre selected, and its position then regulates the positions of the centres of the remaining arcs. Thus, we may, at the first, select any convenient point for the centre of an arc connecting B C, or C D, instead of A B; then proceed as shewn with the other arcs.——Figs. 142, 143, shew curves of different shapes, drawn through given points These methods are useful in *Mechanical* and *Architectural Drawing*, but, as a rule, are not suitable for *Ornamental Design*.

(E) ANGLES IN SEGMENTS.

PROBLEM 85. From a given Circle, to cut off a Segment containing angles equal to a given angle P.

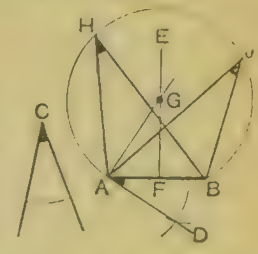


1.—Through any point **Q**, draw a tangent **R S.**

2.—Make angle **S Q T = P.** From *any* points, **V, W,** in the arc of the SEGMENT **Q V W T**, draw lines to **Q** and **T.** Then the ANGLES, **V, W,** are equal to **P.** (*See* § 143.)

Note.—The angles in the *smaller* segment, on the chord Q T, are equal to the angle R Q T. Thus, if *any* chord is drawn in a circle, the angles in the two segments together equal two right angles. If the given angle is a *right angle*, Q T becomes a diameter. (*See* § 81).

PROBLEM 86. Upon a given line A B, describe a Segment of a Circle, to contain angles equal to a given angle C.



1.—Make angle **B A D = C.** Bisect **A B** by **E F.**
2.—Draw **AG** *perpendicular* to **A D.**
3.—With centre **G,** and radius **GA,** describe the arc **A H J B** From *any* points, **H, J,** in this arc, draw lines to **A** and **B.** Then the SEGMENT **A H J B** contains ANGLES at **H** and **J** equal to **C.**

Note (a).—If the given angle is *obtuse*, the centre of the segment falls *below* A B, on the bisector E F. If the given angle is a *right angle*, the required segment is a *semi-circle* on A B.

(b).—By this problem, an *isosceles* or *scalene* triangle can be constructed on A B, as a base, and having a given vertical angle = C.

General Notes about Circles.

147.—A chord is said to *subtend*—or be opposite to—the arc to which it belongs.——Equal chords subtend equal arcs in the same circle, and *vice versa.*——A diameter bisects all chords perpendicular to it.——Chords equidistant from the centre are equal.——The shortest chord passing through any given point in a circle, is that which is perpendicular to the diameter which passes through the same point.——Chords, drawn at equal angles from a point in the circumference, cut off equal arcs in the circumference, as in Problems 52 (*see note* b), 55, 57.——The line joining the centres of two intersecting circles, is perpendicular to their common chord, and bisects it. (*See* Fig. 107.)

148.—Figs. 144 to 153 shew how various combinations of arcs, or arcs and straight lines,

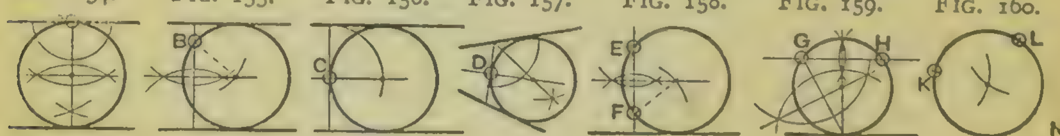
FIGS. 144. 145. 146. 147. 148. 149. 150. 151. 152. 153.

can be made out of the problems given in this chapter. Thus, Fig. 144 is from Problem 64;—145, from Problem 67;——146, from Problem 68;——147, from Problem 69;——148, from Problem 73;——149, from Fig. 134;——150, from Problem 77;——151, from Problem 78;——152, from Problem 81; and 153, from Problem 82.

149.—If the problems in this chapter be understood, many others, not given here, can be solved. The working lines may arrange themselves *differently*, although the principle of construction is the same. *The last remark applies to the whole range of geometrical constructions.* In endeavouring to solve a new problem, make a careful diagram of the figure as it should appear when completed. This will frequently suggest a solution—sometimes by *reverse* working.

150.—The construction lines explain the working of the following Problems. Describe a

FIG. 154. FIG. 155. FIG. 156. FIG. 157. FIG. 158. FIG. 159. FIG. 160.



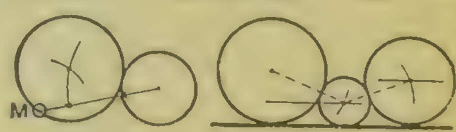


FIG. 161. FIG. 162.

CIRCLE:—1st. Touching 2 parallel lines, and passing through point A, in one of them (Fig. 154);——2nd. Touching 2 parallel lines, and passing through B (Fig. 155);——3rd. Touching 2 parallel lines, and passing through C, midway between them (Fig. 156);——4th. Touching 2 converging lines, and passing through D, midway between them (Fig. 157);——5th. Touching a line, and passing through points E, F, in a line perpendicular to the given line (Fig. 158);——6th. Touching a line, and passing through points G, H, in a line parallel to the given line (Fig. 159);——7th. Of a given radius = N, and passing through 2 given points K, L (Fig. 160);——8th. Of a given radius = N, passing through a given point M, and touching a given circle (Fig. 161);——9th. Draw 2 or more CIRCLES of given radii, touching each other successively, and a given line (Fig. 162).

Work out the Exercises on CIRCLES and TANGENTS.

CHAPTER VII. RATIO AND PROPORTION.

CLASS-SHEET 9.

DEFINITIONS, etc.: 151.—See PROBLEM **16**, and PROBLEM **34**.

152.—**Ratio** is the relationship which exists between two numbers, lines, or quantities. When we compare two numbers together, and say "as **3** is to **6**" (stated thus—**3 : 6**), we form a RATIO. We see that **6** is *double* as much as **3**; we therefore say that "**4** is to **8**" (**4 : 8**) is in the same RATIO as "**3** is to **6**" (**3 : 6**), because **8** is the *double* of **4**. Again, **3 : 9** is in the same ratio as **4 : 12**; etc. The order of the numbers may be reversed, and we can say **9 : 3** is the same ratio as **12 : 4**; or **12 : 4** is in the same ratio as **9 : 3**; etc. It is the same with regard to LINES. Thus, in Fig. 163, suppose line **A** = 1", **B** = 3", **C** = 2", **D** = 6"; then the lines **A, B,** have the same ratio as lines **C, D,** because **B** is

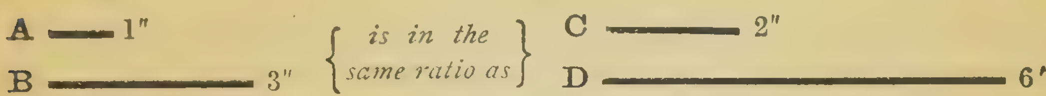


FIG. 163.

just as much longer, *proportionately*, than **A**, as **D** is longer than **C**;—that is, **A : B** equals **C : D**. Or, **1" : 3"** equals **2" : 6"**. (*Euclid* V. *Def.* 3.)

153.—In **Proportion,** we combine two *equal* ratios. Thus, we say "**12** is to **4** as **9** is to **3**" (written **12 : 4 :: 9 : 3**), because **12** is three times greater than **4**, and **9** is three times greater than **3**. It is the same with regard to LINES. Thus, in Fig. 164, suppose line **E** = 9", **F** = 3", **G** = 12", **H** = 4"; then the line **E** bears the same proportion to line **F**, that line **G** bears to line **H**;—that is, **E : F :: G : H** (or **9" : 3" :: 12" : 4"**). These four lines are said to be IN PROPORTION, or to FORM A PROPORTION. (*Euclid* V. *Defs.* 6, 9.)

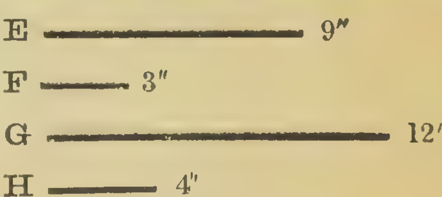


FIG. 164.

154.—The four numbers, or lines, in a PROPORTION, are called the **Terms** of the proportion, the first and last terms being named the **Extremes,** and the two middle terms, the **Means.** Thus, in the proportion **5 : 10 :: 7 : 14**, the 1st and 4th terms (**5** and **14**) are the EXTREMES, and the middle terms (**10** and **7**) are the MEANS. In Fig. 164, lines **E, H,** are the EXTREMES, and lines **F, G,** the MEANS. The 4th, or *last term*, is called the **Fourth-proportional.** (*Euclid* VI. 12.)

155.—If the two middle terms, or means, are *alike*, thus, **2 : 4 :: 4 : 8**, then the last term is called the **Third-proportional.** It is the same with regard to LINES. Thus, in Fig. 165, suppose **J** = 2", **K** = 4", **L** = 8"; then **J : K :: K : L**, and **L** is the THIRD-PROPORTIONAL. Or, we may reverse the order, and say **L : K :: K : J**; then **J** is the THIRD-PROPORTIONAL. Lines **J, K, L,** are said to be *in proportion*, since line **J** bears the same proportion to **K**, that **K** bears to **L**. Lines **J, L,** are the *extremes*; and line **K** is the **Mean-proportional.** (*Euclid* VI. 11, 13.)

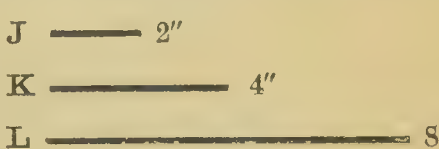


FIG. 165.

156.—In a proportion of numbers, if we multiply the two *extremes* together we get the same product as when we multiply the two *means* together. Thus, in the

proportion 2 : 4 :: 5 : 10, we find that $2 \times 10 = 20$, and $4 \times 5 = 20$. Or, in the proportion 3 : 6 :: 6 : 12, we see that $3 \times 12 = 36$, and $6 \times 6 = 36$. With respect to LINES : In Fig. 164, the *rectangle* contained by the two *extremes*, E and H, has the same amount of surface or AREA, as the *rectangle* contained by the *means*, F and G. Again, in Fig. 165, the *rectangle* contained by the *extremes*, J and L, has the same AREA, as the *square* constructed upon line K—the *mean-proportional*. (*Euclid* VI. 16.)

157.—A knowledge of proportion is necessary for the construction of SIMILAR FIGURES, for working problems in AREAS, and many diagrams in SCALE-DRAWING.

PROBLEM 87.—To divide the given line, P R, proportionally to the divisions on the given line S T. 1ST METHOD.

1.—Draw **S A** *equal* to **P R**, and at *any* angle with **S T.** Join **A T.**

2.—From the divisions on **S T**, draw **4 4**, **3 3**, **2 2**, and **1 1**, *parallel* to **A T.** (*Use a set-square.*)

3.—The divisions on **S A** bear the *same proportion* to one another, as the divisions on **S T** bear to one another.

4.—Transfer the divisions on **S A** to the line **P R.** Then **P R** is divided PROPORTIONALLY to the divisions on **S T.**

Note (a).—The divisions on SA have the same *ratio* to one another, as those on S T. Thus, should T 4 = $\frac{1}{9}$ of S T, then A 4 = $\frac{1}{9}$ of S A ; etc.

(b).—The triangles S 1 1, S 2 2, &c., are similar triangles (*see* § 80). If we draw *parallel* lines, at *irregular* distances from one another, and then draw lines in *any* direction across these parallels, as at D, E (Fig. 166), the divisions on D have the *same proportion* to one another, as those on line E ; and both lines, D and E, are divided proportionally to the spaces between the parallels. (*See note* b, *Prob.* 16.) On this principle, we can draw between any 2 parallel lines, F, G (Fig. 167), other lines parallel to them, and having spaces between them, in the same proportion to one another as the distances set off upon a given line H J. Draw H J, at *any* angle, having given points K, L. With centre H, and radius H J, strike arc J M. Join H M. With centre H, and radii H L, H K, get N, O. Through N, O, draw parallels with a set-square. By this method we can draw *any* number of parallels between F, G, at EQUAL distances, by setting off the required number of equal parts on a line H J.

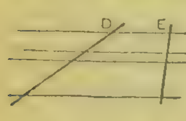


FIG. 166.

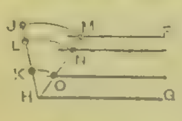


FIG. 167.

(c).—*When marking off a number of divisions from one line to another, set them off from* ONE END *of the line.* Thus, from P, make P 4 = S 4 (on line S A), P 3 = S 3 ; &c.

2ND METHOD :—**Divide the given line A B, proportionally to the divisions on the line C D** (FIG. 168) :—Draw **S T = C D**, and *parallel* to **A B**, and at *any* convenient distance from it. Mark off divisions **1, 2, 3, 4,** from **C D** on to **S T.** Join the ends of the lines **S T, A B,** by lines meeting at **R.** From **R,** draw lines through the divisions on **S T** to **A B.** Then line **A B** is divided PROPORTIONALLY to the divisions on S T, or C D. —Proceed in the same way, when the given divided line is the *longer* of the two. The 1st Method should, as a rule be used in preference to the 2nd Method.

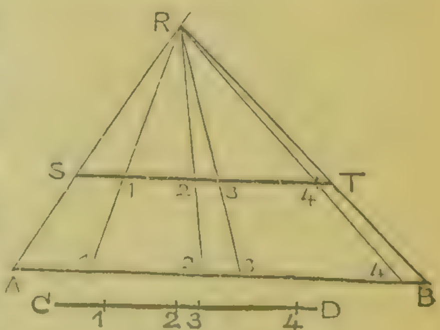
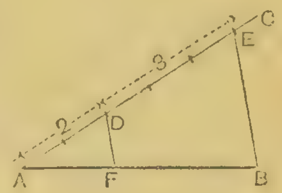


FIG. 168.

(d).—In PERSPECTIVE drawing, the application of the 2nd Method enables us to measure proportional distances, perspectively, on straight lines parallel to the picture plane.

PROBLEM 88.—To divide a given line into two or more parts, having a given proportion to one another. Say divide A B into two parts, having the ratio to one another of 2 : 3.



1.—Draw **A C**, at *any* angle. Mark off 2 parts, of *any* convenient length, from **A** to **D**; and 3 parts, of the *same* length, from **D** to **E**.

2.—Join **E B**. Draw **D F**, parallel to **E B**. Then **A F** is to **F B** as **2** is to **3** (written **A F : F B : : 2 : 3**).

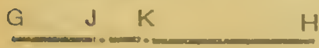
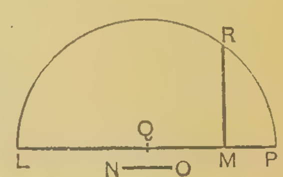


FIG. 169.

Note.—The solution is obtained by dividing A B into 5 equal parts, and making A F=2, and F B=3 of them. Proceed in the same way to divide a line into *any* number of parts, having a given proportion to one another. If required to divide a line G H (Fig. 169) into 3 parts, so that G J = twice J K, and K H = twice G J, then, by the above Prob., make G J = 2 parts, J K = 1 part, and K H = 4 parts. If required to divide A B in 2 parts, or segments, having the same ratio as two *given lines*, make A D, D E = the given lines; &c.

PROBLEM 89.—To find the Mean-Proportional between two given lines, L M, N O.



1.—Produce **L M** to **P**, making **M P = N O**.

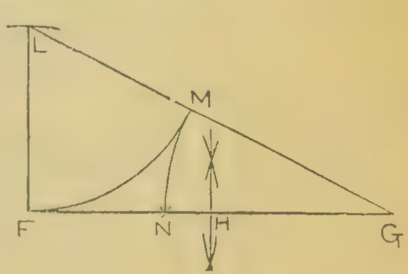
2.—Bisect **L P** at **Q**. With centre **Q**, and radius **Q L**, describe a semi-circle.

3.—At **M**, erect a *perpendicular*, to meet the semi-circle at **R**.

4.—Line **M R** is the required MEAN-PROPORTIONAL: that is, **L M** bears the same proportion to **M R**, as **M R** does to **M P**; or, **L M : M R : : M R : M P**.

Note.—If L M=8″, and M P=2″, then M R=4″, because 8″ : 4″ : : 4″ : 2″. (*See* §§ 155, 156.) L M, M P, are the *extremes*, and M R is the *mean*.

PROBLEM 90.—Divide the line F G in Extreme and Mean-Proportion, or Ratio.



1.—Bisect **F G** at **H**. At one end erect a *perpendicular* **F L = F H**.

2.—Join **L G**. With centre **L**, and radius **L F**, describe the arc **F M**.

3.—With centre **G**, and radius **G M**, describe arc **M N**. Then line **F G** is divided in EXTREME and MEAN-PROPORTION at point **N**: that is, whatever proportion **F G** bears to **N G**, then **N G** bears the same proportion to **F N** (**F G : N G : : N G : F N**).

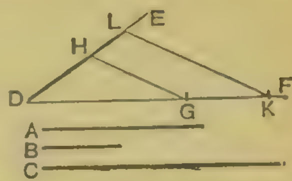
Note (a).—A right line is said to be cut in extreme and mean-ratio, when the whole line bears the same proportion to the greater segment, as the latter does to the smaller segment. Thus, F G, F N, are the *extremes*, and N G is the *mean-proportion*. Line F G is also said to be divided *medially*. (*See* § 155.)

(b).—Two diagonals of a regular pentagon cut each other in extreme and mean-ratio;—each larger segment = length of side of pentagon. If the radius of a circle is cut in extreme and mean-ratio, the greater segment = side of a regular decagon inscribed in the circle.

PROBLEM 91.—To find a Fourth-Proportional to three given lines, A, B, C.

1.—Draw **D E, D F,** at *any* convenient angle. Make **D G = A,** and **D H = B.** Join **G H.** Make **D K = C.**

2.—Draw **K L,** parallel to **G H.** Then **D L** is the required FOURTH-PROPORTIONAL: *i.e.,* **D G : D H :: D K : D L,** or **A : B :: C : D L.**



Note (a).—D G H, D K L are *similar triangles* (*see* § 80); therefore, D G has the same proportion to D H, that D K has to D L. Thus, if A = 4″, B = 2″, C = 6″, then D L = 3″, because 4″ : 2″ :: 6″ : 3″.

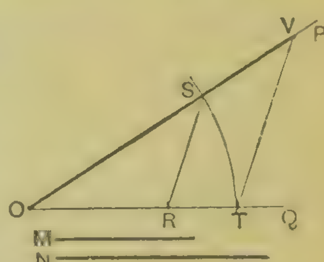
(b).—The proportion may be stated with the given lines in *any* order. Thus, we can work the proportion A : C :: B is to the required line. If we commence with the longest line, and end with the shortest—thus, C : A :: B is to the required line, we get the *shortest* possible line in the proportion—called the 4TH-PROPORTIONAL LESS. By reversing the order—thus, B : A :: C is to the required line, we get the *longest* possible line in the proportion—called the 4TH-PROPORTIONAL GREATER. Test the above by drawing to scale.

(c).—**The foregoing problem enables us to obtain the exact proportion for the size of an enlarged or reduced drawing, etc.** Thus, suppose we have a drawing or painting, rectangular in shape, the sides being respectively equal to A and B; and we wish to make an *enlargement*, each longer side to correspond with C. Then D L is the length of each of the shorter sides of the enlarged drawing, &c. Proceed in the same way if the size is to be *reduced.*

PROBLEM 92.—To find a Third-Proportional to two given lines, M and N.

1.—Draw **O P, O Q,** at *any* convenient angle. Make **O R = M.** Make **O S** and **O T = N.** Join **R S.**

2.—Draw **T V,** parallel to **R S.** Then **O V** is the required THIRD-PROPORTIONAL; *i.e.,* **O R : O S :: O T : O V,** or **M : N :: N : O V.**



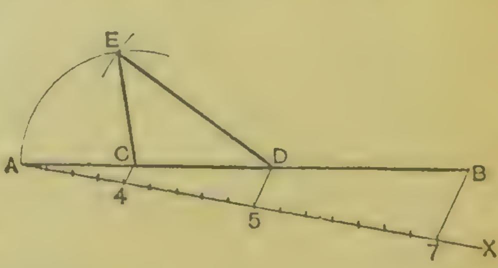
Note (a).—This prob. is simply a variation of Problem 91. The distance N, which is marked off *twice*, at O S and O T, is the *mean-proportional.* If M = 4″, and N = 6″, then O V = 9″ (4 : 6 :: 6 : 9).

(b). O V is the 3RD-PROPORTIONAL GREATER, because it is the *longest* 3rd-proportional line that can be obtained with M and N. If we mark off the *shorter* line, M, *twice*, on O P and O Q, then M is the *mean-proportional*; and we get the *shortest* 3rd-proportional that can be got with M and N, and it is called the 3RD-PROPORTIONAL LESS. (*See* § 158.)

PROBLEM 93.—Construct a Triangle, having a perimeter equal to A B, and the sides to be in the given proportion 4 : 5 : 7.

1.—Draw **A X,** at any angle with **A B.** Then because 4 + 5 + 7 = 16, set off 16 equal parts from **A** on **A X.** Mark points **4, 5, 7.** Join **7 B.** Draw **4 C, 5 D,** parallel to **7 B.**

2.—With centre **C,** and radius **C A** and with centre **D,** and radius **D B,** strike arcs cutting at **E.** Join **E C, E D.** Then **C D E** is the required TRIANGLE.



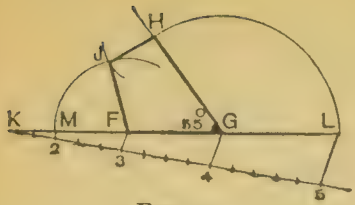


FIG. 170.

Note (a).—A B is divided in the required proportion at C and D. Thus, A C=4 parts, C D=5 parts, D B=7 parts. Therefore the triangle has sides in the proportion 4 : 5 : 7.

(b).—Fig. 170 shews an application of the above principle in constructing a **trapezium F G H J**, with a given perimeter **K L**, a given angle of 55° at G, and the sides in a given proportion of 2 : 3 : 4 : 5. The side **H J=K M.** Adopt a similar method for rectilineal figures having *any* number of sides. The size of the given angle, or angles, must depend upon the shape of the figure.

General Notes about Proportion.

158.—If lines be drawn parallel to one side of a triangle, they cut the other two sides—or these produced—*proportionally.* Thus, in the triangle A B C (Fig. 171), if D E, F G, be drawn parallel to one of the sides C B, to meet A C, A B, or these sides produced, then A D, D C, C F, are in the same proportion as A E, E B, B G. It is on this principle that Probs. 87 (1*st Method*), 88, 91, 92, are constructed. A E D, A B C, A G F are *similar triangles;* therefore, A E has the same proportion to A D, as A B has to A C, or A G to A F. Also, A C : C F :: A B : B G; and A D : D F :: A E : E G; and A E : E D :: A B : B C; &c. (*Euclid* VI. 2, 4.)

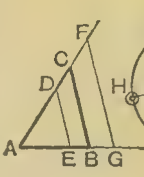


FIG. 171.

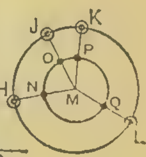


FIG. 172.

159.—In Prob. 89, if we join L R, R P, we get a right-angled triangle. Thus, in all right-angled triangles, a line drawn from the right angle, perpendicular to the hypotenuse, is a mean-proportional to the segments L M, M P, on the hypotenuse. (*Euclid* VI. 8, *Cor.*)

160.—Fig. 172 shews how to divide the circumference of one circle, in proportion to the divisions on the circumference of another circle. Suppose the larger circumference has divisions, H, J, K, L. Required, to divide in the same proportion, the circumference of a smaller circle. With the same centre, M, describe the 2nd circle, and draw lines H M, J M, K M, L M. Then the arcs, N O, O P, P Q, Q N have the same proportion to one another, as the arcs H J, J K, K L, L H have to one another. The sectors of one circle are in the same proportion as the sectors of the other circle. Proceed on the same principle, if the given divided circle is the smaller of the two.

Work out TEST-PAPER C, and the Exercises on RATIO and PROPORTION.

CHAPTER VIII. SIMILAR FIGURES.

CLASS-SHEETS 9 AND 10.

DEFINITIONS, etc. :—**161.**—See § 80. One figure is said to be **Similar** to another, when it is the same *shape*, but enlarged or reduced, by scale, exactly in the *same proportion.* Similar figures have each the same number of sides and angles. SIMILAR RECTILINEAL FIGURES HAVE THEIR CORRESPONDING ANGLES EQUAL, AND THE SIDES ABOUT THE EQUAL ANGLES ARE PROPORTIONALS. In Figs. 173, 174, we have two *similar* triangles. Angle A =

FIGS. 173. 174. 175. 176. 177. 178. 179. 180.

angle D, angle B = E, and C = F. The sides A B, A C, have the same proportion to each other, as the sides D E, D F. It is the same with the sides A B, B C, and D E, E F; and the sides B C, C A and E F, F D. The *corresponding* sides are said to be **Homologous**; that is, A B, B C, C A, are, respectively, homologous to D E, E F, F D. In *similar triangles*, the homologous sides are those which are opposite to the equal angles. (*Euclid* VI. *Def.* 1.)

162.—Figs. 175, 176, shew examples of *similar irregular polygons.* The equal angles, and the corresponding, or homologous sides, are indicated by the marks attached to them.

163. All *triangles*, having their corresponding angles equal, are similar. All *squares* are similar: so are all *regular polygons*, having the same number of sides. *Rectangles* are not similar, unless the sides be in proportion. Rectilineal figures, having more than 3 sides, may have their sides in proportion; but, if the corresponding angles are not equal, the figures are not similar. Thus, Figs. 177, 178, have their sides in proportion; but, because the corresponding angles are unequal, the figures are not similar. Again, Figs. 179, 180, are not similar; for although the corresponding angles are equal, the sides are not proportionals.

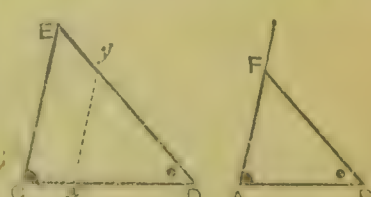
164.—Observe the distinction between the terms "EQUAL" and "SIMILAR." Fig. 175 is *similar* to Fig. 176, but it is not *equal* to it. One figure is said to be *equal* to another, when it contains exactly the same amount of surface, or *area*,—no matter how different the *shapes* may be. Thus, a *triangle* may be equal to a *square*,—but it cannot be similar to it. Figures which are precisely the same in AREA AND SHAPE, are said to be **Equal and Similar**, or IDENTICAL; or they are called CONGRUENT FIGURES. (*See Prob.* 61.)

165.—Figs. 181, 182, 183, 184, shew several applications of one of the principles on which similar figures can be constructed, in each case, by the use of *radiating lines:* at G, by radials drawn from an angle; at H, from a point in one side; at J, from a point inside the figure; and at K, from a point outside the figure. Notice that, in each instance, the corresponding sides are parallels. The figures may be *regular* or *irregular*, and *larger* or *smaller* than the given figures.

FIGS. 181. 182. 183. 184.

PROBLEM 94.—On a given line A B, to construct a Triangle, similar to the triangle C D E.

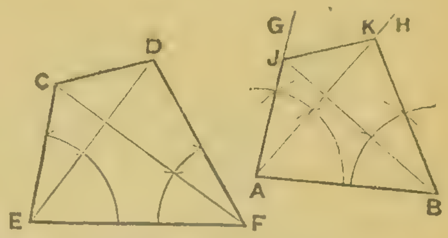
1ST METHOD:—Make the angle **B A F = D C E**, and the angle **A B F = C D E.** Then the TRIANGLE **A B F** is SIMILAR to the triangle **C D E.**



Note.—The angle A F B = angle C E D. The sides C D, D E, E C, are in the same proportion as sides A B, B F, F A.

2ND METHOD:—If we draw a line *parallel* to one side of a triangle, meeting the remaining sides, or these sides produced, we get a *similar* triangle. Thus, cut off **D x = A B**, and draw **x y** parallel to **C E**; then TRIANGLE **x D y** is SIMILAR to triangle **C D E.** If **A B** is longer than **C D**, produce **D C**, **D E**, indefinitely, and get the line **x y** *outside* of the triangle **C D E**, and *parallel* to **C E.** (*See Probs.* 16, 87, and § 158.)

PROBLEM 95.—On a given line to construct a Rectilineal Figure, similar to ANY given rectilineal figure. Say, on A B, construct a Trapezium, similar to the given trapezium E C D F. (1ST METHOD.)



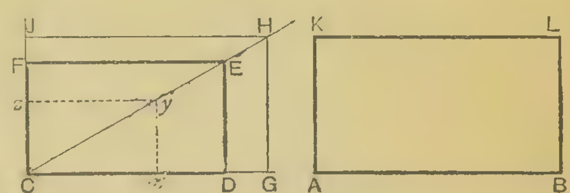
1.—Draw the diagonals **E D, C F.**
2.—Make angles **B A G, B A H,** equal to the angles at **E,** and angles **A B K, A B J,** equal to the angles at **F.** Draw **J K.** Then the FIGURE **A J K B** is SIMILAR to **E C D F.**

Note (a).—The above method is the same as that used for Prob. 94 (1st Method); *i.e.*, on A B we have constructed *triangles*, *similar* to those on E F (Triangle A J B is similar to E C F, and A K B is similar to E D F). *Any* rectilineal figure—no matter how many sides—can be thus divided into *triangles*, constructed upon *any* side as a base. Fig. 186 shews how an irregular pentagon has been constructed upon L M, *similar* to Fig. 185,—the angles at L and M being made equal to those at N and O. If lines L M, N O, are in *one straight line*, or *parallel*, the corresponding sides of the figures are *parallel*.

FIGS. 185. 186. 187. 188.

(b).—Fig. 188 shews how to construct, upon a given line P Q, a figure similar to Fig. 187—an irregular shape with a *curved line*. At convenient distances, points are marked upon the curve. These points become the vertices of the triangles. The curve must be drawn as a freehand line through the points obtained. Adopt the same method, when constructing figures similar to *any* arrangement of straight or curved lines, whether *connected*, *unconnected*, or *intersecting*.

PROBLEM 96.—On a given line, to construct a Rectilineal Figure, similar to ANY given rectilineal figure. Say, on A B, construct a Rectangle, similar to the given rectangle C D E F. (2ND METHOD.)



1.—Produce sides **C D, C F.** Draw diagonal **C E,** and produce it.
2.—Make **C G = A B.** Draw **G H** parallel to **D E,** and **H J** parallel to **E F.** Then the RECTANGLE **C G H J** is SIMILAR to **C D E F.** Make the rectangle **A B L K** equal in all respects to **C G H J** (*with a set-square*).

Note (a).—If the given line is *shorter* than **C D,** mark off its length on **C D,** as at **C x.** Draw **x y, y z,** parallel to **D E, E F.** Then **C x y z** is the required RECTANGLE. This construction is on the same principle as that used in the 2nd Method, Prob. 94.

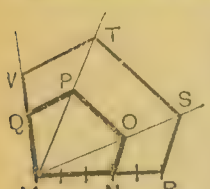
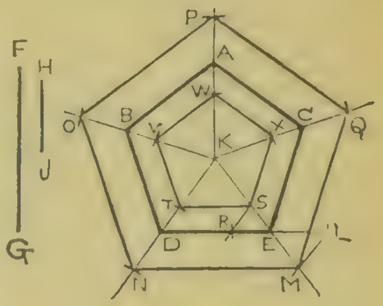


FIG. 188a.

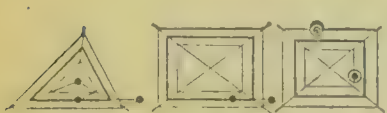
(b).—The above method enables us to determine the exact proportion for the size of an enlarged or reduced drawing, painting, &c. (*See note* c, *Prob.* 91.)

(c).—M N O P Q (Fig. 188a) is a given **Irregular Polygon.** Construct a similar polygon whose sides shall be to those of the given polygon as 5 : 3. Divide M N into 3 equal parts. Produce M N to R, making N R = 2 of the equal parts on M N. Then M R : M N :: 5 : 3. From M draw radials through O, P, Q. Draw R S, S T, T V, parallel to N O, O P, P Q. Then M R S T V is similar to M N O P Q, and its sides are to those of M N O P Q as 5 : 3. (*See Fig.* 181.)

PROBLEM 97.—To construct, inside or outside ANY Regular Polygon, a polygon similar to the given one. The sides of the required polygon to be parallel to, and equidistant from, the given polygon.



1.—Let **A B D E C** be a given *pentagon.*
2.—From **K**, the centre, draw radials through the angles **A, B, D, E, C.**
3.—FIRST :—Construct a *larger* pentagon, having sides equal to **F G.**
4.—Produce **D E** to **L**, making **D L = F G.** From **L**, draw **L M**, *parallel* to **K D.**
5.—With centre **K**, and radius **K M**, mark off points **N, O, P, Q.** Join these points. Then **N O P Q M** is the required PENTAGON.
6.—SECONDLY :—Construct a *smaller* pentagon, having sides equal to **H J.**
7.—On **D E**, mark off **D R = H J.** Draw **R S**, *parallel* to **K D.**
8.—With centre **K**, and radius **K S**, get points **T, V, W, X.** Join these points. Then **S T V W X**, is the required PENTAGON.

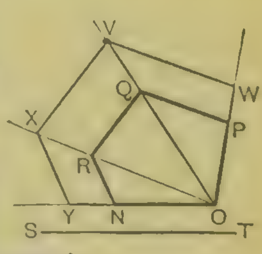


FIGS. 189. 190. 191.

Note (a).—A rectilineal figure can be constructed, inside or outside *any* given rectilineal figure, and *similar* to it ; but the sides can be made *equidistant*, only in the case of a *triangle*, *square*, *rhombus*, or *regular polygon.* For a **triangle**, Fig. 189, find its *centre*, and draw the radials ; &c. Fig. 190 shews the construction for a **rectangle.** For *any regular* or *irregular* rectilineal figure, *any* point may be selected, from which to draw the radials, as at J, in Fig. 183. For practice, construct similar figures—having the length of one side given—inside and outside a given triangle, a square, a rhombus, a rhomboid, a trapezium, and an irregular pentagon.

(b).—Fig. 191 shews how *any* rectilineal figure may be constructed, similar to a given figure, with one of its sides passing through a *given point*, inside or outside the given figure.

PROBLEM 98.—To construct a Rectilineal Figure, similar to ANY given figure, the length of ANY Diagonal being given. Say, construct an Irregular Pentagon, similar to N O P Q R, when the length of the diagonal corresponding with O Q = S T.



Produce **O P, O Q, O R, O N.** Make **O V = S T.** Draw **V W, V X, X Y**, *parallel* to **Q P, Q R, R N.** Then **Y O W V X** is the required IRREGULAR PENTAGON.

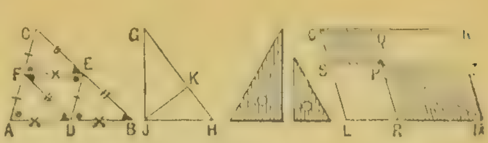
Note.—If S T is less than O Q, mark the distance from O on O Q ; &c.——Proceed on the same principle, if, instead of having the length of a *diagonal* given, we have the length of a line drawn from *any* point in a *side*, to any point in another *side*, or to any *angle.* (*See Fig.* 182.) The figures may have *straight* or *curved* boundaries.

General Notes about Similar Figures.

166.—In similar rectilineal figures, the *perimeters* are, to one another, in the same ratio as the *corresponding sides* are to one another. (*Euclid* VI. 20.)

167.—In circles, the circumferences are in the same ratio as the diameters.

168.—If in *any* given triangle, A B C (Fig. 192), we bisect each side, at D, E, F, and join these points, we divide the triangle into 4 *equal and similar* triangles. The equal sides and angles are indicated by the marks attached to them. Lines D E, E F, F D, are *parallel* to A C, A B, B C. Line F E = half A B, D F = half B C, and D E = half A C. Therefore, if only the *middle points* of the sides of a triangle are given, we can construct the triangle, by joining these points, getting a small triangle D E F, and then drawing, through D, E and F, lines *parallel* to D E, E F, F D.



FIGS. 192. 193. 194. 195.

169.—If, in any right-angled triangle, we draw a perpendicular from the right angle to the hypotenuse, we get 2 triangles G K J, J K H (Fig. 193), similar to each other and to the triangle G J H. (*Euclid* VI 8.) Rectilineal figures are similar to each other (provided the corresponding angles are equal, and the sides about the equal angles are proportionals), although one may be reversed, or turned over, in its position. Thus, any 2 set-squares of 60° (Fig. 194) are similar triangles, although placed in reversed positions. In the same way, A B C D, T V W X (Fig. 116), are *equal and similar* figures, but reversed in position.

170.—The parallelograms about the diagonal of *any* parallelogram, are similar to the whole, and to each other. Thus, in the parallelogram, L M N O (Fig. 195), if we draw a diagonal O M, and mark any point P in it, and through P draw Q R, S T, parallel to O L, L M, then the parallelograms S P Q O, R M T P, are *similar* to each other, and to L M N O. (*Euclid* VI. 24.)

Work out the Exercises on SIMILAR FIGURES.

CHAPTER IX. SCALES & SCALE-DRAWING.

(PART II.)

CLASS-SHEET 10.

DEFINITIONS, etc. :—171.—See CHAPTER III, §§ **82 to 93.**

172.—**Scales** are based upon the principles of PROPORTION. With scales we can measure the sides of *rectilineal* figures. When we apply them to *curvilineal* figures, we measure the *diameters*, and not the *curves*.

173.—Note the difference between the meaning of the terms **scale** and **size.** SCALE refers to *length* or *breadth:* SIZE refers to the amount of *surface.* Thus, the square A B C D is *double* the scale, but *four times* the size of the square A E F G (Fig. 196). Hence, care must be observed not to confound the two terms. Frequently we meet with such a statement as this, that A B C D is double the *size* of A E F G, instead of double the *scale.* In the same way, a diagram made 6″ to 1′ is *said* to be drawn "*half full-size*" instead of to "*half-scale*"; and one drawn 3″ to 1′ is *said* to be made "*quarter full-size,*" instead of to "*quarter-scale*"; and so on. By force of custom, such imperfect statements have to be accepted. (*See* § 85.) When a diagram is made exactly the *same size* as the object represented, we can then correctly say that it is drawn "*full-size.*"

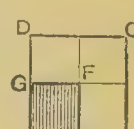


FIG. 196.

174.—An acquaintance with Arithmetic is of great service in the construction of some scales. **Decimal Notation** should be understood. Thus, one numeral after a *decimal point* (·), as **·3,** reads "decimal 3," or "point 3," and stands for $\frac{3}{10}$ (three-*tenths*). Two numerals, as **·72**—"decimal (or point) seven two"—stand for $\frac{72}{100}$ (seventy-two *hundredths*). Three numerals, as **·538**—"decimal five three eight"—stand for $\frac{538}{1000}$ (five hundred and thirty-eight *thousandths*); &c. Hence, **4·03″** signifies $4\frac{3}{100}$ in. Notice that **·25** stands for $\frac{25}{100}$ or $\frac{1}{4}$; **·5,** for $\frac{5}{10}$ or $\frac{1}{2}$; and **·75,** for $\frac{75}{100}$ or $\frac{3}{4}$.

175.—The Representative Fraction.—If a diagram is drawn to *half* scale, say a scale of 6 in. to 1 ft., we state that its REPRESENTATIVE FRACTION is $\frac{1}{2}$. This signifies the proportion existing between the diagram, and the actual object. Thus, in a diagram drawn to 6″ = 1′, a line, 2″ long by *scale*, represents a *real* length of 4″; and so on. In the same way, a diagram drawn *two-thirds* the scale, has a representative fraction of $\frac{2}{3}$. A diagram to *one-hundredth* the scale has a representative fraction of $\frac{1}{100}$; &c.

176.—Examples of Representative Fractions.—In CHAPTER III, Fig. 68, the scale of "1 in. to a ft." has a REPRESENTATIVE FRACTION of $\frac{1}{12}$, because 1 in. is $\frac{1}{12}$ of 1 ft.

Fig. 69, "1 in. to 5 ft." (*i.e.*, 1 in. to 60 in.) the REP. FRACTION is $\frac{1}{60}$.
Fig. 70, "1 in. to 3 ft." (*i.e.*, 1 in. to 36 in.) the REP. FRACTION is $\frac{1}{36}$.
Fig. 71, "1 in. to 20 yds." (*i.e.*, 1 in. to 720 in.) the REP. FRACTION is $\frac{1}{720}$.

In the last example 1 in. indicates 20 yds., or 720 in.; therefore each measurement, by *scale*, is $\frac{1}{720}$ the *actual length*. In the following scales:—

"$\frac{1}{2}$ in. to 1 ft." (*i.e.*, 1 in. to 24 in.) the REPRESENTATIVE FRACTION is $\frac{1}{24}$.
"3 in. to 1 ft." (*i.e.*, 3 in. to 12 in., or 1 in. to 4 in.) the REP. FRACTION is $\frac{1}{4}$.
"1 in. to 1 mile" (*i.e.*, 1 in. to 63360 in.) the REP. FRACTION is $\frac{1}{63360}$.
"1 ft. to a yd." (*i.e.*, 1 ft. to 3 ft.) the REPRESENTATIVE FRACTION is $\frac{1}{3}$.
"1 in. to $\frac{1}{2}$ in." (*i.e.*, 2 in. to 1 in.) the REPRESENTATIVE FRACTION is $\frac{2}{1}$.
"5 in. to 1 ft." (*i.e.*, 5 in. to 12 in.) the REPRESENTATIVE FRACTION IS $\frac{5}{12}$.

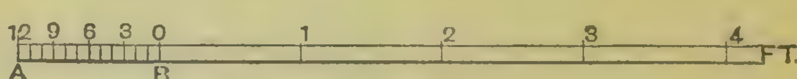
In the last example we have to retain 5 as the numerator, because 5 is not contained in 12, without a remainder. Each dimension by *scale* is $\frac{5}{12}$ the *actual* length. It will be seen in the above examples that, in each case, we reduce the greater unit of measurement to the same denomination as the lesser unit. Thus, in "1 in. to 1 ft." we reduce feet to inches, and get the REPRESENTATIVE FRACTION $= \frac{1}{12}$.

177.—A Scale can be given in three ways:—1st. By *drawing* it; 2nd. By a *statement*, as 3″ = 1′, or 3″ to 1′; 3rd. By the *representative fraction*, as $\frac{1}{4}$.

178.—For intricate work, make the scale upon the drawing-paper. Then, with atmospheric changes, it contracts or expands in the same proportion as the drawing.

PROBLEM 99.—Construct a Scale of $\frac{1}{24}$, to measure feet and inches (Fig. 197).—1 in. represents 24 in.; therefore $\frac{1}{2}$ in. represents 1 ft. Draw

FIG. 197.
Scale of $\frac{1}{24}$.



parallel lines. Set off spaces of $\frac{1}{2}$ an in., from A. Each space represents 1 ft. by scale. Divide the 1st space, A B, into 12 equal parts, to represent inches. Figure the scale as shewn. Then the distance from *zero* to 4 = 4′, and from 2 to 9 = 2′ 9″, by scale; &c. Each *scale* measurement is $\frac{1}{24}$ the *real* length.

PROBLEM 100.—Construct a Scale of $\frac{1}{36}$, to measure yards and feet (Fig. 198).—Draw the parallel lines. Because 1 in. represents 36 in. or 1 yd., set off, from X, distances of 1 in., each representing 1 yd. by scale.

FIG. 198.—*Scale of* $\frac{1}{36}$

Divide the 1st space, **X A**, into 3 equal parts, to represent feet. The distance **A B** = 2 yds.; and **C D** = 1 yd. 2 ft., by *scale*. Each *scale* measurement = $\frac{1}{36}$ *real* length. We can divide **X D** into 12 equal parts, to represent inches.

PROBLEM 101.—The line E F represents 1 mile 3 furlongs. Set off, from E, the distance representing 1 mile. Mark the subdivisions for furlongs. Construct the Scale to measure 5 miles (Fig. 199).—As 1 mile 3 furlongs = 11 furlongs, divide **E F** into 11 equal parts.

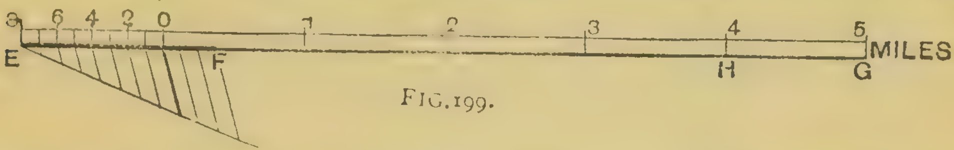


FIG. 199.

Then 8 of these parts, from **E** to **O** = 8 furlongs, or 1 mile. Set off 5 distances from **O** = **E O**. Then **O G** = 5 miles; **H 6** = 4 miles 6 furlongs; and so on.

179.—DIAGONAL SCALES.—In a SIMPLY DIVIDED, or PLAIN SCALE, we only get *two* dimensions, as inches and 10ths, feet and inches, or miles and furlongs; &c. In a **Diagonal Scale** we can get *three* dimensions of measurement, as units, 10ths and 100ths; or yards, feet and inches; or miles, furlongs and perches; &c. A diagonal scale gives *very minute* measurements, which it is impossible to get in a plain scale. To shew the *principle of construction*, we will first draw a diagonal scale, giving only *two* dimensions.

PROBLEM 102.—Construct a Diagonal Scale, ½ in. to the yard, shewing yards and feet (Fig. 200).—Draw a horizontal line **J K**, any length. Mark points **0, 1, 2,** at ½" apart, to represent *yards*. At **J** raise a vertical line of *any* length. As a yard contains 3 ft. set off 3 *equal* spaces of *any* convenient length, from **J** to **L, M, N.** From **L, M, N,** draw horizontals. Raise verticals from **0, 1, 2.** The distances between the *verticals* represent YARDS. The *diagonal* **0 N** divides the horizontals into FEET. Thus, **P Q** = 1 ft., **R S** = 2 ft., **T N** = 3 ft. In like manner **V Q** = 2 yds., 1 ft.; &c.

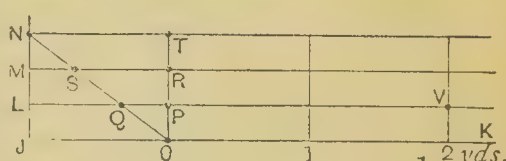


FIG. 200. ½" *to a yard* = $\frac{1}{72}$

Note.—0 P Q, 0 T N are *similar triangles*. Therefore, because 0 P is ⅓ of 0 T (by construction), P Q must be ⅓ of T N, *i.e.*, 1 ft. In like manner R S = ⅔ of T N, *i.e.*, 2 ft. All diagonal scales are made on this *principle*. In the above scale the representative fraction is $\frac{1}{72}$, because there are 72 half-inches in 1 yd.

PROBLEM 103.—Construct a Diagonal Scale of ¾" to 1 yd., to measure yds., ft., and in. (Fig. 201).—Draw a horizontal line **A B**, of indefinite length. Set off spaces of ¾" from **A** to **0, 1, 2,** &c., to represent the required number of YARDS. Divide **A 0** into 3 equal parts to represent FEET. At **A**, raise a vertical line of *any* length, and because there are 12 IN. in 1 ft., set off upon it, from **A**, 12 *equal* spaces of *any* convenient length. Draw horizontals from

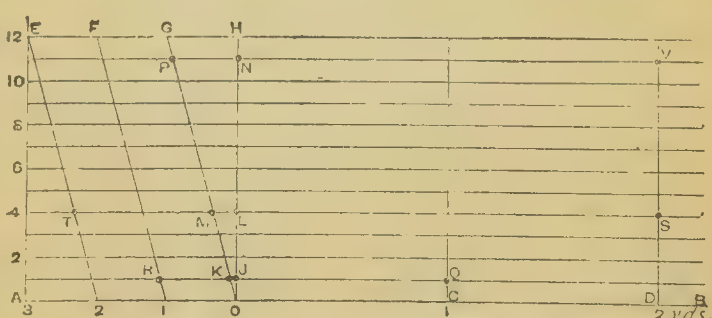


FIG. 201. *Scale of* ¾" *to* 1 *yd.* = $\frac{1}{48}$

these 12 points. Raise verticals from **0, C, D.** Divide **E H** into 3 equal parts, at **F, G**, which also represent FT. Draw the *parallel* oblique lines **2 E, 1 F, 0 G.** Figure the scale as shewn.

Note (a).—The horizontal distances between the parallel *oblique* lines, are all equal. Thus, between **0 G** and **1 F**, or **1 F** and **2 E**, each horizontal distance = 12 in. But, the horizontals between **0 G** and **0 H** *increase*, each 1′ in length, from **0** to **G H**, as indicated by the numerals on the line **A E.** Thus **J K** = 1″; the space above that = 2″; the next = 3″; then **L M** = 4″; and so on, up to **N P**=11″, and **H G**=12″. In the same way **Q J**=1 yd.; **Q K** =1 yd. 1 in.; **Q R** = 1 yd. 1 ft. 1 in.; **S L** = 2 yds.; **S M** = 2 yds. 4 in.; **S T** = 2 yds. 2 ft. 4 in.; and **V P**=2 yds. 11 in.

(b).—The representative fraction is $\frac{1}{48}$, because $\frac{3}{4}''$ is contained 48 times in 1 yd. The above scale measures *three* dimensions, *viz.*, yds. ft. and in. We always set off the *largest* dimensions—in this case yds.—on the right of 0; the dimensions *next* in length from 0 to A; and the *smallest* dimensions determine the number of horizontals. Thus, in a **Diagonal Scale of** $\frac{1}{2}''$ **to a mile, to shew miles, furlongs, and chains** (Fig. 202), we set off spaces of $\frac{1}{2}''$ on a horizontal line. Those on the right of 0 represent the *largest* dimensions—miles. We divide the $\frac{1}{2}''$ space on the left of 0 into 8 equal parts, to represent the dimensions *next* in length—furlongs. As a furlong contains 10 of the *smallest* dimensions—chains, we obtain the 10 equal spaces on a vertical line, and complete the scale. Then **0 1**=1 mile; **1 6**=1 mile 6 furlongs; line **X**=9 chains; **Y**=2 miles 5 furlongs 6 chains; **Z**=1 mile 4 furlongs 2 chains. The representative fraction of this scale is $\frac{1}{126720}$; because $\frac{1}{2}''$ represents 1 mile, and there are 126,720 half-inches in a mile.

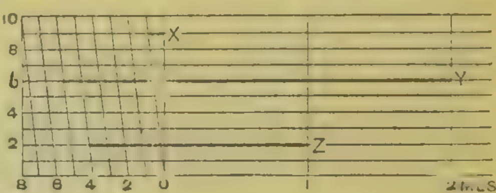


FIG. 202. $\frac{1}{2}''$ *to a mile*$=\frac{1}{126720}$

PROBLEM 104.—To construct a Diagonal Scale of 1″ to a ft., to shew feet, 10ths, and 100ths of a foot (Fig. 203).—On a

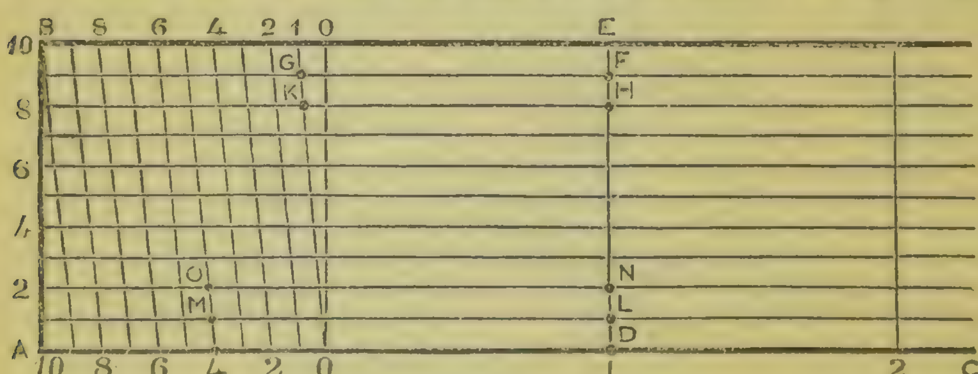


FIG. 203.—*Scale of* 1″ *to a ft.*

horizontal line, **A C**, set off *inch* spaces to represent FEET. Divide **0 A** into 10 equal parts for the TENTHS. As each tenth contains 10 hundredths, set off 10 equal parts—*any* length—on **A B**, to enable us to get the HUNDREDTHS. Complete the scale by drawing the horizontal, vertical, and oblique lines. Then **0 1, 1 2** = feet; **D 4**=1 ft. 4 tenths (or **1·4**, *i.e., one, decimal four.*—See § 174);

0 6 = $\frac{6}{10}'$ (·6′)	**F G** = $1\frac{9}{100}'$ (1·09′)	**L M** = $1\frac{41}{100}'$ (1·41′)
E 1 = $1\frac{1}{10}'$ (1·1′)	**H K** = $1\frac{8}{100}'$ (1·08′)	**N O** = $1\frac{42}{100}'$ (1·42′)

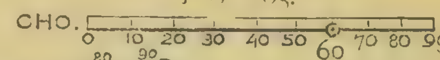
Note (a).—The inch spaces set off horizontally, may be assumed to represent *any* given distance, as *yards, miles, leagues*, &c.: or they may be used as a scale of 1″ *to the inch*, to take measurements *full-size*. The scale in each case gives 10ths and 100ths, whatever denomination of measurement is selected. The unit of measurement may have any given scale-length instead of the inch.

(b).—In Fig. 203, the scale marks 100ths of an inch. If the verticals are ½" apart, we get 200ths of an inch. If ¼" apart, we get 400ths of an inch; and so on. It is evident that we could never get such minute measurements with any *plain scale*. Diagonal scales are of great value; and if the principle of their construction be understood, they can be made in great variety, to suit different requirements. We usually find one or more diagonal scales marked upon the flat protractor.

180.—A SCALE OF CHORDS is frequently marked on each side of a flat protractor, and it is indicated by the letters "C" or "C H O" (Fig. 205). This scale is used to measure *angles.*

PROBLEM 105.—To construct a Scale of Chords.—Draw a right angle **P Q R** (Fig. 204). With centre **Q**, and ANY radius, strike arc **P R**. Trisect the arc, by Prob. 18. By trial with compasses, divide each trisection into 3 equal parts. The entire arc is then divided into 9 equal parts, each containing 10°. Draw the CHORD **R P**. With centre **R**, and with the distances to the divisions upon the arc as radii, strike arcs cutting the *chord* **R P**. The line **R P** is then a SCALE OF CHORDS, *i.e.*, we have marked upon it from **R**, the lengths of the *chords* between **R** and the divisions upon the arc. Line **R P** is a chord of **90°**, because it joins the extremities of an arc containing 90°; and so forth. The smaller divisions between R and 10, 10 and 20, &c., on the arc and chord **R P**, are not shewn, but the *whole* of the 90 divisions are supposed to be made upon the arc and the chord **R P**. Observe that the divisions on the *chord* **R P** get less in length from **R** to **P**. Draw 2 parallel lines the same length as chord **R P**, as in Fig. 205; and transfer the divisions upon chord **R P**, to these parallels—from 0 (zero) to 90—and we obtain the required **scale of chords.**

FIG. 205.



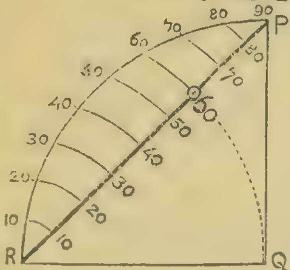


FIG. 204.

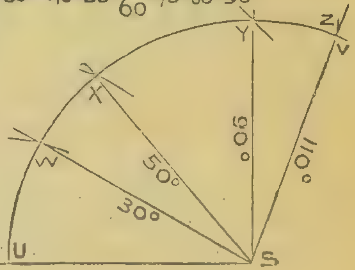


FIG. 206.

Note (a).—How to use the Scale of Chords.—Notice that the distance R to 60, on R P, or 0 to 60 in Fig. 205, is the length used for the *radius of the arc.* It is required to draw lines from S, making *any* angles with S T (Fig. 206). *Whatever* angle is required, *always* take the length of the chord of 60°—*i.e.*, from 0 to 60 (Fig. 205)—as a radius, and with S as centre, strike arc **U V.**——For an angle containing **30°**, measure the length of the *chord* of 30°, from 0 to 30, and mark it off from U to W. Draw the line W S. Then the angle U S W contains 30°.——For **50°**, measure from 0 to 50, and set it off from U to X. Draw X S. Angle U S X=50°.——For **90°**, measure from 0 to 90, and set it off from U to Y. Draw Y S. Angle U S Y = 90°.——For **110°**, first get point Y, for 90°. Then because 90°+20°=110°, *add* the length of the *chord* of 20°, from Y to Z. Draw Z S. Then U S Z=110°. Or, we can obtain *any* angle—say 110°—by setting off in succession, any 2 chords *together* equal to 110°, as 50°, and 60°, or 70° and 40°; &c.

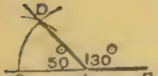


FIG. 207.

(b).—An angle *greater* than 90° can be got by *one* measurement. Thus at A draw a line at 130° with A B. Produce B A to C. Make C A D=50°. Then D A B must = 130°; because 180°−50°=130° (*see* § 49). Fig. 207 is constructed with a *smaller* scale of chords than Fig. 205.

(c).—An angle *already* drawn can be measured with a scale of chords. Thus, suppose angle T S X (Fig. 206) is *given.* With centre S, and radius of a chord of 60°, draw the arc. Measure the length of the chord U X which the scale (Fig. 205) tells us equals 50°. A scale of chords is not so convenient for general use as a *protractor.*

(d).—By using a *scale of chords*, or a *protractor*, we can divide the circumference of a circle into a number of equal parts, and thus construct certain REGULAR POLYGONS

PROBLEMS IN SCALE-DRAWING.

PROBLEM 106.—Fig. 208 is the Plan of a Building, drawn to a scale of $\frac{1}{3}$ in. to 10 ft. Copy it to a scale of $\frac{1}{2}$ in. to 10 ft.—Construct the required scale of $\frac{1}{2}''=10'$. The walls meet at *right angles.* Draw **A B** = 50′. Raise perpendiculars **A C**=25′, **B F** = 20′. Draw **C D** = 28′, **D E**=5′. Join **E F**, and thus complete the outer boundary of the building. It will be seen by the scale attached to the plan that the walls are 3′ thick. In the copy make all the walls 3′ thick, by the new scale. Set off 3′ from *each corner*, from **A**, on **A C** and **A B**; from **B**, on **B A** and **B F**; &c.; and join these points. If the angles at **A**, **B**, **C**, &c., are bisected, each *bisector* should pass through an *interior angle* of the wall; because the walls are of *equal thickness.* Draw these bisectors. In architecture, carpentry, &c., the bisector of a right angle is called a *mitre.*

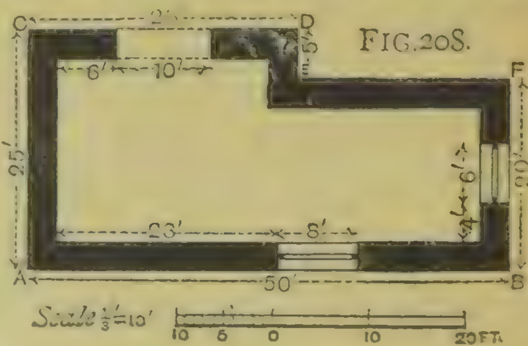


FIG. 208.

Scale $\frac{1}{3}''=10'$

For the door (10′ wide), find the centre of the wall, **C D**, and set off 5′ on each side of it. For the window (6′ wide) in wall **B F**, find the centre, and set off 3′ on each side of it. For the window in wall **A B**, set off 23′ from the interior angle, then 8′.

Note.—As $\frac{1}{3}''=10'$, then $1''=30'$, or $360''$. Therefore 1″ in the plan is as 360″ in the actual building, so the *representative fraction* is $\frac{1}{360}$. By the same mode of calculation, we ascertain that the *representative fraction* for the copy done to a scale of $\frac{1}{2}''=10'$, is $\frac{1}{240}$.

PROBLEM 107.—Fig. 209 is drawn to a scale of 1 in. to 40 ft. Make a copy of this diagram to a scale of 1 in. to 15 ft.—Construct a scale of 1 in. to 15 ft. Draw the line **D L**, of indefinite length, and set off upon it, by the scale just made, the distances of 11′, 20′, 8′, 13′. At these points obtained, raise perpendiculars 9′, 22′, 6′, 19′, 12′ high. Join the points **P**, **Q**, **R**, **S**, **T**. Proceed by a similar method if **D L** is given above, or by the side of, the given figure.

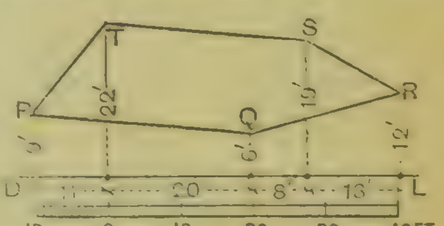


FIG. 209. *Scale of* $1''=40'$

Note.—In the above diagram, as $1''=40'$, or $480''$, the *representative fraction is* $\frac{1}{480}$. The *representative fraction* for the copy is $\frac{1}{180}$. A line, such as D L, from which we set off measurements, is called a DATUM LINE, or BASE LINE. This line may, or may not, touch the diagram.

PROBLEM 108.—To enlarge or reduce a diagram by means of Proportional Lines.—Make a reduced copy of Fig. 210. Let the distance **C D** (Fig. 211) correspond with the length **A B**. Bisect **C D** at **G**. Draw a vertical *centre-line* from **G**. Draw **Q 1**, **Q 2** (Fig. 212), at *any* angle. Make **Q R** = **A B**, and **Q S** = **C D**. Join **R S**. Make **Q T** = **E F**. Draw **T V** parallel to **R S**. Then **Q V** is the *proportional length* for **G H**. (*See Prob.* 87, 1*st Method.*)

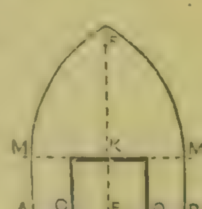


FIG. 210.

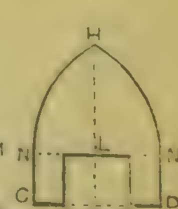


FIG. 211.

To get point **L**, make **Q W** = **E K**. Draw **W X** parallel to **R S**. Then **Q X** is the propor-

tional length for **G L.** Draw a horizontal line through **L.** Draw **C N, D N.** With centres **N, N,** and radius **N N,** describe the arcs which meet at **H.** Make **Q Y = E O.** Draw **Y Z** parallel to **R S.** Make **G P, G P,** each = **Q Z.** Draw perpendiculars from **P, P.** The DIAGRAM **C D H** is then exactly in the same proportion as **A B F.** (*The above diagrams should be drawn at least three times the scale*).

FIG. 212.

Note (a).—Proceed in the same way with an *enlarged* copy. When making a *symmetrical* diagram, get the *centre-line*, as G H, first. Then set off the equal parts on each side of it.

(b).—To construct **proportional scales.** Draw Q R, Q S, as before. Divide Q R into any number of small *equal* parts. Divide Q S into the *same number* of equal parts. We can use the large scale for the large diagram, A B F, and the small scale for the reduced diagram, C D H.

FIG. 213.

(c).—The principle of construction explained in the above Prob., can be applied to the enlargement or reduction of an irregular **curvilineal figure.** Thus, make an enlarged copy of Fig. 213. Draw a *datum line* **D L.** From *any* number of points, conveniently placed on the curve, draw perpendiculars to **D L.** Let the width of the curve between **X Y** be increased *one-third.* Draw another datum line, and on it mark a distance $\frac{1}{3}$ greater than **X Y.** Then, as in Fig. 212, get *proportional lines*, and on the new datum line mark off distances in proportion to those on **D L.** Raise *proportional* perpendiculars. Through the points obtained draw the required CURVE. The greater the number of points on the curve—the more accurate will it be in shape.

PROBLEM 109.—To reduce or enlarge any Figure by means of Squares.—In Fig. 214 we have an irregular figure—say a map of an island. Make a *reduction* to a scale $\frac{1}{4}$ less. Draw over Fig. 214 a series of vertical and horizontal lines, at convenient distances apart, and forming squares. Make the rectangle **E F G H** $\frac{1}{4}$th less in scale than **A B C D,** *i.e.*, **E F** = $\frac{3}{4}$ of **A B,** and **E H** = $\frac{3}{4}$ of **A D.** (*See Prob.* 96.) Divide **E F G H** into the *same* number of squares as there are in **A B C D.** Number the lines as shewn. Then carefully copy the portion in each square of **A B C D,** in each corresponding square of **E F G H.**

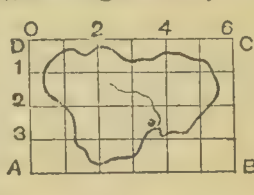


FIG. 214.

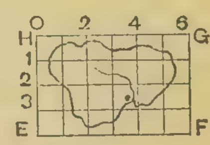


FIG. 215.

Note.—The lines are not obliged to form *squares.* Thus, A B C D, E F G H, may be cut up into *similar rectangles*, by dividing A B, E F, each into *any* number of equal parts, say 12, and A D, E H, each into *any* number of equal parts, say 9. Draw vertical and horizontal lines to form *similar rectangles.* The greater the number of divisions—the greater the accuracy. If the diagram may not be drawn upon, construct the squares or rectangles upon *tracing-paper*, and place it over the diagram. The above method is exceedingly useful to Engravers and others, when making reductions or enlargements of pictures, &c.

PROBLEM 110.—To construct a Figure from given dimensions. Make a corrected drawing of Fig. 216. Scale, 1 in. = 1 ft. Use a Diagonal Scale.—(N.B. *This figure is purposely drawn inaccurately. A correct diagram must be made in accordance with the given dimensions.*) The diagonal scale of 1 in. = 1 ft. is shewn in Fig. 203. Make **A B** = 2·18′, by scale. Make angle **B A E** = 80°. Get the length of each side, &c., from the diagonal scale, and complete the figure by the method explained in Probs. 58, 59.

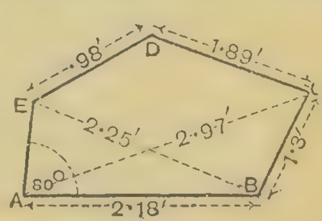


FIG. 216.

Note.—Proceed in the same way if the dimensions are given in yards, miles, &c.

Work out TEST-PAPER D, and the Exercises on SCALES.

CHAPTER X. INSCRIBED AND CIRCUMSCRIBED FIGURES.

CLASS-SHEETS 11 AND 12.

DEFINITIONS, etc.:—181.—Several Problems dealing with INSCRIBED FIGURES were, for convenience of arrangement, introduced in previous Chapters. Refer to PROB. **40**; § **122**; and PROBS. **48, 49, 50, 51.**

FIG. 217.

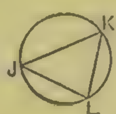


FIG. 218.

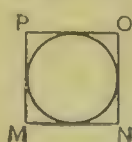


FIG. 219.

FIG. 220.


FIG. 221.

182.—A *rectilineal* figure is **Inscribed** in another *rectilineal* figure, when all its angles touch the sides of the larger figure. Thus, A B C D (Fig. 217) is INSCRIBED in E F G H. (*Euclid* IV. *Def.* 1.)

183.—A *rectilineal* figure is **Circumscribed** about another *rectilineal* figure, when all its sides pass through the angular points of the smaller figure. Thus, E F G H (Fig. 217) is CIRCUMSCRIBED about A B C D. (*Euclid* IV. *Def.* 2.)

184.—A *rectilineal* figure is **Inscribed** in a *circle*, when all its angles touch the circumference of the circle. Thus, the triangle J K L (Fig. 218) is INSCRIBED in the circle. (*Euclid* IV. *Def.* 3.)

185.—A *rectilineal* figure is **Circumscribed** about a *circle*, when all its sides touch the circumference of the circle. Thus, the square M N O P (Fig. 219) is CIRCUMSCRIBED about the circle. (*Euclid* IV. *Def.* 4.)

186.—A *circle* is **Inscribed** in a *rectilineal* figure, when its circumference touches all the sides of the rectilineal figure. Thus, the circle in Fig. 219 is INSCRIBED in the square M N O P. (*Euclid* IV. *Def.* 5.)

187. A *circle* **Circumscribes** a *rectilineal* figure, when the circumference passes through all its angular points. Thus, the circle in Fig. 218 CIRCUMSCRIBES J K L. (*Euclid* IV. *Def.* 6.) **Described about** means the same as *circumscribed about.*

188.—From the above it will be seen that, in some cases, we can get *four* combinations with 2 figures. Thus, in Fig. 220, at Q we have a triangle *in* a circle; at R, a triangle *about* a circle; at S, a circle *in* a triangle; and at T, a circle *about* a triangle. These four arrangements cannot be obtained with *any* 2 figures. Thus,—

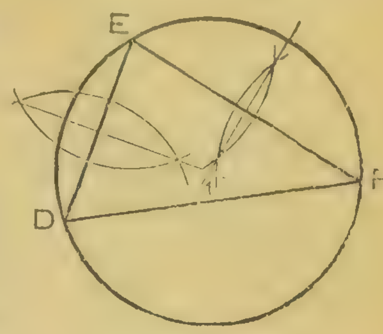
We cannot inscribe a Rhombus within a Square:
We cannot inscribe a Trapezion within a Square:
We cannot circumscribe a Square about a Rhombus:
We cannot circumscribe a Square about a Trapezion:
We cannot inscribe a Rhombus within a Circle:
We cannot circumscribe a Circle about a Rhombus:
We cannot inscribe a Circle within a Rectangle:
We cannot circumscribe a Rectangle about a Circle.

189.—A circle that touches one side of a triangle, and the other two sides produced (Fig. 221), is called an **Exscribed or Escribed** circle.

PROBLEM 111. To circumscribe a Circle about a given Triangle D E F.

1.—Bisect *any* two of the sides, as **D E, E F,** by perpendiculars intersecting at **1.**

2.—With point **1** as centre, and radius **1 D, 1 E,** or **1 F,** draw the required CIRCUMSCRIBED CIRCLE.



Note (a).—By this problem we find a point 1, which is EQUIDISTANT *from any* 3 *given points.* Therefore, the problem might be given thus :—From 3 given points **D, E, F,** draw 3 *equal* lines, which shall meet in one point. Draw **D E, E F,** and the bisectors cutting at **1.** Join **1 D, 1 E, 1 F.** Prob. 47 ; note c, Prob. 47 ; and Prob. 111, are identical in their working.

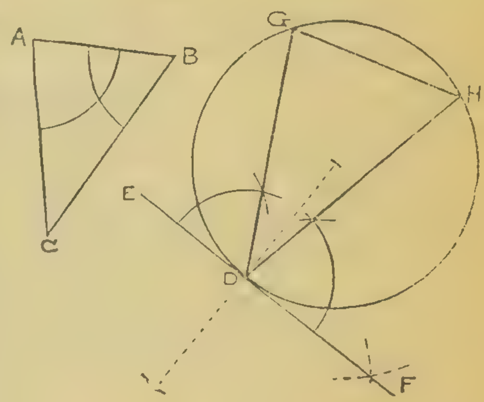
(b).—For a *right-angled* triangle, the bisection of the hypotenuse gives the centre.

PROBLEM 112.—In a given Circle to inscribe a Triangle, similar to a given Triangle, A B C, one of the angles to touch a given point D.

1.—Through **D,** draw *tangent* **E F,** as shewn by dotted lines.

2.—Make angle **E D G** *equal* to the angle at **B.** Make angle **F D H** equal to the angle at **A.**

3.—Join **G H.** Then **D G H** is the required TRIANGLE.



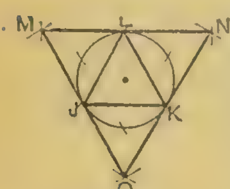


FIG. 222.

Note (a).—G H D is *similar* to A B C. By Prob. 85, we have cut off a segment D G H, which contains an angle at G = F D H or A. In the same way, the segment G H D contains an angle H = B. The remaining angle H D G must = C.

(b).—1st. Inscribe, 2nd. Describe, an EQUILATERAL TRIANGLE in, and about, a given CIRCLE (Fig. 222. *Special Methods*) :—1st. Mark the radius 6 times round the circumference. Join the alternate points **J, K, L.**——2nd. With centres **J, K, L,** and radius **J K,** strike arcs cutting at **M, N, O.** Join these points.

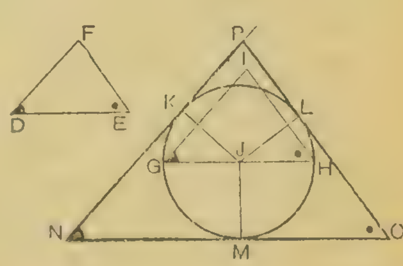
(c).—1st. Inscribe, 2nd. Describe, an EQUILATERAL TRIANGLE in, and about, a given EQUILATERAL TRIANGLE (*Special Methods*) :—1st. To *inscribe* in **M N O,** bisect each side at **J, K, L.** Join the points.——2nd. To *describe* about **J K L,** with centres **J, K, L,** and radius **J K,** strike arcs cutting at **M, N, O.** Join the points.

PROBLEM 113.—About a given Circle, circumscribe a Triangle similar to a given triangle D E F.

1.—Draw a diameter **G H.** On **G H,** construct a triangle **G H I** *similar* to **D E F.**

2.—From **J,** the centre of the circle, draw **J K, J L, J M,** *perpendicular* to the sides of the triangle **G H I.**

3.—Through **K, L, M,** draw **N P, P O, N O,** *parallel* to **G I, I H, G H.** (*Use a set-square.*) Then **N O P** is the required TRIANGLE.



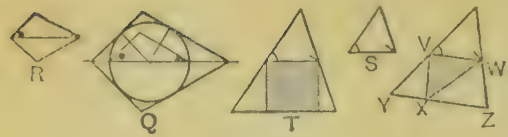
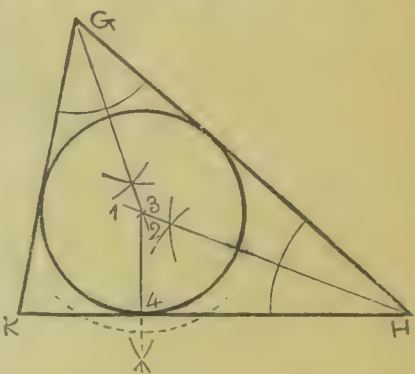


FIG. 223.

Note (a).—The diameter, G H, may correspond with *any* side of the given triangle. If one of the angles, adjacent to G H—say at G—is a *right angle*, the perpendicular from J *coincides* with J G: if *greater* than a right angle, the perpendicular from J will cut G I, *produced*. Lines N O, O P, P N, are *tangents* to the circle.——Should one of the sides of the circumscribed triangle be required to pass through a *given point* M—say the side corresponding with D E—draw a tangent through M, and make G H *parallel* to it; &c.

(b). By this method we can circumscribe about a circle, a figure similar to any rectilineal figure that *can* be described about it, as a RHOMBUS, or TRAPEZION. At Q, Fig. 223, we see the construction for a *trapezion* similar to R. For an *equilateral triangle*, *square*, or *regular polygon*, the special methods are more convenient.——On the same *principle*, we construct a triangle similar to S, about a square T, or about a triangle V W X (draw Y Z *parallel* to V W).

PROBLEM 114.—To inscribe a Circle in a Triangle K H G.



1.—Bisect *any* two of the angles, by lines **H 1, G 2,** intersecting at **3.** Point **3** is the *centre* of the triangle.

2.—From **3,** draw a *perpendicular* to either of the sides, say line **3 4.**

3.—With centre **3,** and radius **3 4,** draw the required CIRCLE.

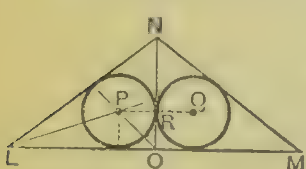


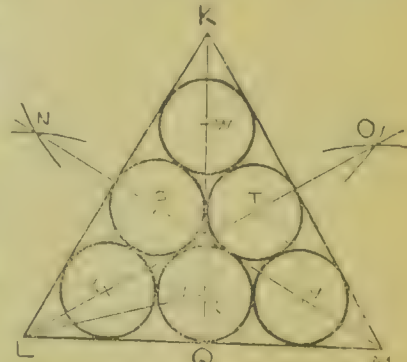
FIG. 224.

Note (a).—The construction of this problem is the same as that of Prob. 73. The sides of the triangle are *tangents* to the circle, which is the *largest* circle that can be cut out of the triangle.—— To CIRCUMSCRIBE a circle about a triangle we bisect the *sides*; to INSCRIBE a circle in a triangle we bisect the *angles*.

(b).—To inscribe 2 EQUAL CIRCLES, touching each other, in an ISOSCELES TRIANGLE, each circle to touch 2 sides of the triangle (Fig 224):—Draw the altitude **N O.** Inscribe a circle in triangle **L O N.** Draw **P Q** parallel to **L M,** making **R Q = R P.** With centre **Q,** and same radius as for circle **P,** draw the circle.

N.B.—PROBS. **115, 116, 117, 118,** ARE BASED UPON PROB. 114.

PROBLEM 115.—To inscribe six equal Circles in an Equilateral Triangle, K L M.



1.—Draw **L O, M N,** the *bisectors* of the angles **K L M, L M K,** and through **P,** their point of intersection, draw **K Q.** The triangle **K L M** is then divided into 3 *equal and similar* triangles, **L P M, M P K, K P L.**

2.—Find the centre of one of the triangles, say **L P M.** The angle at **P** is already bisected by **P Q.** Bisect angle **P L M** by **L R.** Point **R** is the *centre of one of the circles.*

3.—With centre **P,** and radius **P R,** mark points **S** and **T.** With centre **R,** and radius **R T,** mark point **V.**

4.—With centre **P,** and radius **P V,** obtain points **W** and **X.** Then, with *centres* **X, S, W, T, V, R,** and *radius* **R Q,** inscribe the required SIX CIRCLES.

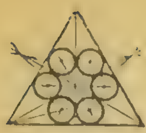
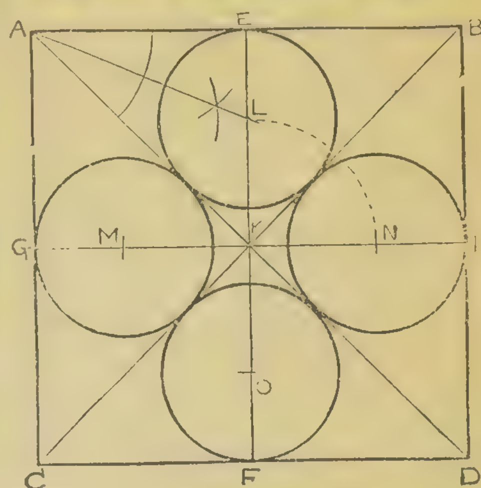


FIG. 225.

Note.—W, X, V, can be got by drawing through R, S, T, lines *parallel* to L M, L K, K M.——If we have to inscribe only *three* equal circles, touching one another, and *one* side of the triangle, the circles R, S, T, are those required.——Fig. 225 shews another arrangement of 6 equal circles in an equilateral triangle. If we insert another equal circle in the centre, and one at each angle of the triangle, we inscribe *ten* equal circles.

PROBLEM 116.—Inscribe four equal Circles within a Square, C D B A—each circle to touch only one side of the square



1.—Draw the *diagonals* **A D, B C,** and the *diameters* **E F, G H.**

2.—The *diagonals* divide the square into 4 *equal and similar* triangles. Inscribe a circle in each triangle.

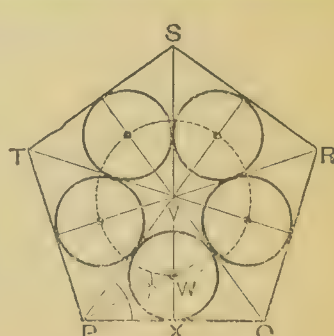
3.—In the triangle **A B K** the angle **A K B** is already bisected by **K E.** Bisect angle **K A B** by **A L.** Point **L** is the *centre of one of the circles.*

4.—With centre **K,** and radius **K L,** mark points **N, O, M.** With centres **L, N, O, M,** and radius **L E,** inscribe the required FOUR CIRCLES.

FIGS. 226. 227. 228.

Note.—If with centre K, and radius K E, we draw a circle, it *circumscribes* the four circles.——Fig. 226 gives 2 circles inscribed in a square. Fig. 227 shews that by repeating the construction of the last Fig., we inscribe 8 circles in a square. Fig. 228 shews that by repeating the construction given in Fig. 224, we inscribe 4 circles in a **rhombus.**

PROBLEM 117.—In ANY Regular Polygon to inscribe as many equal Circles as the polygon has sides—each circle to touch only one side of the polygon.



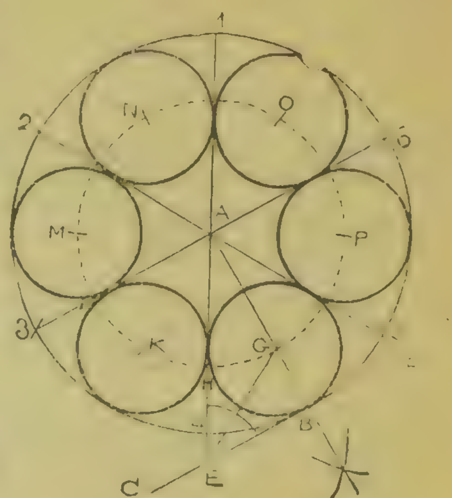
1.—Say in the PENTAGON **P Q R S T.** Bisect each side and draw lines from the middle points to the opposite angles. These lines intersect at **V,** and divide the pentagon into 5 equal *isosceles triangles,* **P Q V, Q R V,** &c.

2.—Bisect angle **V P Q,** by **P W.** Then **W** is the *centre of one of the circles.* With centre **V,** and radius **V W,** draw a circle, which gives the *centres* of the inscribed circles. With radius **W X,** inscribe the required FIVE CIRCLES.

FIG. 229.

Note.—When the polygon has an *even* number of sides, draw all the longest diagonals and diameters.—— To inscribe *half* as many Circles in a Regular Polygon as the polygon has sides, say in an Octagon—each circle to touch *one* side of the polygon (Fig. 229) : —This can be done only when the polygon has an *even* number of sides. Bisect each *side* and draw the diameters. Produce a side and a diameter to meet at Y. Bisect angle Y, and proceed as shewn.

PROBLEM 118.—Within a given Circle to inscribe ANY number of equal Circles—say six.



1.—Divide the circumference into as many *equal* parts, as there are to be *circles* —in this case *six*. Mark off the *radius* at **1, 2, 3,** &c.

2.—Draw the diameters **1 4, 2 5, 3 6,** which divide the circle into 6 equal *sectors*.

3.—Bisect one of the sectors. Say, bisect **4 A 5** by the line **A B.**

4.—Through **B,** draw **C D,** perpendicular to **A B.** Produce **A 4, A 5** to **E** and **F.**

5.—Bisect angle **A E F** by **E G.** Point **G** is the *centre of one of the circles*. With centre **A,** and radius **A G,** draw the dotted circle.

6.—Make **H K, L M,** &c., each equal **H G.** With centres **G, K, M,** &c., inscribe the required SIX CIRCLES.

Note (a).—The above problem shews how to inscribe a CIRCLE in a given **sector 4 A 5.** The *triangle* E A F is drawn in full to shew more clearly the principle of construction.——A line joining the centres of any two of the adjacent circles, gives their *point of contact*.——With centre A, and radius G B, another circle can be drawn touching the 6 circles, thus getting 7 circles. We can inscribe 6 *smaller* circles in the central space.

(b).—Suppose **3 4 5** to be any given ARC, and **A E** a given LINE. It is required to describe a circle which shall touch **A E,** and the arc at a given point **B.** Join **B A.** Draw the tangent **B C.** Bisect angle **B E A** by **E G.** Then **G** is the centre, and **G B** the radius, of the required CIRCLE.

PROBLEM 119.—To inscribe a Circle within a Trapezion M O N L.

1.—Join **M N.** Bisect angle **M O N** by **O P.**

2.—Draw **P Q,** perpendicular to *any* side.

3.—With centre **P,** and radius **P Q,** inscribe the required CIRCLE.

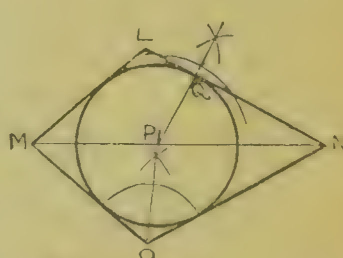


FIG. 230.

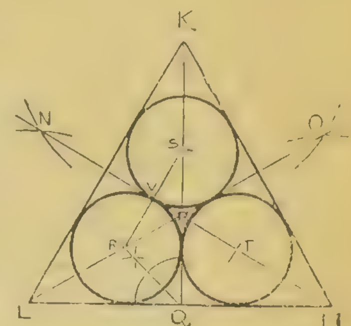
Note (a).—A circle cannot be *circumscribed* about a trapezion, except the angles at L and O are *right angles*; and then the bisection of **M N** gives the centre.——To inscribe a SEMI-CIRCLE in a triangle **M L N**:—Bisect angle L, and get point **P.** Draw **P Q,** perpendicular to L N; &c. Such semi-circles cannot be drawn on *all* the sides of some triangles.

(b).—To inscribe a CIRCLE in a RHOMBUS **A B C D** (Fig. 230):—The intersection of the *diagonals* gives the centre of the circle **E.** Draw a perpendicular **E F** from the centre to one side. With centre **F,** and radius **E F,** inscribe the CIRCLE.

(c).—To circumscribe about a given CIRCLE, E, a RHOMBUS, having two opposite angles, each equal to a *given angle* D A B:—Draw two diameters to the given circle, making, at their intersection, an angle equal to the given angle D A B. Through the extremities of these diameters, draw lines perpendicular to the diameters. Produce these tangents until they meet in four points. We then construct a circumscribed RHOMBUS, having two opposite angles, each equal to the given angle D A B.——The only parallelograms that can be circumscribed about a *circle* are the *square* and the *rhombus*.

N.B.—PROBS. 120, 121, ARE BASED UPON PROB. 119.

PROBLEM 120. To inscribe three equal Circles within a given Equilateral Triangle K L M -each circle to touch two sides of the triangle.



1.—Bisect angles **L** and **M**, by lines **L O** and **M N**, which intersect at point **P**.

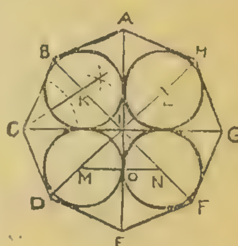
2.—Through **P**, draw **K Q**. The triangle is then divided into 3 *equal and similar trapezions.*

3.—Bisect angle **L Q P**, by the line **Q R**. Point **R** is the *centre of one of the circles.* Make **P S, P T**, each equal to **P R**. Join **R S.**

4.—With centres **R, S,** and **T**, and radius **R V**, inscribe the required THREE CIRCLES.

Note (a).—In Problems 115, 120, a set-square of 30° will give the lines L O and L M.
(b).—*We can inscribe equal tangential* CIRCLES *in* ANY REGULAR POLYGON—*each circle to touch* TWO *sides of the polygon—by first dividing the polygon into as many equal* TRAPEZIONS, *as the polygon has sides, and then proceeding as above.*

PROBLEM 121.-In ANY Regular Polygon, having an even number of sides, inscribe half as many equal Circles as the polygon has sides - each circle to touch 2 sides of the polygon.



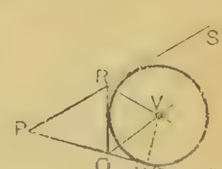
1.—Say in an OCTAGON. Draw the diagonals. The alternate diagonals divide the octagon into 4 *equal and similar trapezions.*

2.—Bisect the angle **B C I**, by line **C K**. Then point **K** is the *centre of one of the circles.*

3.—Make **I L, I N, I M**, each equal **I K**. Join **M N.** With centres **K, L, N, M,** and the radius **M O**, inscribe the required FOUR CIRCLES.

Note.—For practice, insert 3 such equal circles in a regular hexagon, 5 in a regular decagon, and 6 in a regular dodecagon.

PROBLEM 122.—To draw an Exscribed Circle to touch any side of a given Triangle P Q R.



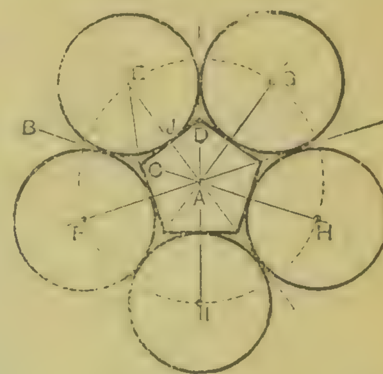
1.—Say to touch **Q R**. Produce **P R, P Q,** indefinitely, to **S** and **T.**

2.—Bisect the *exterior* angles **Q R S, R Q T**, by lines intersecting at **V**. From **V**, draw a perpendicular to **P S** or **P T**, as **V W** With centre **V**, and radius **V W**, draw the EXSCRIBED CIRCLE. (*See Prob.* 73.)

Note.—If the exscribed circle is to touch the *base* of an *Isosceles* or *Equilateral Triangle*, the centre of the circle can be obtained where the bisector of *one* of the *exterior* angles, cuts the *altitude produced.*

N.B.—PROBS. **123, 124,** ARE BASED UPON PROB. **122.**

PROBLEM 123.—To draw a number of equal Circles around a Regular Polygon, touching one another, successively, and the sides of the polygon.



1.—Say about a PENTAGON. Through **A**, the *centre* of the polygon, draw lines of indefinite length through the angles.

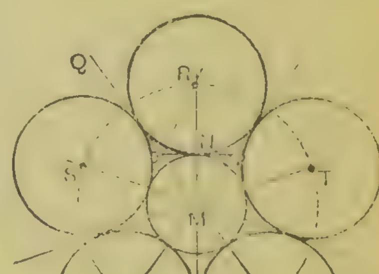
2.—Bisect angle **B C D** by **C E.** With centre **A**, and radius **A E,** draw a circle.

3.—With centres **E, F, G,** &c., and radius **E J,** draw the required CIRCLES.

FIG. 231

Note.—C D A is an *isosceles* triangle, and the altitude A J is produced through E. (*See note, Prob.* 122.)——For polygons having an *even* number of sides, the sides must be bisected, and radials drawn from the centre through these middle points, as well as through the angles.——For a HEXAGON (Fig. 231), the centres of the circles can be found by producing the sides to meet in points J, K, L, &c. Or, J, K, L can be got by intersecting arcs, with the angles for centres, and a side for radius.——Equal circles can also be drawn around an **equilateral triangle, a square,** or a **rhombus.** Radials must be drawn from the centre of each figure.

PROBLEM 124.—To draw ANY number of equal Circles around a Circle, touching one another, successively, and the given circle.



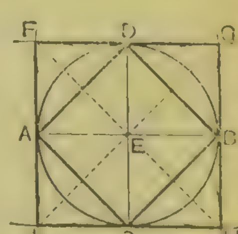
1.—Say draw 5 CIRCLES. Divide the circumference of the given circle into *twice* as many equal parts as there are circles required. (*See note* a, *Prob.* 52.)

2.—From **M**, the centre, draw radials through these points. Through **N**, draw the *tangent* **O P**, *i.e.*, make **O P** *perpendicular* to **M N** (*with a set-square*). Bisect angle **Q O P** by **O R.**

3.—With centre **M**, and radius **M R**, draw a circle. With centres **S, R, T,** &c., and radius **R N,** draw the required CIRCLES.

Note.—This problem is identical with Prob. 123, the tangent O P being one side of a pentagon that would circumscribe the circle M.

PROBLEM 125.—To inscribe, or circumscribe, a Square in or about a given Circle.

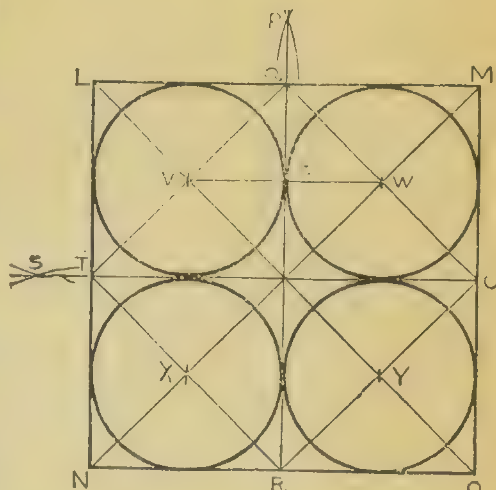


1st.—TO INSCRIBE A SQUARE:—Draw 2 diameters **A B, C D,** at right angles to each other. Join **A D, D B, B C, C A.** Then **A D B C** is the required INSCRIBED SQUARE.

2nd.—TO CIRCUMSCRIBE A SQUARE:—Draw **A B, C D,** as before. With centres **A, D, B, C,** and radius **E A,** strike arcs intersecting at **F, G, H, I.** Join **F G, G H, H I, I F.** Then **F G H I** is the required CIRCUMSCRIBED SQUARE.

Note.—To INSCRIBE A CIRCLE within the SQUARE **F G H I**:—Draw the diagonals **F H**, **I G**. Draw **E A** perpendicular to **F I**. With centre **E**, and radius **E A** inscribe the CIRCLE. —To CIRCUMSCRIBE A CIRCLE about the SQUARE **A D B C**:—Draw the diagonals **A B**, **C D**. With centre **E**, and radius **E A**, draw the CIRCUMSCRIBED CIRCLE. By the same method, we can circumscribe a *circle* about a RECTANGLE.

PROBLEM 126. To inscribe four equal Circles within the Square N O M L—each circle to touch two sides of the square.



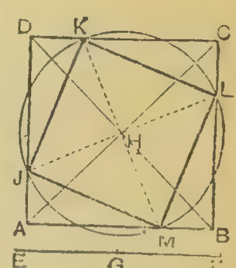
1.—Draw the diagonals **L O**, **N M**. With centres **N**, **L**, and **M**, and any radius, strike arcs intersecting at **S** and **P**.

2.—From **S** and **P**, draw lines through the centre of the square. The diameters **T U**, **Q R**, divide **N O M L** into 4 *equal squares*.

3.—Draw **T Q**, **Q U**, **U R**, **R T**. Join **V W**. With centres **V**, **W**, **Y**, **X**, and radius **V Z**, inscribe the required FOUR CIRCLES.

Note.—*To circumscribe* a SQUARE *about the* SQUARE **T Q U R**:—Draw the diagonals **Q R**, **T U**. With centres **T**, **Q**, **U**, **R**, and a radius equal to *half* the diagonal **Q R**, strike arcs intersecting at **L**, **M**, **O**, **N**. Join **N O**, &c. Then **N O M L** is the required *circumscribed* SQUARE.

PROBLEM 127.—In a given Square A B C D, inscribe a Square, having diagonals equal to the given line E F.

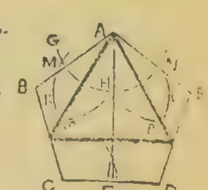


1.—Bisect **E F** at **G**. Draw the diagonals **A C**, **D B**. With centre **H**, and radius **G E**, draw a circle cutting the sides of the square at **J**, **K**, **L**, **M**.

2.—Join **J K**, **K L**, **L M**, **M J**. Then **J K L M** is the required SQUARE. The diagonals **J L**, **K M**, = **E F**.

Note.—If required to inscribe in a given SQUARE, **A B C D**, another SQUARE, having *one of its angles to touch a given point* **K**, draw diagonals **A C**, **D B**. With centre **H**, and radius **H K**, describe the circle. Draw the required SQUARE **J K L M**.——Inscribed and circumscribed squares, one within or about the other, are *concentric*, because they have the *same centre*. J D K, K C L, L B M, M A J, are *equal right-angled triangles*.

PROBLEM 128.—To inscribe an Equilateral Triangle in a Square, or ANY Regular Polygon—say in the Pentagon A B C D E.

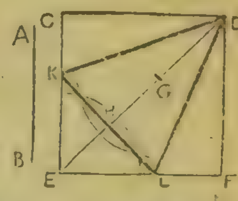


1.—Bisect angle **B A E** by **A F**. With centre **A**, and *any* radius, strike arc **G**. With *same* radius, and centre **H**, strike arcs **K** and **L**. With centres **M** and **N**, and *same* radius, obtain points **O** and **P**.

2.—Through **O** and **P**, draw **A O**, **A P**. Complete the EQUILATERAL TRIANGLE. (*See Prob.* 20.)

Note.—To inscribe an equilateral triangle in a regular *hexagon*, either join the alternate angles, or join the middle points of the alternate sides.

PROBLEM 129.—In a given Square E F D C, inscribe an Isosceles Triangle, having a given base of convenient length—say equal to A B.



1.—Draw **E D.** Make **E G** equal to the base **A B.**

2.—Bisect **E G** at **H,** by **K L.** Join **D K, D L.** Then **K L D** is the required TRIANGLE.

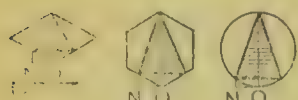
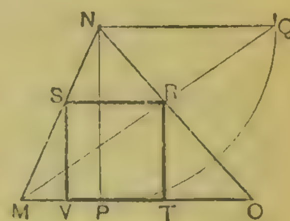


FIG. 232.

Note (a).—Fig. 232 shews how to inscribe an isosceles triangle, having a base = M, in a RHOMBUS, POLYGON, or CIRCLE. In each case **N O** = ½ **M** The rest of the construction is obvious. Prob. 129 can be solved in a similar way.——If the length of one of the equal *sides* is given, as **D K,** with centre **D,** and radius **D K,** get **K** and **L**; &c. Proceed in the same manner with an *isosceles triangle* in a RHOMBUS, POLYGON, or CIRCLE.

(b).—To *circumscribe* a SQUARE about an ISOSCELES TRIANGLE **K L D** :—Draw the altitude **D H,** produce it to **E,** making **H E** = **H K.** Then **D E** is a *diagonal* of the required SQUARE. If the *vertical angle* of the isosceles triangle is not *less* than 90°, the problem is unworkable.

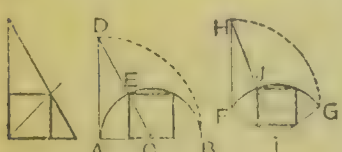
PROBLEM 130.—To inscribe a Square in ANY Triangle, M O N.



1.—Draw the *altitude* **N P.** From **N,** draw **N Q** = **N P,** and perpendicular to **N P.**

2.—Join **M Q.** Point **R** gives the position of one of the angles of the square.

3.—Draw **R S,** parallel to **M O; R T, S V,** parallel to **N P.** Then **V T R S** is the required SQUARE.



FIGS. 233. 234. 235.

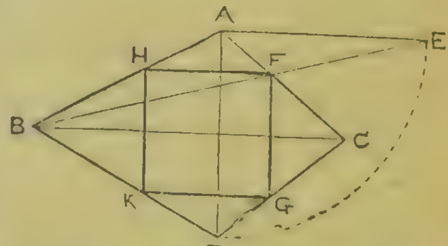
Note (a).—In a right-angled triangle (Fig. 233), the bisector of the right angle gives the position of an angle of the square, on the hypotenuse. This is the *largest* square that can be inserted in a right-angled triangle.——Fig. 234 shews the construction for inscribing a SQUARE in a **semi-circle** or other **segment.** Bisect the chord **A B** at **C.** Make **A D** = **A B,** and perpendicular to it. Join **C D.** An angle of the square falls on **E.**

(b).—Fig. 235 shews how a SQUARE is inscribed in a **quadrant** or other **sector.** Join **F G.** Make **F H** = **F G,** and perpendicular to it. Join **H I,** which gives point **J**; &c.

N.B.—PROBS. **131, 132,** ARE BASED UPON PROB **130.**

PROBLEM 131.—Inscribe a Square in a Trapezion A B D C.

1.—Draw the diagonals **A D, B C.**



2.—Draw **A E,** *perpendicular* to **A D,** and make **A E** *equal* to **A D.**

3.—Join **E B.** Then point **F** is the position of one angle of the square.

4.—Draw **F G, F H,** parallel to **A D, B C.** Draw **H K,** parallel to **A D.** Join **K G.** Then **K G F H** is the SQUARE.

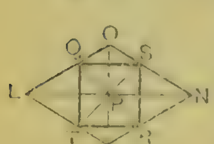
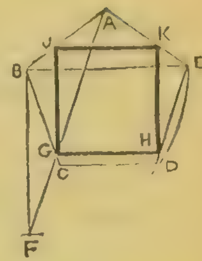


FIG. 236.

Note.—To inscribe a SQUARE in a RHOMBUS **L M N O** (Fig. 236):—This can be solved by the above problem, but the following *special* method is simpler in construction :—Draw the diagonals **L N, O M.** Bisect angles **L P O, O P N,** by the lines **Q R, S T.** Join **T R, R S, S Q, Q T.** Then **T R S Q** is the inscribed SQUARE. Adopt the same method, when inscribing a *square* in a *square.*

PROBLEM 132. To inscribe a Square within a Regular Pentagon A B C D E.



1.—Join the alternate angles **B, E.** Draw **B F,** perpendicular, and equal, to **B E.**

2.—Join **F A.** Then **G** is the position of one angle of the square.

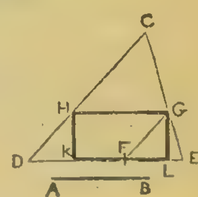
3.—Draw **G J,** parallel to **B F;** and **G H, J K,** parallel to **B E.** Join **H K.** Then **G H K J** is the required SQUARE.

FIG. 237.

Note (a).—If we produce B C, E D, to meet in a point X, the figure X B A E is a *trapezion.* Therefore, the construction of this problem is identically the same as in Prob. 131.

(b).—To inscribe a SQUARE in a HEXAGON **A B C D E F** (Fig. 237):—Draw one of the longest diagonals **A D,** and bisect it at **I,** by the line **G H.** Bisect angles **G I A, A I H,** by lines **K L, M N.** Join **K N, N L,** &c. Then **M L N K** is the required SQUARE. Adopt the same construction for ALL polygons having an *even* number of sides.

PROBLEM 133.—In a given Triangle, D E C, inscribe a Rectangle, having two sides equal to a given line A B.



1.—Set off **D F,** equal to **A B.** Draw **F G,** parallel to **D C.**

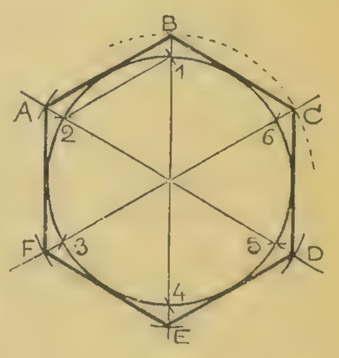
2.—Draw **G H,** parallel to **D E.** Draw **H K, G L,** perpendicular to **D E.** Then **K L G H** is the required RECTANGLE.

Note (a).—D F G H is a *rhomboid,* having sides D F, H G=A B.

FIGS. 238. 239. 240.

(b).—As applications of the above principle of construction, Fig. 238 shews the construction for inscribing a *rectangle,* with a given side L, in a SQUARE; Fig. 239, in a TRAPEZION; and Fig. 240, in a HEXAGON. In each figure, the thick line indicates where the length L is to be set off. Adopt a similar construction for a RHOMBUS, and any REGULAR POLYGON.——To inscribe a *rectangle* in a CIRCLE, see Prob. 40.——By the construction shewn in Fig. 239, we can inscribe in *any trapezium,* a parallelogram having two sides of a given length. This parallelogram will not be *rectangular,* except the diagonals of the trapezium intersect at *right angles.*

PROBLEM 134.—About a given Circle to circumscribe ANY Regular Polygon—say a Hexagon.



1.—Divide the circumference into as many equal parts, as the polygon is to have sides—in this case *six.* (*By Prob.* 49.)

2.—Draw radii to **1, 2,** &c., and produce them.

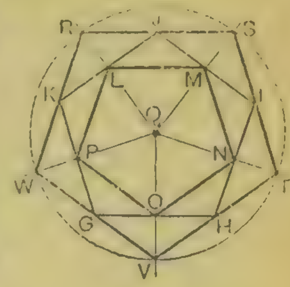
3.—Join **1 2.** Draw **A B,** parallel to **1 2,** as a *tangent* to the circle.

4.—Take the distance from the centre of the circle to **A,** and mark it off from the centre to points **C, D, E, F.**

5.—Join **B C, C D, D E,** &c. Then **A B C D E F** is the required HEXAGON.

Note.—If an OCTAGON, or a DODECAGON be required, divide the circumference by Probs. 50 and 51. For other polygons, divide the circumference by Prob. 48.——A line drawn from the centre of a polygon, perpendicular to one of its sides, is the radius of an *inscribed* circle. The distance from the centre to an angle is the radius of a *circumscribed* circle. A polygon and its inscribed or circumscribed circle have the same centre.

PROBLEM 135.—Within or about ANY Reg. Polygon—say the Pentagon G H I J K—construct a similar Reg. Polygon.



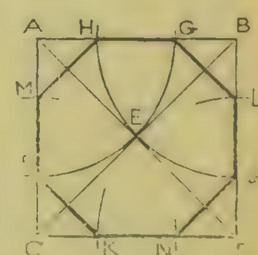
1st.—To INSCRIBE a pentagon:—Bisect each side, at **L, M, N, O, P.** Join **L M, M N,** &c. Then **L M N O P** is the required *inscribed* PENTAGON.

2nd.—To CIRCUMSCRIBE a pentagon:—From the centre **Q,** draw radials through **L, M, N,** &c. Through **J,** draw **R S,** parallel to **L M.** With centre **Q,** and radius **Q R,** strike a circle, cutting the radials at **T, V, W.** Join **S T, T V,** &c. Then **R S T V W** is the required *circumscribed* PENTAGON.

FIG. 241.

Note.—As the above polygons have a common centre, they are said to be *concentric* polygons.——If we set off an *equal* distance of *any* length, from each angle of a regular polygon, as at X, O, O, &c. (Fig. 241), and join these points, we inscribe a similar polygon. Point X may be a *given* point.——To inscribe a similar polygon (of an *even* number of sides), having a given length for its *longest* diagonal:—With a radius Y X, equal to *half* the given diagonal, and centre Y, strike a circle which will give points O, O, &c. (Fig. 241). This construction is the same as that used in Prob. 127.

PROBLEM 136.—Inscribe an Octagon in a Square C D B A.



1.—Draw the diagonals **A D, B C,** intersecting at **E.**

2.—With centres **A, B, D** and **C,** and radius **A E,** strike arcs **F G, H J, K L, M N.**

3.—Join **M H, G L, J N, K F,** and we obtain the required OCTAGON.

Note.—By this problem we can construct an OCTAGON, having a given *diameter*. First, construct a *square* having sides equal to the given diameter. Then inscribe the required octagon in the square.

General Notes about Inscribed and Circumscribed Figures.

190.—If we adhere strictly to Euclid's definitions concerning INSCRIBED and CIRCUMSCRIBED figures we should not say "INSCRIBE *a square in a triangle*" (as in Prob. 130), because one *side* of the square falls upon one of the *sides* of the triangle. It has, however, become so customary to use the terms "inscribe" and "circumscribe" in this loose manner, that it was felt necessary to adopt them in their wider meaning for several of the foregoing problems.

191.—If we draw through the centre, X, of *any* parallelogram A B C D, lines E F, G H, at *right angles* to each other, and join their extremities, we construct a RHOMBUS (Fig. 242). Thus, a rhombus can be inscribed in a parallelogram, with one of its angles touching a *given point*, E, or G.

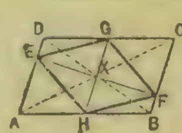


FIG. 242.

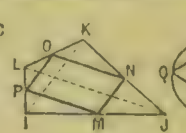


FIG. 243.

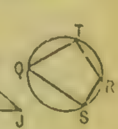


FIG. 244.

192.—If we join the middle points of the sides of *any* quadrilateral figure, I J K L, as at M, N, O, P (Fig. 243), we construct a PARALLELOGRAM, which is half the size of the whole figure. The sides of M N O P are parallel to the diagonals of I J K L. If the diagonals of the given figure intersect at *right angles*, the inscribed figure is a *rectangle*.

193.—The *opposite* angles of *any* quadrilateral figure inscribed in a circle, are together equal to two right angles. (*Euclid* III. 22.) Thus, in Fig. 244, the angles Q + R = 180°, and S + T = 180°. It is only about such proportioned quadrilateral figures that we can circumscribe circles. (*See note, Prob.* 85.)

194.—Many problems, connected with *inscribed and circumscribed figures*, not given in

this chapter, can be solved if this and previous chapters be well understood. The diagrams shown in Figs. 245 to 251 should be worked out by the student.

FIGS. 245. 246. 247. 248. 249. 250. 251.

Work out the Exercises on INSCRIBED AND CIRCUMSCRIBED FIGURES.

CHAPTER XI. FOILED FIGURES.

CLASS-SHEET 12.

DEFINITIONS, etc.:—195.—Figs. 252, 253, 254, are examples of **Foiled Figures,** such as we find in Gothic tracery and other ornamental forms. If there are 3 lobes, or *foils*, the shape is called a **Trefoil** (Fig. 252); 4, a **Quatrefoil** (Fig. 253); 5, a **Cinquefoil** (Fig. 254). Figures having more than 5 foils are said to be **Multifoiled** or **Polyfoiled.**

FIGS. 252. 253. 254.

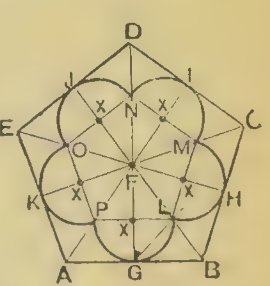
196.—The pointed junction, turned inwards, of each pair of foils is called a **Cusp,** as at A (Fig. 254).

197.—The foils may consist of simple or compound curves, and they are usually —but not necessarily—formed with arcs of circles. Arcs and straight lines are frequently used in combination, as at M, Fig. 265.

PROBLEM 137.—In ANY Regular Polygon, inscribe as many equal Semi-circles (having adjacent diameters) as the polygon has sides—each arc to touch one side of the polygon.

1.—Say in the PENTAGON **A B C D E.** Bisect each side, at **G, H,** &c., and from these middle points draw lines to the opposite angles, intersecting at the centre **F,** as lines **G D, H E,** &c.

2.—Make angle **F G L** = **45°.** With centre **F,** and radius **F L,** get points **M, N,** &c. Join **L M, M N,** &c. With centres **X, X,** &c., and radius **X L,** strike the required SEMI-CIRCLES.



Note (a).—The arcs of the 5 semi-circles form a **cinquefoil.** The sides of the polygon are *tangents* to the arcs. When 2 lines, as M N, O N, meet in a point N, they are said to be *adjacent:* hence, the semi-circles have *adjacent diameters*

(b).—The principle on which this problem is constructed, is that we divide the polygon into as many equal ISOSCELES triangles as the polygon has sides; and *in each triangle we inscribe a* SEMI-CIRCLE, *as in* A B F. For polygons that have an *even* number of sides, join the opposite angles, and also the middle points of the opposite sides.

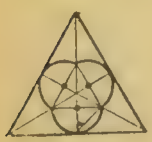


FIG. 255.

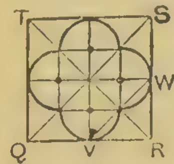
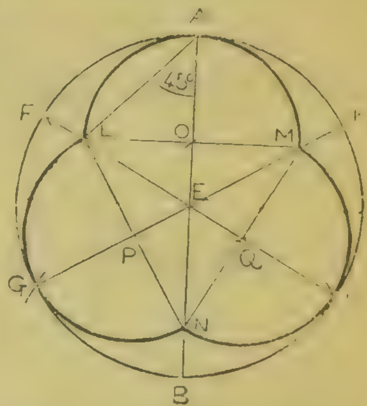


FIG. 256.

(c).—Adopt the same construction for inscribing a **trefoil** of equal semi-circles in an EQUILATERAL TRIANGLE (Fig. 255); or for a **quatrefoil** of equal semi-circles in a SQUARE, Q R S T (Fig. 256). In a square the angle of 45° may be obtained by joining V W.

PROBLEM 138.—In a given Circle to inscribe ANY number of equal Semi-circles, having their diameters adjacent.



1.—Say inscribe 3 SEMI-CIRCLES. Divide the circumference into *twice* as many equal parts as there are to be semi-circles, *i.e.*, 6 equal parts, at **A, K, H, B, G, F.**

2.—Join **A B, F H, G K.** Make the angle **E A L** = 45°. With centre **E,** and radius **E L,** mark points **M** and **N,** on the *alternate* diameters.

3.—Join **L M, M N, N L.** With centres **O, P, Q,** and radius **O L,** inscribe the required SEMI-CIRCLES.

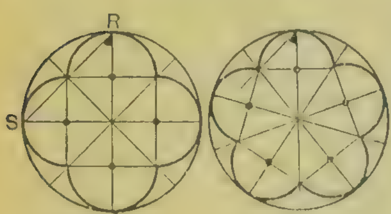


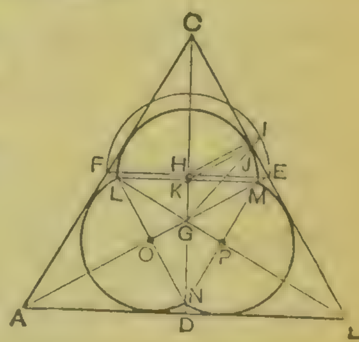
FIG. 257. FIG. 258

Note (a).—The arcs of the 3 semi-circles form a **trefoil.** The semi-circles *touch* the circumference of the circle at points A, G, H.

(b).—The principle on which this problem is constructed is identical with that of Prob. 137. We divide the circle into as many equal SECTORS as there are to be semi-circles, and *in each sector we inscribe a* SEMI-CIRCLE, *as in* E F A K.

(c).—Fig. 257 shews how to inscribe a **quatrefoil,** and Fig. 258, a **cinquefoil,** of equal semi-circles, in a circle. For a *quatrefoil,* the line R S gives an angle of 45°.

PROBLEM 139.—Within an Equilateral Triangle, A B C, to inscribe three equal Semi-circles, having adjacent diameters —each arc to touch two sides of the triangle.



1.—Bisect the sides, at **D, E, F.** Draw **D C, E A, F B,** intersecting at the centre **G.** Join **F E.** With centre **H,** and radius **H F,** describe a semi-circle.

2.—Draw **H I,** perpendicular to **C B.** Join **I G,** cutting **C B** at **J.** Draw **J K,** parallel to **H I.**

3.—Through **K,** draw **L M,** parallel to **F E.** Make **G N = G L.** Join **L N, N M.** With centres **K, O, P,** and radius **K L,** or **K J** inscribe the required SEMI-CIRCLES.

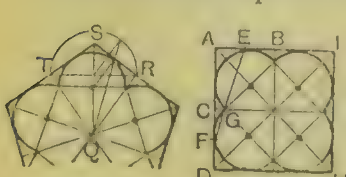


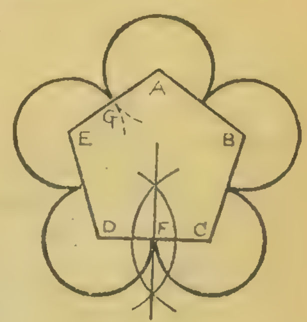
FIG 259. FIG. 260.

Note (a).—The arcs of the 3 semi-circles form a **trefoil.** The sides of the triangle are *tangents* to the arcs.——The principle on which this problem is constructed, is that we divide the given figure into as many equal and similar TRAPEZIONS, as the figure has sides, as G E C F, &c., and *in each trapezion we inscribe a* SEMI-CIRCLE.

(b).—Adopt a similar construction for **ANY Regular Polygon.** Thus, for a PENTAGON (Fig. 259), divide it into 5 *trapezions,* inscribe a semi-circle in each, as in **Q R S T,** and obtain a **cinquefoil.** For a polygon that has an *even* number of sides, join the middle points of the opposite sides, and join the opposite angles.

(c).—By Prob. 139 we can also inscribe 4 equal semi-circles in a **Square**, but the *special* method for a SQUARE, shewn in Fig. 260, is simpler in construction. Bisect **A B**, **C D**, at **E** and **F**. Join **E F**, which gives point **G**, the end of one of the diameters. Complete the diagram, and get a **quatrefoil.**

PROBLEM 140.—About ANY Regular Polygon, to construct a Foiled Figure having tangential arcs of circles.



1.—Say about the given PENTAGON **A B C D E.** Bisect one side **D C**, at **F.**

2.—With centres **A, B, C, D, E,** and radius **D F,** describe the 5 arcs, which form a CINQUEFOIL.

FIGS. 261. 262. 263.

Note (a).—The arcs are *tangential*; because, if continued, as at G, they do not intersect.——If required to construct a foiled figure of a given number of foils, having tangential arcs, each arc to have a *given radius*, say a CINQUEFOIL, with radius for arcs = D F, construct the necessary polygon having sides *twice* the length of the given radius, and proceed as above.

(b).—Figs. 261, 262, shew how a **trefoil**, with tangential arcs, can be constructed about an EQUILATERAL TRIANGLE; and a **quatrefoil,** about a SQUARE. Fig. 263 shews how a foiled figure of equal *semi-circles*, having *adjacent diameters*, can be constructed *about* a square or a polygon.

General Notes about Foiled Figures.

198.—Problems connected with Inscribed, Circumscribed, and Foiled Figures, play a most important part in Ornamental Design. They enable the architect and decorator to construct the leading lines for tracery, and other ornamental forms.

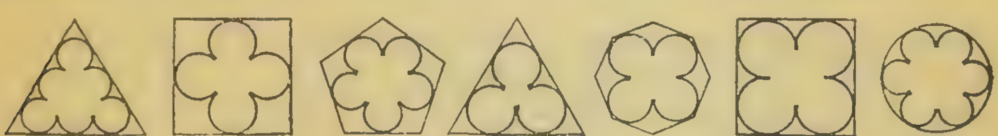


FIG. 264. A B C D E F G

199.—The diagrams in Fig. 264 shew how foiled figures can be constructed by means of some of the problems in Chapter X. Thus, in Probs. 115, 116, 117, 120, 121, 126, if we draw lines *joining the centres of the circles*, and erase all within these lines, we obtain foiled figures similar, respectively, to A, B, C, D, E, F. By Prob. 118, if we join the centres of the small circles, we can obtain a figure containing *any* number of equal foils within a circle, as at G. The figures thus obtained are formed with **tangential arcs of circles.**——A *trefoil* of tangential arcs can be made out of Fig. 245.——By joining the centres of the circles in Figs. 246, 247, we obtain foiled figures formed with arcs of *different* radii. Arcs of circles can be arranged so as to produce foiled shapes of *infinite variety.*

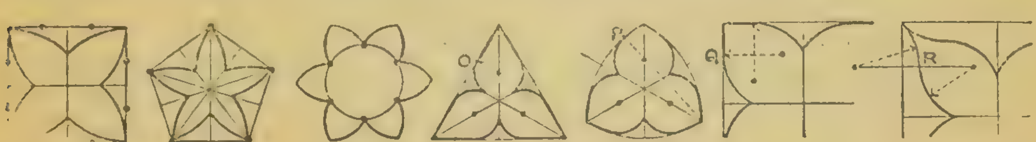


FIG. 265. H I J K L M N

200.—In Fig. 265 we have several examples of *pointed foils.* In each case the centres of the arcs are indicated by black dots. In K, the arc touches the straight line at O. In L, the small arc touches the large arc at P. At M and N, are shewn portions of quatrefoils. At M, the arc touches the straight line at Q. At N, each side of the foil is formed by a compound curve, or ogee, the arcs touching at R.

Work out the Exercises on FOILED FIGURES.

CHAPTER XII. THE ELLIPSE.

CLASS-SHEET 13.

DEFINITIONS, etc.:—201.—An Ellipse is a plane figure, contained by one continuous curved line (Fig. 266). Its greatest length, A B, is called the **Major** or **Transverse Axis,** or the **Longest Diameter.** Its greatest width, C D, is called the **Minor** or **Conjugate Axis,** or the **Shortest Diameter.** The major and minor axes bisect each other at *right angles* in a point E, called the **Centre.** *Any* line drawn through E, and terminated by the curve at each extremity, is a *diameter*, as H M, or N O; but A B is the longest, and C D, the shortest diameter that can be drawn in an ellipse. *All* diameters are bisected at E, and each diameter divides the ellipse into two equal and similar parts.

202.—On the major axis, A B, equidistant from E, are situated two *fixed* points F and G. Each of these points is called a **Focus** (plural *foci*). The positions of the foci are regulated by the length of each axis. They are so placed that, if from any point, H, in the curve we draw lines to each focus, *the two lines are together equal to the length of the major axis.* Thus, H F, H G, together equal A B.

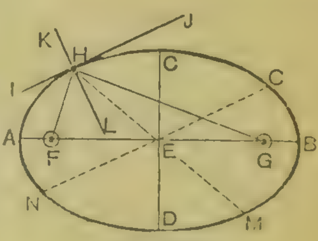


FIG. 266.

203.—A Tangent to an ellipse is a straight line that touches the curve in *one* point, as the line I J, at H, the *point of contact.*

204.—A Normal to an ellipse is a straight line, K L, that is perpendicular to a tangent, I J, at the point of contact H. A normal is frequently called a PERPENDICULAR to the ellipse. The angle, F H G, is *bisected* by the normal K L.

205.—The major and minor axes of an ellipse divide the figure into four equal and similar parts, as A E C, C E B, B E D and D E A.

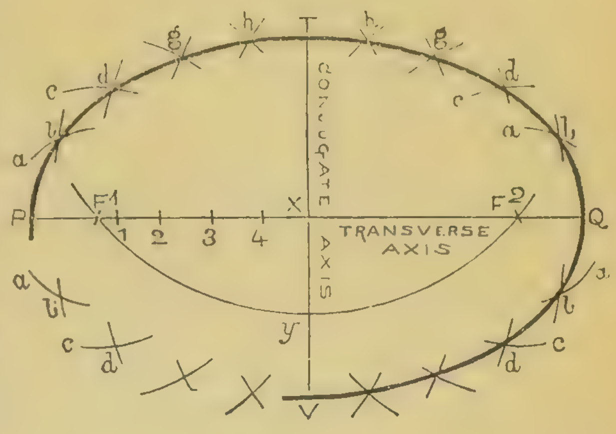
206.—If from *any* point H in the curve, we draw a diameter H M, and then draw another diameter N O, *parallel* to the tangent drawn through H, such lines are called **Conjugate Diameters.**

207.—Ellipses can be drawn in *infinite* variety, as to length and width. The major and minor axes may have any imaginable difference in length. They can never be *equal*, but the nearer they are to each other in length, the closer does the ellipse approach the form of a circle. Only *one* elliptic curve belongs to any two given major and minor axes. By holding a penny, first in nearly a vertical position, and then gradually turning it round, until hardly any of the surface is visible, the circular edge of the coin presents every conceivable change in the form of an ellipse. *No portion* of the curve of an ellipse coincides with an arc of a circle.

208.—An ellipse is often inaccurately called an *oval;* but the latter is a distinct curvilineal figure, something like an ellipse, but broader at one end than at the other, like the shape of an egg.

209.—A Periphery is the boundary line of a *curvilineal figure.* Thus, the periphery of a circle is its circumference; and the periphery of an ellipse is its curved boundary.

PROBLEM 141.—To find the Foci of an Ellipse; and then to draw the Elliptic Curve by means of intersecting arcs, the major axis P Q, and minor axis T V, being given.



1.—With **T**—one end of the *minor* axis—as centre, and **X Q**—*half* the *major* axis—as radius, strike arc **y**, cutting the major axis at F^1, F^2. *These points are the required* FOCI.

2.—Between F^1 and **X**, mark *any* number of points **1, 2, 3, 4.**

3.—With centres F^1, F^2, and radius **P 1**, strike arcs **a, a, a, a.** With the *same* centres, and radius **Q 1**, cut arcs **a, a, a, a,** at **b, b, b, b.**

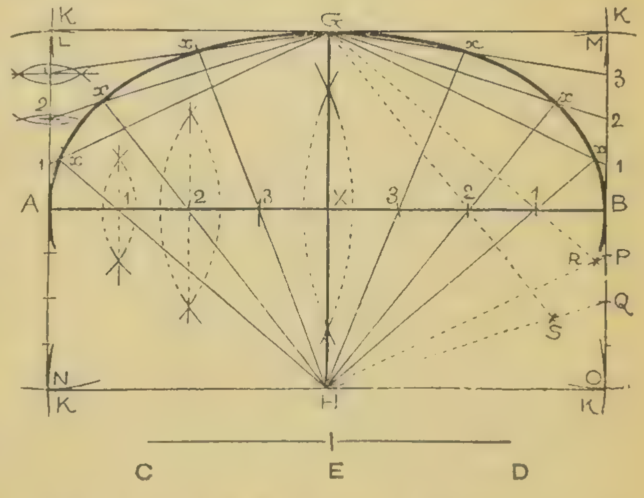
4.—With each *focus* as centre, and radius **P 2**, strike arcs **c, c, c, c.** With the *same* centres, and radius **Q 2**, cut these arcs at **d, d, d, d.**

5.—In the same way use points **3** and **4** to get **g, g, h, h.** Through points **b, d, g, h,** draw the curve of the ELLIPSE.

Note (a).—The points 1, 2, 3, 4, may be at *any* distance apart, but it is more convenient to let the divisions *decrease* in length towards F^1. Do not make the arcs too long, as this causes confusion.

(b).—The major axis and foci being given, to find the minor axis :—Bisect **P Q** at **X.** With **X P** or **X Q** as radius, and the *foci* as centres, strike arcs cutting at **T** and **V.** Join **T V.** Then **T V** is the MINOR AXIS.

PROBLEM 142.—Draw the curve of an Ellipse, by means of intersecting lines. The lengths of the major and minor axes, A B, C D, are given.



1.—Bisect **C D** at **E.** Bisect **A B** at **X**, by a perpendicular. Make **X G, X H,** each equal to **E C** or **E D.**

2.—With centres **G, H,** and radius **X A,** strike arcs at **K, K, K, K.** With centres **A, B,** and radius **X G,** cut these arcs at **L, M, N, O.**

3.—Join **L M, M O, O N, N L.** Divide **A L, A N, B M, B O, A X, B X,** each into the *same* number of equal parts —say four.

4.—Draw lines from **G** to **1, 2, 3,** on **A L, B M.** From **H,** through **1** (on **A X**), draw a line to meet **1 G** at **x.** Through **2,** draw a line from **H** to meet **2 G** at **x.** Through **3,** draw a line to meet **3 G** at **x.**

5.—In the same way get points **x, x, x,** on the other side. Also get similar points for the lower half of the ellipse, as shewn by dotted lines at **R** and **S.** Through **x, x, x, R, S,** &c., draw the curve of the ELLIPSE.

Note (a).—The divisions on A L, A X, &c., may be *unequal*, provided those on A X are *proportional* to those on A L. A *French curve* may be used for drawing the elliptic curve through the points x, x, x, &c.

(b).—By this problem, an ellipse may be inscribed in any RECTANGLE. By joining A G, G B, B H, H A, we get a rhombus. Therefore an ellipse can be *circumscribed* about a RHOMBUS; or a RHOMBUS can be *inscribed* in an ellipse.

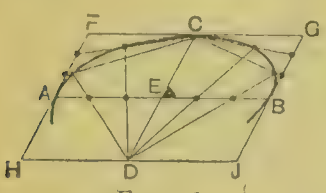


FIG. 267.

(c).—To draw the curve of an Ellipse, when two **conjugate** diameters A B, C D, making a stated angle at E, are given (Fig. 267):—Get F G, H J, parallel and equal to A B; and F H, G J, parallel and equal to C D. Get the divisions on A F, A E, &c., and obtain the curve as in Prob. 142. The *major* and *minor axes* of this ellipse can be obtained by Prob. 144. If we join A C, C B, B D, D A, we get a rhomboid. We can therefore inscribe an ellipse in, or circumscribe one about, a given RHOMBOID. By the same principle of construction we can inscribe an ellipse in any RHOMBUS.

(d).—Prob. 142 and Fig. 267 shew how we may connect the ends of any two lines A N, G M, or A H, C G, or B J, C F, by an *elliptic curve*. These are good methods to adopt for lines used in *ornamental design*, and for the contours of *mouldings*, *vases*, &c. Two or more elliptic curves may be used in combination.

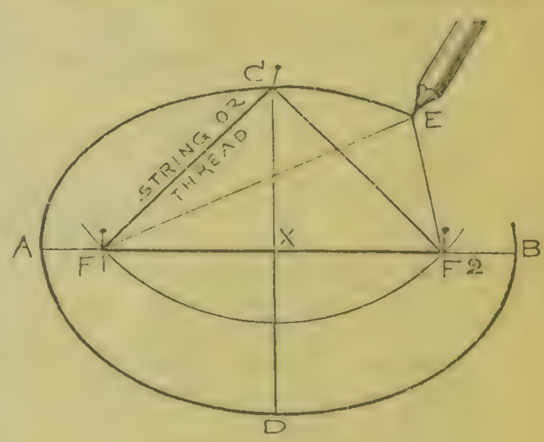
(e).—*Any* two lines, bisecting each other, may be considered as a *given* pair of CONJUGATE DIAMETERS of an ellipse. These lines may intersect at *any* angle. They may be of *equal* length, provided they do not intersect at 90°. (Draw the ellipse as in Fig. 267.) When the lines intersect at 90°, they become the *major and minor axes*, and these axes *must* be of *unequal* length. (*See* § 207.) If we attempt to describe an ellipse with two *equal* axes at 90°, we obtain a *circle*. Try this, as an experiment, by Prob. 142. Points x, x, x, will be *equidistant* from the centre X: therefore the curve obtained must be a *circle*.

PROBLEM 143.—To draw an Ellipse, by means of a piece of thread, the major axis A B, and minor axis C D, being given.

1.—Find the *foci* **F**¹, **F**², as in Prob. 141. Insert a *pin*, or a *needle*, in point **C**, and in each *focus*.

2.—Tie a piece of *fine thread* tightly around the three pins, and keep it *close* to the paper.

3.—Remove the pin from **C.** Place the point of a pencil, *uprightly*, inside the thread, and by moving the point of the pencil around the foci — *keeping the thread stretched to its full extent*—the pencil describes the curve of the ELLIPSE.



Note (a).—Before commencing to draw, the knot of the thread should be turned round so as to come between the foci. *This problem is of great practical value to designers, joiners, masons, smiths, gardeners, &c.*

(b).—Thread tied around *any* 3 points enables us to draw an ellipse, but not one of *given* dimensions. By tying a piece of thread around 4 pins, inserted in the angles of a *trapezion*, J K L M (Fig. 93), and letting the pencil take the place of the pin at M, we can describe the curve of an **oval**, but J L, M K, are not its diameters. Nicely proportioned *ovoid* figures can be described by making J L = 2″, N K = 2$\frac{3}{4}$″, M N = $\frac{3}{4}$″; or by making J L = 2″, N K = 2$\frac{1}{4}$″, M N = $\frac{5}{8}$″. These ovals are each made up of portions of 6 elliptic curves joined together in a continuous line.

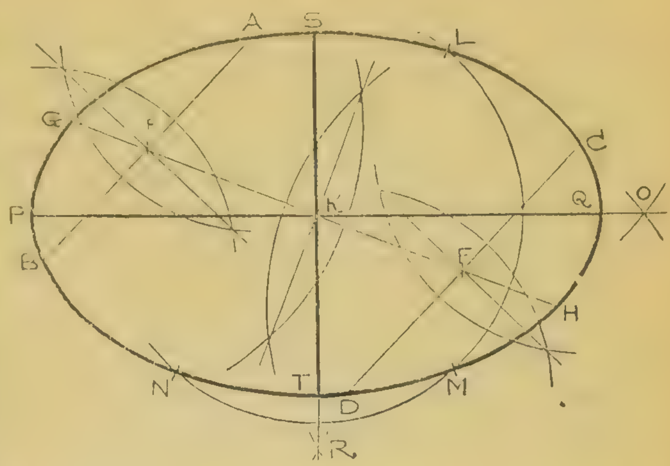
PROBLEM 144.—The curve, or portion of the curve of an Ellipse being given, to find the centre, and the major and minor axes.

1.— Draw two *parallel chords* **A B, C D.** Bisect them at **E** and **F.**

2.—Through **E, F,** draw **G H,** which is a *diameter.* Bisect it at **K,** which is the CENTRE of the ellipse.

3.—With centre **K,** and *any* convenient radius, describe the arc **L M N.**

4.—With centres **L** and **M,** and *any* radius, describe arcs cutting at **O.** From **O,** through **K,** draw **P Q,** which is the MAJOR AXIS.



5.—With centres **M** and **N,** and *any* radius, describe arcs cutting at **R.** From **R,** through **K,** draw **T S,** which is the MINOR AXIS.

Note (a).—Instead of getting arcs at O and R, we can join L M, M N, and then draw the axes through K, parallel to L M, M N. For convenience, the given ellipse may be drawn with a piece of thread, as in Prob. 143.

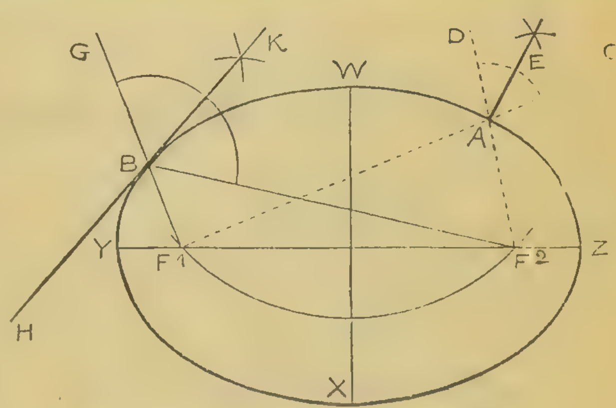
(b).—If a small portion of the curve is given, the chords, A B, C D, must be drawn closer together. If only *one* end of G H meets the curve, draw *another* pair of parallel chords, and get *another* diameter, then the intersection of the two diameters gives the CENTRE. The portion of the curve given should contain at least one end of each axis.

PROBLEM 145.—At any point A, in the curve of an Ellipse, to draw a Normal: and through any point B, in the curve, to draw a Tangent.

1.—Draw the ellipse with a piece of thread (Prob. 143).

2.—From each *focus*, draw a line through **A,** to **D** and **C.**

3.—Bisect angle **D A C,** by **A E.** The line **A E** is the required NORMAL (or PERPENDICULAR).



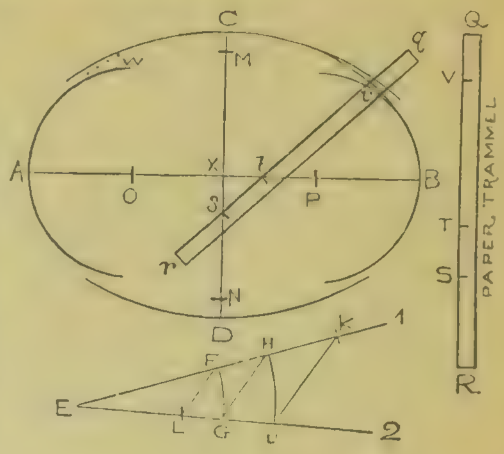
4.—From the *foci* draw lines to **B.** Produce one of the lines, say to **G.** Bisect the angle **G B F^2,** by **H K.** Then the line **H K** is the required TANGENT.

FIG. 268.

Note (a).—The *normal* may also be obtained by bisecting the angle F^1 A F^2. To draw a NORMAL at either extremity of the major or minor axis, simply *produce* the axis. A TANGENT at either extremity of the major or minor axis, must be drawn at *right angles* to the axis.

(b).—In the construction of a *semi-elliptical* arch (Fig. 268), the lines of separation between the stones (*voussoirs*) should be *normals* to the semi-elliptical curve.

PROBLEM 146. -To get the curve of an Ellipse, approximately, with arcs of circles, and by the use of a paper trammel.



1.—The lines **A B, C D,** are the major and minor axes.

2.—Draw **E 1, E 2,** at any angle. Make **E G = X C,** and **E H = X A.** Join **G H.** With centre **E,** and radii **E G, E H,** strike the arcs **G F, H J.** Draw **F L, J K,** parallel to **G H.**

3.—Make **D M, C N,** equal to **E K,** and **A O, B P,** equal to **E L.** With centres **N** and **M,** and radius **N C,** strike arcs passing through **C** and **D.** With centres **O, P,** and radius **O A,** strike arcs at **A** and **B.** These 4 arcs give, *approximately*, parts of the ELLIPSE.

4.—On one edge of a straight slip of paper **Q R,** set off **V S** equal to **A X,** and **V T** equal to **C X.** Then we can use **Q R** as a *trammel*. Adjust the trammel **Q R,** in such a manner, that point **t** rests somewhere on the *major* axis; and point **s,** on the *minor* axis. Wherever point **v** comes, will be *a point situated in the curve of the* ELLIPSE. Mark several points as at **w,** &c., and through these points draw curves connecting the arcs.

Note (a).—E L is a *third-proportional less*, and E K is a *third-proportional greater*, to the lines E G, E H. A French curve may be used to connect the arcs through the points at w, &c. The *entire* curve can be drawn by means of points obtained with a trammel.—— When an ellipse has a short minor axis, the points, M and N, fall *outside* the ellipse, on the minor axis *produced*.

(b).—This method is exceedingly useful when representing *circles in perspective*; and also in mechanical drawing, when describing ellipses.

General Notes about Ellipses.

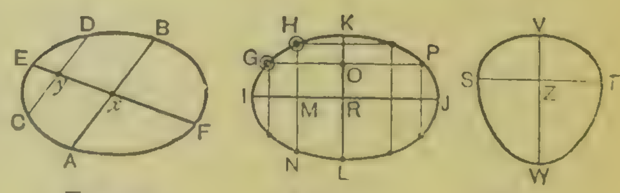


FIG. 269. FIG. 270 FIG. 271.

210.—*Any* diameter, A B, of an ellipse being given, to find its CONJUGATE diameter (Fig. 269):—Draw any chord C D, *parallel* to A B. Bisect A B, C D, at x, y. Then E F, drawn through x, y, is a *conjugate* diameter to A B. The minor axis is called "*the* conjugate axis," because of its relationship to the major axis. The major and minor axes are a *pair* of conjugate diameters. (*See* § 206.)

211.—If from any points, G, H, in the curve of an ellipse (Fig. 270), we draw lines parallel to the major axis I J, or to the minor axis K L, and make the distance M N = M H, or O P = O G, we get other points, N, P, in the elliptic curve. A line y C, or y D, drawn from *any* point y, in a diameter E F, and parallel to a conjugate diameter A B, is called an ORDINATE. M H, O P, are also *ordinates*. The whole line C D, H N, or G P, is a *double ordinate*.

212.—To draw an ellipse when the foci, and one point in the curve, are given:—Draw a line of indefinite length through the foci. Draw a line from each focus to the given point. The *sum* of these two lines gives the length of the *major* axis. With *half* the major axis as radius, and the *foci* as centres, strike arcs intersecting at points, which give the ends of the *minor* axis. Obtain the curve of the ellipse.

213.—To draw tangential lines to an ellipse from a given point outside the curve :—Call the given point 1, and place it in any position with regard to the ellipse. With 1, as centre, and the distance to the NEARER *focus*, as radius, describe about half of a circle cutting the ellipse in two places. With the FURTHER *focus* as centre, and the *major axis* as radius, cut the arc in points 2 and 3. From points 2 and 3, draw lines to the *further* focus. These lines cut the ellipse in two points. Call these points, 4 and 5 : they are the required *points of contact*. Draw two lines from the given point 1, *through* points 4 and 5, and these lines are the required TANGENTS.

214.—To draw an ellipse—having *one* diameter given—SIMILAR to any given ellipse :—In two *similar* ellipses, *any* two conjugate diameters of one ellipse have the same proportion to each other as the *corresponding* conjugate diameters of the other ellipse have to each other. Therefore, by Prob. 91, find a *fourth-proportional* to the given diameter, and the two diameters of the given ellipse. This fourth-proportional gives us the length of the other diameter of the required ellipse. Place the two diameters, bisecting each other, and at the required angle. Then construct the ellipse. If one ellipse be placed within, or without, another *similar* ellipse, with the corresponding conjugate diameters *coinciding*, the curves of the two ellipses are *not* equidistant from each other.

215.—If with centre R, and radius R L (Fig. 270), we draw a circle, it is *inscribed* in the ellipse. If with centre R, and radius R J, we draw a circle, we *circumscribe* a circle about an ellipse.

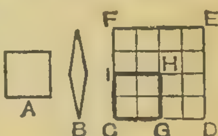
216.—If on one side of a given line S T (Fig. 271), as a *major* axis, we draw a semi-ellipse, having any convenient distance, Z V, for half its minor axis, and on the other side of S T, as a *minor* axis, we draw another semi-ellipse, having any convenient distance, Z W, for half its major axis, we obtain a perfect OVAL having *given* diameters, S T, V W. The curve of each semi-ellipse can be obtained with intersecting arcs or lines, or with a thread.

Work out TEST-PAPER E, and the Exercises on ELLIPSES.

CHAPTER XIII. AREAS OF PLANE FIGURES.

CLASS-SHEET 13.

DEFINITIONS, etc. :—217.—The amount of SURFACE enclosed by the boundary line, or lines, of any figure, is termed its **Area**, or SUPERFICIAL CONTENT. The area is not always determined by the length of the *boundary*. Thus, the square A, and rhombus B, in Fig. 272, have *equal perimeters*, but the area of the square is greater than that of the rhombus. Areas of plane figures are calculated in *squares*. Thus, in Fig. 273, suppose the square, C D E F, to have sides of 4 inches, or 4 feet. We then say that the square contains 16 *square inches*, or 16 *square feet* ($4 \times 4 = 16$). If the sides are 3 ft. long, the square contains 9 *square inches* ($3 \times 3 = 9$), and so forth. In a similar way, a *rectangle* having sides of 4 ft. and 6 ft., contains 24 *square feet* ($4 \times 6 = 24$).

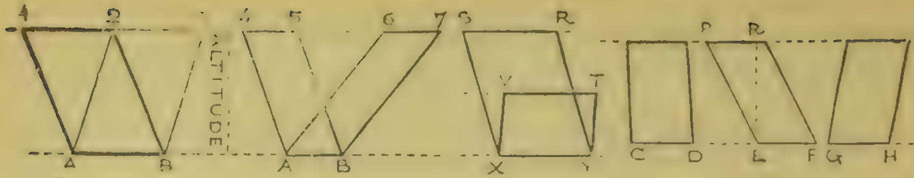


FIGS. 272. 273.

218.—It must be remembered that there is a distinction between the terms "4 *inches square*," and "4 *square inches*." Thus, in Fig. 273, C D E F is "4 inches square," *i.e.*, each *side* = 4 in. ; but the *smaller* square, C G H I, contains "4 square inches." Again, a square of 100 ft. sides is "100 ft. square," but it contains "10,000 square ft." ($100 \times 100 = 10,000$).

219.—A figure of *any* shape—rectilineal or curvilineal—is said to contain so many *square inches*, or *square feet*, &c. Thus, a polygon, a circle, or an ellipse, may have an area of so many square inches, &c.

220.—Figures that have *equal* areas, no matter how dissimilar in shape, are called **Equivalent Figures.** Thus, a *square* containing 10 square inches is *equal* to a *triangle*, *polygon*, or *circle* having an area of 10 square inches ; and they are EQUIVALENT FIGURES.



FIGS. 274. 275. 276. 277. 278. 279.

221.—Parallelograms upon the same base and of the same altitude, or perpendicular height (i.e., between the same parallels), are equal in area. (EUCLID I. 35.)

THUS, the parallelograms, A B 2 1, A B 3 2 (Fig. 274), upon the *same base*, A B, and of the *same altitude*, are equal to each other in area. In this case it is clearly seen, because each parallelogram is *double* the size of the *triangle* A B 2. Again, in Fig. 275, the parallelograms, A B 5 4, A B 7 6, upon the *same base*, A B, and between the *same parallels*, are equal. Fig. 294 helps to explain the above. Thus, the rhomboid, F G M L, must be equal to the rectangle, F G K H, because the triangle cut off at M K G, is replaced by another precisely equal to it at L H F.

THEREFORE, upon one side of a SQUARE, RECTANGLE, or RHOMBUS, if we produce the *opposite* sides to get the *parallels*, we can construct an equal RHOMBOID having any stated angle. Or, on the base of a RHOMBUS, or RHOMBOID, we can construct a RECTANGLE of equal area. (N.B.—*The various cases, mentioned in this and the following sections, should be worked out.*)

222.—Parallelograms upon equal bases, and between the same parallels, are equal in area. (EUCLID I. 36.)

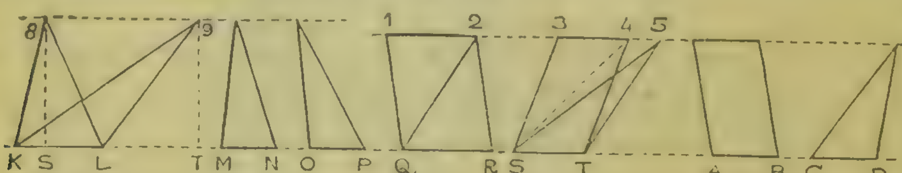
THUS, in Figs. 277, 278, 279, the parallelograms upon the equal bases, C D, E F, G H, are equal to one another. The *perpendicular* line, E R, is the *altitude*, and not E P.

THEREFORE, upon an equal base, provided we keep the *same altitude*, we can construct a RECTANGLE equal to a RHOMBUS or RHOMBOID.

223.—A parallelogram upon the same (or equal) base as a given parallelogram, but 2, 3, or 4 times, &c., the altitude, is 2, 3, or 4 times, &c., the area of the given parallelogram.

THUS, in Fig. 276, the parallelogram X Y R S, having *twice* the *altitude* of X Y T V, is *twice* the *area*; because both are on the *same* (*or equal*) *base* X Y.

THEREFORE, we can construct, upon an equal base, a RECTANGLE, or RHOMBOID, *twice* or *thrice*, &c., the area of a given SQUARE or RHOMBUS. Or we can construct a figure of *less* proportional area, instead of *greater*; thus, X Y T V is half X Y R S. On the same principle, a parallelogram, having the *same altitude* as a given parallelogram, and a BASE *half*, or *twice*, &c., the length, has *half*, or *twice*, &c., the area of the given parallelogram. Thus, in Fig. 287, since A B is 3 times greater than A E, the parallelogram A B C D is 3 times greater in area than A E F G.



FIGS. 280. 281. 282. 283. 284. 285. 286.

224.—Triangles upon the same, or equal bases, and of the same altitude, are equal in area. (EUCLID I. 37, 38.)

THUS, in Fig. 280, the triangles, K L 8, K L 9, are equal, because they are upon the *same base* K L, and between the *same parallels*. The altitudes, S 8, T 9, are equal. Again, in Figs. 281, 282, the triangles are equal, because they are between the same parallels, and M N = O P

THEREFORE, a SCALENE TRIANGLE can be converted into an ISOSCELES, or a RIGHT-ANGLED TRIANGLE, of equal area; or *vice versâ*. Also, a RHOMBUS, H I J K (Fig. 288), can be constructed *equivalent* to a RHOMBOID H L J M. Bisect H J by K I, then H J K = H M J.

FIG. 287.

FIG. 288.

225.—A triangle upon the same (or equal) base as a parallelogram, and of the same altitude, is half the area of the parallelogram. (EUCLID I. 41.)

Thus, in Fig. 283, triangle Q R 2 is obviously *half* the size of the parallelogram Q R 2 1. Again, in Fig. 284, S T 4 is *half* S T 4 3. But S T 5 = S T 4, therefore S T 5 is also *half* the area of S T 4 3. In Figs. 285, 286, as A B = C D, the triangle on C D is *half* the size of the parallelogram on A B.

THEREFORE, we can construct an OBLONG, or RHOMBOID, *double* the area of an EQUILATERAL, ISOSCELES, or RIGHT-ANGLED TRIANGLE.

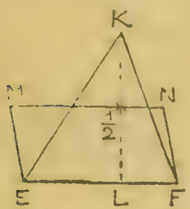


FIG. 289.

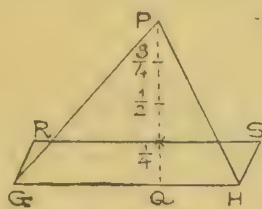


FIG. 290.

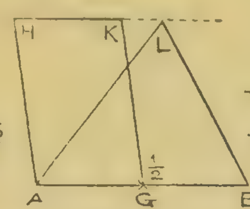


FIG. 291.

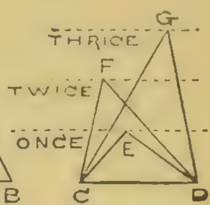


FIG. 292.

226.—A triangle upon the same (or equal) base as a parallelogram, but twice the altitude of the parallelogram, is equal to the latter in area.

THUS, in Fig. 289, as the *altitude* of the *triangle*, E F K, is *double* that of the *parallelogram*, E F N M, and as both are on the *same base*, the figures are *equal in area*. This will be seen more readily on reference to Fig. 293, where the two triangles O, O, and X, X, are evidently equal to each other, and what is *taken away* at X and O in the parallelogram, is contained in the triangle at X and O.

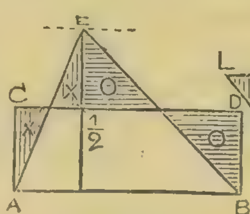


FIG. 293.

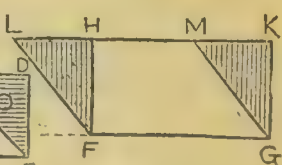


FIG. 294.

THEREFORE, we can construct an ISOSCELES, or SCALENE TRIANGLE, equal in area to a SQUARE, OBLONG, RHOMBUS, or RHOMBOID. Or we can construct an OBLONG, or RHOMBOID, equal in area to *any* TRIANGLE.

227.—A parallelogram upon the same (or equal) base as a triangle, but one-fourth the altitude of the latter, is equal to half the area of the triangle.

THUS, in Fig. 290, the *parallelogram* G R S H, being on the *same base* G H, as the *triangle* G P H, but having only *one-fourth* the *altitude*, is equal to *half* the area of the triangle. In the same way, a parallelogram upon G H, having an altitude $\frac{3}{4}$ of Q P, is $1\frac{1}{2}$ times the area of the triangle.

THEREFORE, by giving different proportions to the ALTITUDES, we can construct TRIANGLES and PARALLELOGRAMS, having different stated proportions to one another. Thus, a triangle upon the same base as a parallelogram, but having *half* the *altitude* of the latter, is *one-fourth* the *area* of the parallelogram.

228.—A parallelogram upon half the base of a triangle, but of the same altitude, is equal in area to the triangle. (EUCLID I. 42.)

Thus, in Fig. 291, as A G is *half* A B, and A G K H, A B L, are of the *same altitude*, the figures are *equal*. The equivalence of the figures is more evident if we make G L one side of the parallelogram.

THEREFORE, an OBLONG, or RHOMBOID, can be made equal to any TRIANGLE of the *same height*, provided the base of the latter is *twice* the length of that of the former.

229.—A triangle, 2 or 3 times, &c., the altitude of a given triangle, and upon the same, or an equal base, is 2 or 3 times, &c., the area of the given triangle.

THUS, in Fig. 292, C D F is *twice* the area, and C D G *thrice* the area of C D E. Or, with *equal altitudes*, we can *increase or decrease the base*, with similar results, as in the case of parallelograms (Fig. 287). Thus, a triangle having a base 2 or 3 times the length of the base of a given triangle, but the *same altitude*, is 2 or 3 times greater in area.

THEREFORE, an ISOSCELES, SCALENE, or RIGHT-ANGLED TRIANGLE, can be constructed ½, 3, or 10 times, &c., the area of *any* given triangle, having the *same altitude*, or the *same base*.

230.—Triangles or parallelograms, between the same parallels, have areas in the same proportion as their bases. (EUCLID VI. 1.)

THUS, the area of A B C (Fig. 295) is in the *same proportion* to that of D E F, as the *base* A B is to D E; and G H I J is to K L M N, as G H is to K L. Again, O P Q is divided into other triangles, and T U V W into other parallelograms, having in each case the *same proportion* to one another, as the divisions, respectively, on O P or T U.

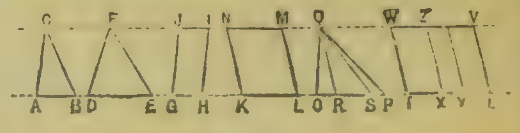


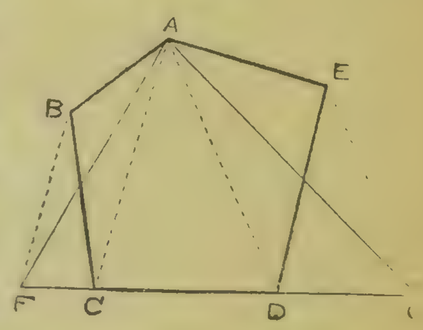
FIG. 295.

THEREFORE, upon T X we can construct a *triangle*, having the same proportion to T U V W, that T X Z W has to T U V W. Or, upon O R we can construct a *parallelogram*, having the same proportion to O P Q, that O R Q has to O P Q.——*Triangles, or parallelograms, on* EQUAL BASES, *have also the* SAME PROPORTION *to one another as their* ALTITUDES.

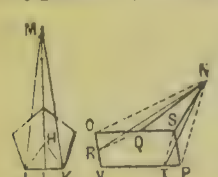
PROBLEM 147.—Construct a Triangle equivalent to ANY Rectilineal Figure—say to the Figure A B C D E.

1.—Produce *any* side, say **C D**, *indefinitely*, at each end. Join **A C, A D.** Draw **B F, E G,** *parallel* to **A C, A D.**

2.—Join **A F, A G.** Then **F A G** is the required TRIANGLE, which is *equal in area* to the given rectilineal figure **A B C D E.**



Note (a).—The triangles C F A, C B A, are upon the *same base* C A, and between the *same parallels* C A, F B. Therefore they are *equal* to each other. An *equivalent* to the portion cut off from the irregular polygon at B, is added at F.



FIGS. 296. 297.

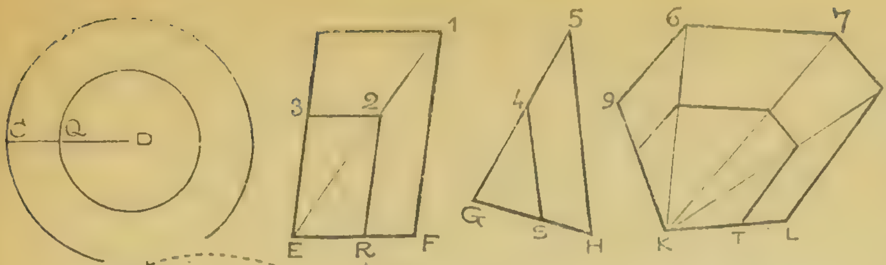
(b).—For *any* REGULAR POLYGON, say a *pentagon* (Fig. 296), find the centre H. Join H J, H K. Bisect J K at L. Through H, draw L M =5 times H L. Get the required TRIANGLE J M K. Or, we can make the triangle with *altitude* H L, and a *base*=5 times J K.

(c).—If we require a figure equal to A B C D E, but to have *one side less*, then F A E D, or A B C G, is the required figure. To construct a figure equivalent to F A E D, but to have *one side more*:—Take *any* convenient point B, outside the figure. Join B F. Draw A C parallel to B F. Draw A B, B C; then A B C D E is the required figure. By the latter process, an ***irregular polygon*** can be constructed equal to a ***triangle.***

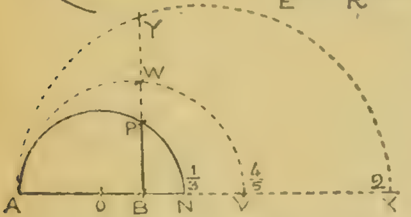
(d).—To construct a triangle equivalent to a given rectilineal figure, and *to retain one of the angles of the given figure*:—Thus, construct a triangle equivalent to A B C D E, the angle B C D to be one of the angles of the triangle. Obtain line A G. Produce C G to the right. Join B G. From A, draw a line A X, parallel to B G, to meet C G, produced. Join B X. Then B C X is the required triangle. The angle at C is ***common*** to both figures.

(e).—We can convert *two or more* rectilineal figures, *similar or not*, into *one* triangle of *equal area*. Thus, convert a RHOMBOID, V P S O, and a TRIANGLE, Q S N, into *one* equivalent triangle. Place the figures together as in Fig. 297. Join N O, N P. Draw Q R, S T, parallel to N O, N P. Join N R, N T. Then, by Prob. 147, convert V T N R into an equal triangle. Fig. 297 shews how to construct a triangle equivalent to a rectilineal figure that has a *re-entrant* angle. Thus, the irregular figure, V O Q N S P, has a re-entrant angle O Q N. We see how *one* line, R N, is substituted for the *two* lines, O Q, Q N. The re-entrant angle, N S P, is treated in the same way.

PROBLEM 148.—To draw a Circle, or ANY Rectilineal Figure, having an area of a given proportion to that of any given Circle, or Rectilineal Figure, respectively.



1.—Let the required figure be constructed, in each case, one-third the area of the given figure. Draw **A B**, equal to the radius, **D C**, of a CIRCLE, or a side, **E F**, of a PARALLELOGRAM, or a side, **G H**, of a TRIANGLE, or a side, **K L**, of ANY POLYGON.



2.—Produce **A B** indefinitely. Set off **B N** = **one-third** of **A B**. Bisect **A N**, at **O**. With centre **O**, and radius **O A**, strike a *semi-circle*. At **B**, erect a perpendicular. Then **B P** is the *mean-proportional* to **A B**, **B N**, and it is the *length* of the required *radius* or *side*.

3.—With centre **D**, and radius **B P**, strike the CIRCLE **D Q**.

4.—Make **E R** = **B P**. On **E R** construct the PARALLELOGRAM **E R 2 3**, similar to the given parallelogram **E 1**.

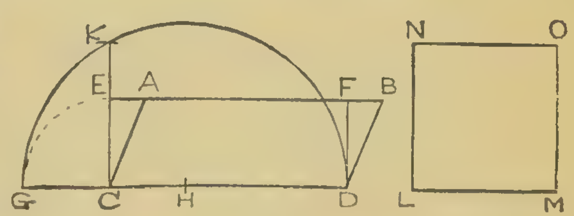
5.—Make **G S** = **B P**. Construct TRIANGLE **G S 4**, similar to **G H 5**.

6.—Make **K T** = **B P**. On **K T** construct an IRREGULAR POLYGON, similar to **K L 8 7 6 9**. Then the CIRCLE **D Q**, the PARALLELOGRAM **E R 2 3**, the TRIANGLE **G S 4**, and the IRREGULAR POLYGON on **K T**, are the required figures.

Note (a).—The line, A B, may represent a *diagonal* or an *altitude* of ANY rectilineal figure, or an axis of an *ellipse*, or the width of any *irregular curvilineal figure*.

(b).—To construct a figure, say FOUR-FIFTHS the area of a given figure :—Make B V = $\frac{4}{5}$ of A B. Draw a semi-circle. Then the *mean-proportional*, B W, is the length of the required side ; &c.—— To get DOUBLE the area :—Make B X = *twice* A B. Draw a semi-circle. Then the *mean-proportional*, B Y, is the length required. Adopt a similar method for ANY proportion. *This problem is most useful when making proportional enlargements or reductions of* DIAGRAMS, MAPS, PLANS, &c.

PROBLEM 149.—To construct a Square, equal in area to ANY Rectilineal Figure. Thus, construct a square equivalent to the Rhomboid C D B A.



1.—Make the *rectangle* **C D F E** equal to **C D B A**, by producing **B A** and erecting perpendiculars **C E**, **D F**. (*See* § 221.)

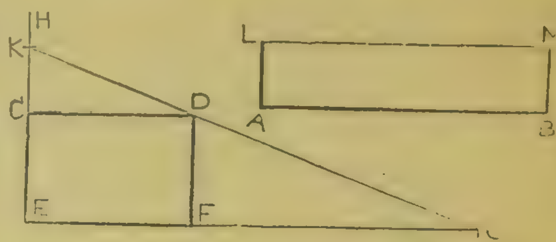
2.—Produce **D C**. Make **C G** = **C E**. Bisect **G D** at **H**. With centre **H**, and radius **H G**, strike a semi-circle. Produce **C E** to **K**. Then **C K** is the *mean-proportional* to **G C**, **C D**.

3.—Make **L M** = **C K**. On **L M** construct the required SQUARE **L M O N**.

Note (a).—We find the *mean-proportional* to a *side* and the *altitude* of a given parallelogram. Thus, we need only obtain D F, and find the mean-proportional to C D, D F.

(b).—We can construct a *square* equal to ANY rectilineal figure. First, convert the given figure into an *equal rectangle* ; then find the required *mean-proportional*. (*See* § 226 *and Prob.* 147.)

PROBLEM 150.—On a given line, A B, to construct a Parallelogram equivalent to ANY Rectilineal Figure—say to the given Rectangle E F D C.



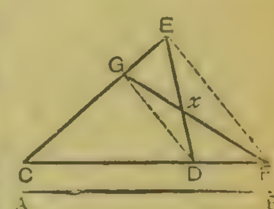
1.—Produce **E F** to **G**, making **F G = A B.** Produce **E C**, indefinitely, to **H.**

2.—Through **D**, draw **G K.** Then **C K** is the *perpendicular height* of the required PARALLELOGRAM **A B M L.**

Note (a).—If the given figure is a *square*, proceed as above. If it is ANY other *rectilineal figure*, first convert it into an equal *rectangle.*

(b). —An equivalent *rhomboid* can be constructed upon A B, having an *altitude* equal to C K. Or an equivalent *triangle*, upon A B, by making its altitude *twice* as long as C K.

PROBLEM 151.—To construct a Triangle, having a given base, A B, and equivalent to ANY Rectilineal Figure—say, equal in area to the triangle C D E.



Produce one side, **C D**, to **F**, making **C F** equal to the given base **A B.** Join **F E.** Draw **D G**, parallel to **F E.** Join **F G.** Then **C F G** is the required TRIANGLE.

Note (a).—G D E, G D F, are *equal* because they are on the *same base* G D, and between the *same parallels* G D, E F. Triangle G D x is *common* to G D E, G D F. Thus, the triangle G x E, which is cut off from C D E, is equal to the added portion D x F; therefore C F G = C D E.——If A B is *less* than C D, set off its length from C to a point F *in* C D. Join F E. From D, draw a line parallel to F E, to meet C E, *produced*, at a point G. Join F G. Then C F G is the required TRIANGLE. When the base of the *required* triangle is *less* than that of the *given* triangle, the *altitude* of the one required is *greater* than that of the one given; and *vice versâ.*

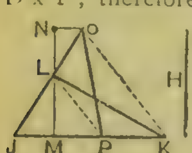
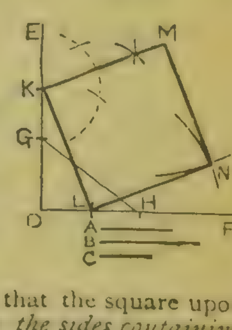


FIG. 298.

(b).—Construct a TRIANGLE, having a given **altitude H**, and *equal* to the given triangle **J K L** (Fig. 298):—Draw the altitude **L M**, and produce it to **N**, making **M N = H.** Produce **J L**, indefinitely. Draw **N O**, parallel to **J K.** Join **O K.** Draw **L P**, parallel to **O K.** Join **O P.** Then **J P O = J K L.** If **H** is *less* than **L M**, then **N** falls *on* **L M.**

(c). —*A triangle can be constructed, having a given* BASE, *or* ALTITUDE, *equal to* ANY *rectilineal figure, by first converting the latter into an* EQUIVALENT TRIANGLE.

PROBLEM 152.—To construct a Square, equal in area to any number of given Squares.



1.—**A, B, C,** are the sides of three given squares.

2.—Draw the lines **D E, D F,** at *right angles.*

3.—Make **D G = A,** and **D H = B.** Join **G H.** Then the *square* upon **G H** *equals the squares* upon **A** and **B.**

4.—Make **D K = G H,** and **D L = C.** Join **K L.** Then the SQUARE, **K L N M,** *equals the three squares* upon **A, B** and **C.**

Note (a).—The construction of this problem is based upon the fact that the square upon the *hypotenuse* of a right-angled triangle, *is equal to the squares upon the sides containing the right angle.* Thus, in Fig. 299, the square O equals squares P and Q. Therefore, in the *right-angled triangle* G D H, the square on G H = squares on D G, D H. Thus a square upon the *diagonal* of a given square, has *double* the area of the given square.

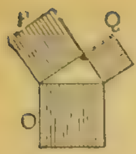


FIG. 299.

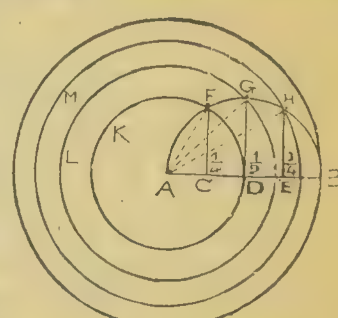
FIG. 300.

(b).—ANY three SIMILAR FIGURES, having their homologous sides upon the sides of a *right-angled triangle* have a like relationship to one another. Thus, in Fig. 300, the polygon R = the two *similar* polygons S and T. It is the same with CIRCLES and ELLIPSES. Thus, the circle or ellipse, described upon G H as a diameter, is equal to the circles or ellipses described, respectively, upon D G, D H, as diameters. We can go on repeating the operation for ANY number of **similar** figures.

(c).—Suppose D H, G H, to be the sides of two squares, or the diameters of two circles, &c., and we wish to construct a square, circle, &c., equal to the DIFFERENCE between their respective areas. Draw D E, D F. Mark off the *shorter* distance D H. With centre H, and radius G H, cut D E at G. Then D G is the side, or diameter, of the required figure.

(d).—To construct a Rectilineal Figure of a GIVEN AREA. 1st. Say a **Square** of $2\frac{1}{4}$ square inches. Draw a rectangle having sides = 1″ and $2\frac{1}{4}$″. Then, by Prob. 149, draw a square = the rectangle. Or, by Prob. 152, draw a square equivalent to the squares upon 3 lines, 1″, 1″, and ½″ long.——2nd. A **Rectangle**, or a **Rhomboid**, of 3 square inches, upon a *given line.* Draw a rectangle having sides = 1″ and 3″. Then, by Prob. 150, construct an equivalent rectang'e (or rhomboid) upon the given line.——3rd. Say a **Triangle** of 5 square inches. Draw a rectangle having sides = 1″ and 5″ (or 2″ and $2\frac{1}{2}$″). Draw a right-angled, an isosceles, or a scalene triangle, equal to the rectangle. By Prob. 151, an equivalent triangle may be constructed, having a *given base*, or *altitude.*

PROBLEM 153.—To divide a Circle into any number of equal or proportional areas, by concentric circles.



1.—Say, divide the given circle into *four* equal parts, or areas.

2.—Draw a radius **A B.** Bisect **A B** at **D,** and describe a *semi-circle.*

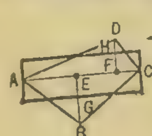
3.—Divide **A B** into *four* equal parts, at **C, D,** and **E.** At these points raise *perpendiculars* to meet the semi-circle.

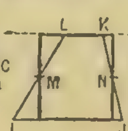
4.—With centre **A,** and radii **A F, A G, A H,** describe the required CIRCLES **K, L, M.**

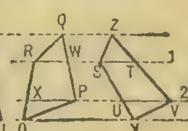
5.—Then the three *rings* and the smallest *circle* **K,** are *equal in area.* Or, circle **K** is *one-fourth* of the given circle ; circle **L** is *half ;* and circle **M** is *three-fourths.*

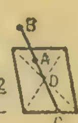
Note (a).—A circle having *half* the radius, or diameter, of a given circle, has a *quarter* the area. Thus, circle A D is *one-fourth* of circle A B.——By dividing A B into *any* number of *unequal* parts, circles can be obtained which have the *same proportion* to one another, as the divisions made upon the radius A B. This problem is useful when calculating for the diameters of pipes for water-supply, &c.

General Notes about Areas.









FIGS. 302. 303. 304. 305. 306.

231.—To construct a rectangle equal to *any* quadrilateral figure, A B C D (Fig. 302):— Draw a diagonal A C. Draw B E, D F, perpendicular to A C. Bisect B E, D F, at G and H. Through A, C, G, H, draw the sides of the required *rectangle,* parallel to B E, A C. (*See* § 226.) For a *trapezoid,* I J K L (Fig. 303), bisect I L, J K, at M and N. Through M and N, draw two sides of the equivalent *rectangle* as shewn.

232.—Draw *any* rectilineal figure, say O P Q R (Fig. 304). Draw *parallels* through the angles. *Anywhere* on the parallels 1, 2, set off S T, U V, = R W, X P. Through T and U, draw V Z, S Y. Join S Z, V Y. Then Y V Z S is equivalent to O P Q R. (*See* § 224.) The angle at S or V may be of a *given size.*

233.—A straight line, drawn through *any* point A, within, or from B, without, or from C, on a side of, *any* parallelogram, through the centre D, divides the figure into *two equal parts.* (Fig. 305).

234.—To cut the *largest possible rectangle* out of a *triangle* E F G (Fig. 306):—Bisect *any* two sides at H, I. Draw the rectangle, H I K J, as shewn. In the same way, cut the largest rectangle out of a *rhombus*, or a *trapezion*. The rectangle has *half* the area of the triangle, rhombus, or trapezion. A *square* is the largest rectangle that can be placed in a *circle*.

235.—A *square* is greater than *any* other quadrilateral of *equal perimeter*. With regular polygons, having *equal perimeters*, that polygon which has the greatest number of sides, encloses the greatest area. A *circle* is greater than *any rectilineal or curvilineal figure* of equal length of boundary. Thus, a given number of hurdles, when arranged in a circle, encloses the greatest possible amount of area.

236.—To construct a RECTANGLE *approximately equal* to a CIRCLE:—Make two sides equal the *radius*, and the other sides equal *half* the length of the circumference. (*See Prob.* 62.) A *triangle* or *square* can be made equal to the rectangle. To draw a CIRCLE *approximately equal* to a SQUARE:—Divide the side of the square into 8 equal parts, and let the diameter of the circle equal 9 of these parts. The circle is only about $\frac{1}{157}$ less than the square.

237.—The areas of SIMILAR FIGURES are to each other as the squares of their corresponding sides. Thus, in Prob. 95, E F D C is to A B K J, as the *square* of E F is to the *square* of A B; &c. With CIRCLES, their areas are to each other as the *squares* of their *diameters*.

238.—A regular polygon, *inscribed* in a circle, has a greater area than an inscribed irregular polygon, of the same number of sides. A regular polygon, *circumscribed* about a circle, has a less area than a circumscribed irregular polygon of the same number of sides.

239.—**To calculate the areas of surfaces arithmetically**:—**1st.** To find the area of a SQUARE or RECTANGLE:—Multiply the lengths of *two adjacent sides*, and the product gives the area. Thus, in a *square* of 6 ft. sides (6×6) we have 36 square ft. In a *rectangle* of 3 in. by 7 in. (3×7) we have 21 square in.——**2nd.** To find the area of a RHOMBUS, or a RHOMBOID:—Multiply the length of *one side* by the *altitude* on that side. Thus, with a *side* of 10 ft., and an *altitude* of 4 ft. (10×4) we have 40 square ft.——**3rd.** To find the area of a TRIANGLE:—As a triangle is equal to a *rectangle* on the *same base* and of *half the altitude*, we multiply the length of the *base* by *half the altitude*. Thus, in a triangle, having a base of 8 ft., and an altitude of 12 ft. (8×6) we have 48 square ft.——**4th.** To find the areas of TRAPEZIA and IRREGULAR POLYGONS:—Divide the given figure into *triangles* (as in Figs. 118, 119). Calculate the area of each triangle separately, and add the products together.——**5th.** To find the area of a REGULAR POLYGON:—As every *regular* polygon can be divided, from its centre, into *equal isosceles triangles*, find the area of *one* of them, and multiply it by the same number as the polygon has sides (see Fig. 117).——**6th.** To find the area of a CIRCLE:—Multiply the *square of the radius* by 3·1416, which gives the area *approximately*. Thus, if the radius is 3 ft., $3 \times 3 = 9$. Then $9 \times 3·1416$ gives us 28·2744 square ft.

Work out the Exercises on AREAS.

CHAPTER XIV. GEOMETRICAL PATTERN DRAWING.

DEFINITIONS, etc.:—240. The following diagrams are based upon *geometrical construction*. They must be copied two or three times the scale. The centre-lines and other lines of construction must be drawn. No CLASS-SHEET is prepared for this chapter: the diagrams should be drawn upon ordinary drawing-paper. They should be copied, at intervals, during the study of the previous chapters. Thus, Figs. 307 to 319 should be copied after Chap. IV.; Figs. 320 to 327, after Chap. V.; Figs. 328 to 347, after Chap. XI.

241.—**Symmetrical Figures** have two sides exactly alike, but reversed in position, as the letters A, H, T, the flower of the speedwell, and Figs. 77, 93, 210, 345, 346. The central line, dividing the figure into two equal and similar parts, is called the *axis of symmetry*, as in the case of a diameter of a circle, and lines J H, Fig. 77, M K, Fig. 93, F E, Fig. 210; &c. **Multisymmetrical Figures** have several *symmetrical* parts, radiating from one point, as in the jasmine, &c.

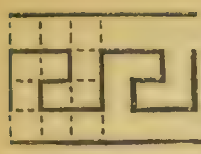


FIG. 307.

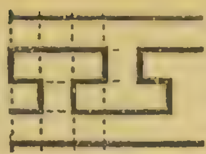


FIG. 308.

FIG. 309.

FIG. 310.

FIGS. 307, 308, 309, 310.—Draw vertical and horizontal lines of construction, in squares. Strengthen the lines of each pattern. When drawing a thick line over a fine one, let the former come *equally* on each side of the fine line.

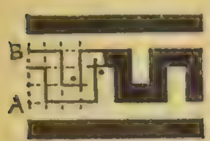


FIG. 311.

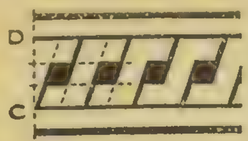


FIG. 312.

FIG. 313.

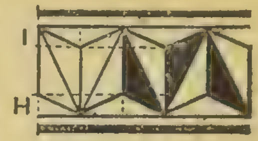


FIG. 314.

FIG. 311.—Divide A B into 3 equal parts. Draw the squares, as before. Paint the darkened portions in this and the following diagrams with Indian ink. When inking lines with the intention of filling the forms with black, let the lines go *beyond* the re-entrant angle, as shewn at the black dots.——**FIG. 312.**—Divide C D into 3 equal parts. Set off parts of the same length, horizontally, from C. Draw oblique lines at 70°.——**FIG. 313.**—Bisect F G at E. On the horizontal line E, set off parts = E F. Through these divisions draw vertical lines; &c.——**FIG. 314.**—Draw horizontal lines at H and I, equal distances apart. Set off equal vertical spaces; &c.

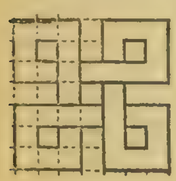


FIG. 315.

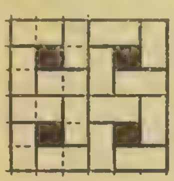


FIG. 316.

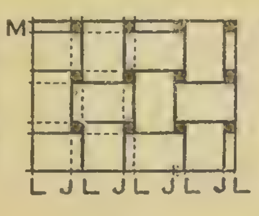


FIG. 317.

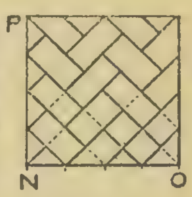


FIG. 318.

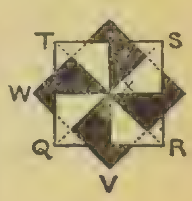


FIG. 319.

FIGS. 315, 316.—Draw vertical and horizontal lines, to form squares.——**FIG. 317.**—Set off equal parts L L, L J. Mark similar divisions on L M. Draw vertical and horizontal lines.——**FIG. 318.**—Set off equal parts on N O, and on the vertical lines at N and O. Draw the oblique lines (at 45°). —**FIG. 319.**—Draw the square Q R S T. Join Q S, R T Through X, draw a vertical and a horizontal line. Make X W, X V, &c. = X Q. Join V W, &c.

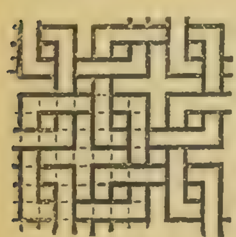


FIG. 320.

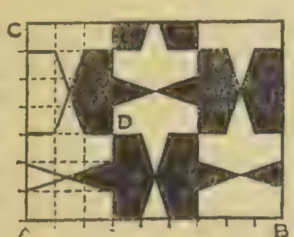


FIG. 321.

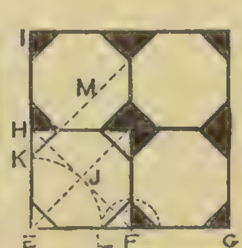


FIG. 322.

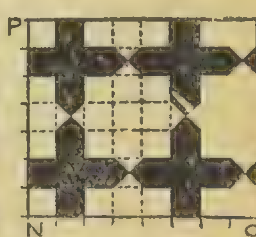


FIG. 323.

Figs. 320 *to* 327, 339, 340, 341, 343, *should be copied to double the scale.* DRAW FOUR TIMES AS MUCH AS IS GIVEN OF EACH PATTERN.—**FIG. 320.**—Draw vertical and horizontal lines, to form squares, as in Fig. 315.——**FIG. 321.**—Set off equal parts on A B, A C. Draw the squares. Strengthen the vertical and horizontal lines passing through D, &c. Draw the oblique lines. The white and black shapes are equal and similar. When a shape is to be painted in *opaque* colour, first outline the form with *thick* lines of the same colour. *These lines must not go beyond the boundary of the shape.*——**FIG. 322** shews octagons in squares. Set off equal parts, E F, F G, E H, H I. Draw the large squares. In the square H F, inscribe an octagon. Draw the diagonals. With centre E, and radius E J, strike L. (*See Prob.* 136.) From the angles of the large squares set off 4 distances = F L. Draw the small squares. The sides of these squares, if produced, form continuous straight lines, at 45°, as at M.——**FIG. 323.**—Set off equal parts on N O, P N. Draw the squares; &c.

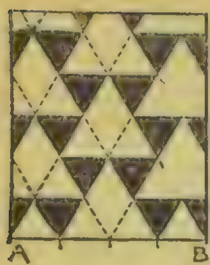


Fig. 324.

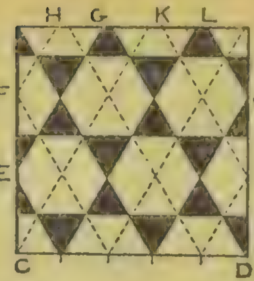


Fig. 325.

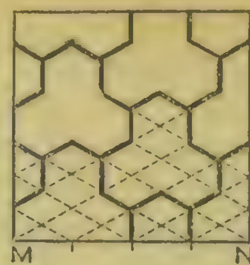


Fig. 326.

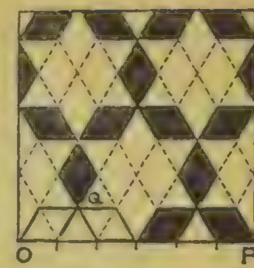


Fig. 327.

FIG. 324.—Equilateral triangles. Set off equal parts on A B. Produce A B, at each end, and continue these equal parts. Draw lines at 60° from these points.——**FIG. 325.**—Equilateral triangles and hexagons. Mark equal parts on C D, and on C D produced, as in Fig. 324. Draw lines at 60°. All these lines can be obtained without producing C D. Draw the oblique lines from the divisions on C D. From E, F, draw E G, F H, and from I, J, draw I K, J L, at 60°. This method can be adopted for Figs. 324, 326, 327.——**FIG. 326.**—Mark equal parts on M N. Draw lines at 30°. Each shape consists of 3 hexagons.——**FIG. 327** is based upon equilateral triangles and hexagons. Mark equal parts on O P. Draw lines at 60°. Draw each set of 3 lines passing through Q, &c.

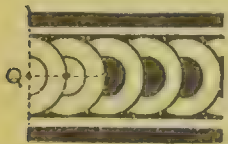


Fig. 328.

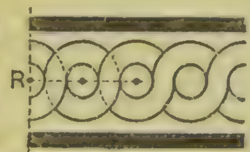


Fig. 329.

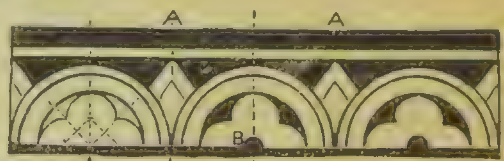


Fig. 330.

FIGS. 328, 329.—Mark equal parts on horizontal lines, Q, R. Draw the large arcs, then the small ones.——**FIG. 330.**—Draw vertical centre-lines at A, B, &c. Draw the semi-circles, then the foils. With centres B, &c., strike arcs meeting under A, A.

Fig. 331.

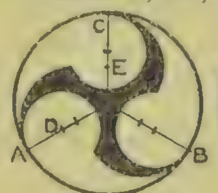


Fig. 332.

Fig. 333.

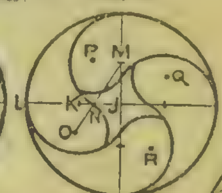


Fig. 334.

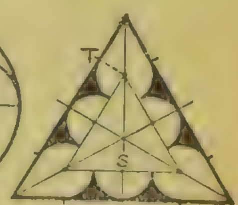


Fig. 335.

FIG. 331.—Inscribe 3 equal circles. Erase portions not required. (*See Prob.* 118.)——**FIG. 332.**—Get 3 equal sectors. With centres D, &c., and radius D A, centres E, &c., and radius E C, strike arcs.——**FIG. 333.**—Draw the two diameters. From F, set off 4 parts = F G. With centres G, &c., and radius G H, strike arcs. If the cusp, I, be given, join I H. The bisector of I H gives G. —**FIG. 334.**—Draw the two diameters. From J, mark 4 equal parts, J K, J M, &c. (J K must be *less than half* J L.) With centres K, M, &c., and radius K L, strike the large arcs. Through N, draw M N, and mark *any convenient* point O. Get similar points, P, Q, R. With centres O, P, Q, R, and radius O N, strike the small arcs. By the methods shewn in Figs. 331, 332, 334, we can insert *any* number of the given forms in circles. By Fig. 333, any *even* number of arcs—from 4 upwards—can be inscribed. Figs. 331, 333, 334, shew lines of construction for Gothic tracery.——**FIG. 335** is based upon Prob. 115, by which, obtain point S. The arc touches the straight line at T.

Fig. 336. *Scale of* $\frac{1}{4}$" *to a ft.*

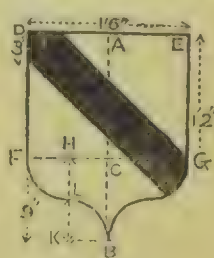


Fig. 337.

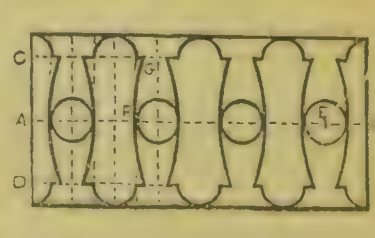


Fig. 338. *Scale of* $\frac{1}{8}$" *to a ft.*

FIG. 336.—Copy this to a scale of $1\frac{1}{4}''$ to a ft. Draw the horizontal and vertical lines. With the centres on lines, M, N, and required radius, strike circles. Draw oblique lines to touch the circles.——**FIG. 337** is drawn $\frac{3}{4}''$ to a ft. Copy it to a scale of $1\frac{3}{4}''$ to a ft. Draw centre-line A B. Get D, E, F, G. Bisect F C at H. Make H K = C B. With centres H, K, and radius H F, strike arcs; &c.——**FIG. 338.**—Copy this to a scale of $1\frac{1}{4}''$ to a ft. Draw the horizontal and vertical lines. Strike the circles and semi-circles. For the large arc F G, use centre E, and radius E F.

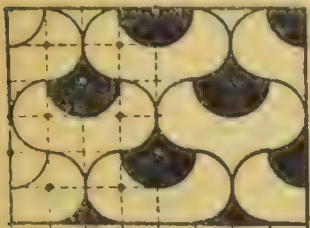


FIG. 339.

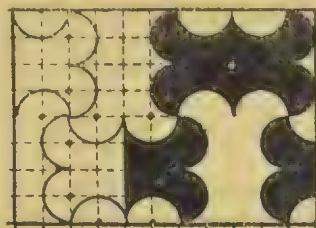


FIG. 340.

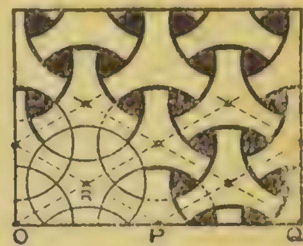


FIG. 341.

FIGS. 339, 340.—Draw squares, and strike semi-circles.——**FIG. 341.**—Set off equal parts. O P, P Q. Draw lines at 30°. Mark points R, &c. Strike circles.

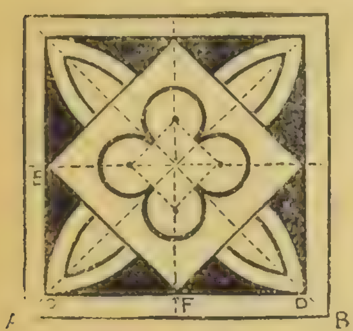


FIG. 342.

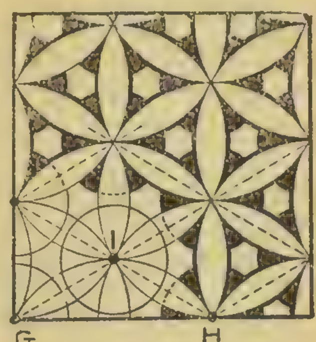
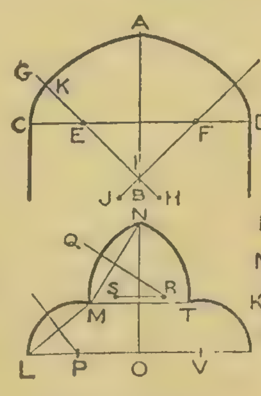


FIG. 343.

FIG. 344.

FIG. 342.—Draw squares A B, C D, E F. E, F, are centres for arcs meeting at C.——**FIG. 343.**—Set off distances = G H. Draw lines at 30°. With centres, I, &c., and radius I G, describe large circles. With same centres, and smaller radius, describe small circles.——**FIG. 344.**—The lines of construction are a sufficient guide.

FIG. 345.—FOUR-CENTRED ARCH. Draw A B, C D. Mark E, in any convenient position upon C D. Make D F = C E. Through E, draw G H *crossing* A B at any convenient angle. Through I, draw F J = E H. With centres E, F, and radius E C, strike arcs. With centres H, J, and radius H K, strike arcs meeting at A.——**FIG. 346.**—POINTED TREFOIL ARCH. Points L, M, N, are given. Draw N O, L V. Join L M, M N. Bisector of L M gives centre P on L O. Centre R may come at any convenient point on the bisector Q R; &c.——**FIG. 347** shews a useful method for drawing a CURVE. Say draw an *ogee*, to connect A and B. Join A B. Mark C *anywhere* on A B. With centres A, C, and *any* convenient radius, get D. From D, through C, draw a line. The bisector of B C gives E. With centres D, E, and radii D C, E C, strike arcs. Mark any number of points on A B, closer together towards A and B, as at G, H, I, J, &c. From these points draw horizontal lines. Draw perpendiculars from these points to meet the arcs. Make G K = G L, H M = H N, I O = I P; &c. Draw the curve through the points obtained. If a *simple* curve is required, as from A to C, or C to B, draw *one* arc, and proceed as shewn. By this method curves can be drawn for mouldings, &c.



FIGS. 345, 346.

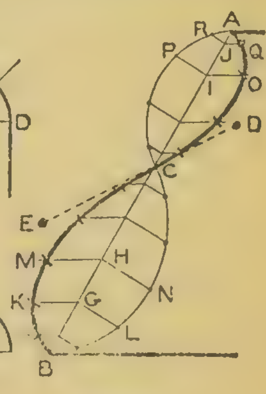


FIG. 347.

CHAPTER XV. ELEMENTARY SOLID GEOMETRY.

CLASS SHEETS 14 TO 20.

*** *The following abbreviations are used in this Chapter* :—ELEV., *for elevation* , HOR., *for horizontal* ; PARL., *for parallel* : PERP., *for perpendicular* ; PROJEC., *for projector* ; PT., *for point* ; VERT., *for vertical* ; H.P., *for horizontal plane* ; V.P., *for vertical plane* ; XY, *for intersecting line.*

DEFINITIONS, etc. :—**242.**—This Chapter should be commenced after Chapter V., and should be studied concurrently with the remaining chapters on Plane Geometry. It is suggested that the Teacher be provided with complete sets of *solid* and *skeleton* models : also with a straight piece of thick wire, and triangles, squares, polygons, and circles, cut out of cardboard or zinc.

A knowledge of elementary Solid Geometry is essential to the student about to study PERSPECTIVE.

243.—**A Vertical Plane** is perfectly upright. A **Horizontal Plane** is perfectly flat, like the surface of water when at rest. Planes, neither vertical nor horizontal, are **Inclined** or **Oblique.** Planes, meeting or intersecting at right angles, are *perpendicular* to each other.

244.—**A Solid** is bounded by *planes*, called FACES or SIDES, and it has *length*, *breadth*, and *thickness*. The boundary lines of the faces are called EDGES. (*Euclid* XI. *Defs.* 1, 2.) When 3 or more plane angles—not in the same plane—meet in one point, they form a **solid angle**, as at A, B, C, L, M, P, &c. Figs. 348, 349, 350. (*Euclid* XI. *Def.* 9.)

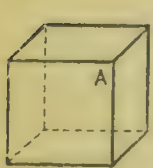


FIG. 348

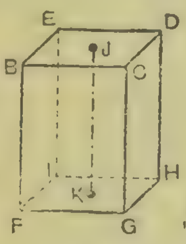


FIG. 349.

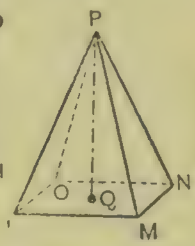


FIG. 350.

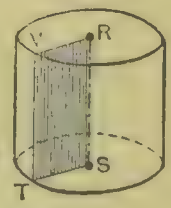


FIG. 351.

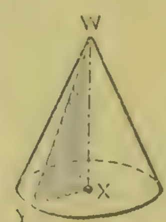


FIG. 352.

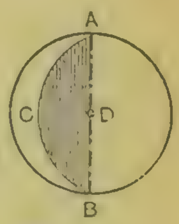


FIG. 353.

245.—**A Cube,** or HEXAHEDRON (Fig. 348), is bounded by six equal square faces. (*Euclid* XI. *Def.* 25.)

246.—**A Prism** (Fig. 349) has two *bases* or *ends*, B C D E, F G H I, which are equal and similar rectilineal figures, parallel to each other. The remaining *sides* or *faces* are parallelograms. (*Euclid* XI. *Def.* 13.)

247.—**A Pyramid** (Fig. 350) has one base, L M N O. The sides of a pyramid are triangles, L M P, &c., whose apexes meet in one point, P, called the *apex* or *vertex* of the pyramid. (*Euclid* XI. *Def.* 12.)

Note.—Prisms and pyramids may have bases of ANY regular or irregular rectilineal form, and they are called *triangular*, *square*, *pentagonal*, *hexagonal*, &c., according to the shape of the base. Thus, Fig. 349 is a square prism, and Fig. 350 is a square pyramid.

248.—**A Circular Cylinder** is described (or generated) by the rotation of a rectangle, R S T V, about one of its sides, R S (Fig. 351). Its bases are equal and parallel circles (*Euclid* XI. *Defs.* 21, 23.)

249.—A Circular Cone is described (or generated) by the rotation of a right-angled triangle, W X Y, about one of the sides, W X, that contain the right angle (Fig. 352). It has one circular base, and the solid tapers to a point called the *apex* or *vertex*. (*Euclid* XI. *Defs.* 18, 20.)

250.—A Sphere is a ball-shaped solid contained by one convex surface, every part of which is equidistant from the *centre*, D (Fig. 353). It is described (or generated) by the rotation of a semi-circle, A B C, about its diameter, A B. A line drawn from the centre to meet the surface is a *radius*. (*Euclid* XI. *Defs.* 14, 16, 17.) A **hemisphere** is the half of a sphere, cut off by a plane passing through the centre.

251.—The **Axis** of a PRISM, or of a CYLINDER, is an imaginary straight line, J K (Fig. 349), or R S (Fig. 351), which joins the centres of the *bases*. The *axis* of a PYRAMID, or of a CONE, is an imaginary straight line, P Q (Fig. 350), or W X (Fig. 352), connecting the *apex* and the centre of the *base*. An *axis* of a SPHERE is *any* diameter, A B, drawn through the centre D. (*Euclid* XI. *Defs.* 15, 19, 22.)

252.—A Right PRISM, PYRAMID, CYLINDER, or CONE, *has the axis perpendicular to the base*, as in Figs. 349, 350, 351, 352. In an **oblique** PRISM, PYRAMID, CYLINDER, or CONE, the axis is *inclined* to the base, as in Figs. 354, 355, 356, 357. An oblique cone is sometimes called a *scalene* cone. Whenever the word prism, pyramid, cylinder, or cone, is used by itself, in this book, a *right* solid is intended.

FIGS. 354. 355. 356. 357.

Note.—In *every* prism, the edges that connect the bases, are *parl. to the axis*. Therefore, in a *right* prism, such edges are *perp.* to the bases. In a *right* pyramid the triangular faces are equal and similar *isosceles* triangles. From the foregoing definitions, we see that *a right regular hexagonal prism, or a pyramid, means one with a regular hexagon for the base, and with the axis perp. to the base.*——The HEIGHT of a *right* prism, pyramid, cylinder, or cone, is the length of the *axis*. In an *oblique* prism, pyramid, cylinder, or cone, the *height* is the *perp. distance* between the 2 bases, or between the apex and the base.——If we cut away the upper portion of a pyramid, or of a cone, the part that is left is said to be **truncated**, and is called the **frustum** of a pyramid, or a cone. The cutting plane may or may not be *parl.* to the base. (Figs. 358, 359.) A truncated prism, or cylinder, is made by a plane *not* parl. to the bases.

FIG. 358.

FIG. 359.

FIG. 360.

FIG. 361.

253.—A Tetrahedron (Fig. 360) is a pyramid having 4 equilateral triangular faces. An **Octahedron** (Fig. 361) has 8 equal equilateral triangular faces. A **Dodecahedron** has 12 equal regular pentagonal faces. An **Icosahedron** has 20 equal equilateral triangular faces. (*Euclid* XI. *Defs.* 26, 27, 28, 29.) The *cube* and the above 4 solids are called the FIVE REGULAR SOLIDS, each of which can be inscribed in, or described about, a *sphere*.

(A) INTRODUCTION.

254.—BY THE LAWS OF PERSPECTIVE we represent an object as it appears to us, when seen from *one fixed position*. The *visual rays* from all visible portions of the object, *converge* to a point within the eye. In a *perspective* representation, the further an object is from us, the *smaller* it appears. Thus, *parallel* lines that recede from the spectator, appear to get *closer together*. In Fig. 362 we have a *perspective* representation of a rectangular block placed upon a shelf. The *parallel* edges E I, F J, converge as they recede from the eye; and the edge I J, although actually equal to E F, appears less in length than E F. Owing to this apparent diminution in length, it is obvious that we cannot apply a *scale* to a *perspective* drawing, for the purpose of ascertaining the *dimensions*.

255.—In **Solid Geometry,** or **Orthographic Projection,** the visual rays are PARALLEL, and an object is represented exactly the same size, by SCALE, *no matter what distance it may be placed from us.* By the aid of plans, elevations, and sections, machinery can be constructed, and a house or a ship can be built.

256.—Suppose we take a piece of drawing-paper, and tack a portion of it, A B X Y, to a vertical wall (Fig. 362), and tack the remaining portion, C D X Y, to a horizontal shelf. Then A B X Y is a **vertical plane,** and C D X Y is a **horizontal plane.** These two planes are at *right angles* to each other, and they meet or intersect in a straight line, **X Y,** which we call the **intersecting line,** or **ground line.** (*This line is sometimes lettered,* I L, *for* INTERSECTING LINE.) G H E F I J is a rectangular block placed upon the shelf, with two of its faces parallel to the wall. If we imagine the eye to be *vertically over every part of the block, at the same time,* and then run the point of a pencil around the lower edges of the block, we draw a rectangle, G K L H, which is the **plan** of the block. Now, suppose the position of the eye to be changed, and let us look forward in a direction perpendicular to A B X Y, the eye being *opposite to every part of the block, at the same time.* Then, pt. E is opposite to, or is **projected** at, pt. e', on the *vertical plane* A B X Y; pt. F is *projected* at f'; pt. G, at g'; and H, at h'. The rectangle, e' f' g' h', is the **elevation** of the block. Such lines as E e', F f', G g', H h', are called **projectors.**

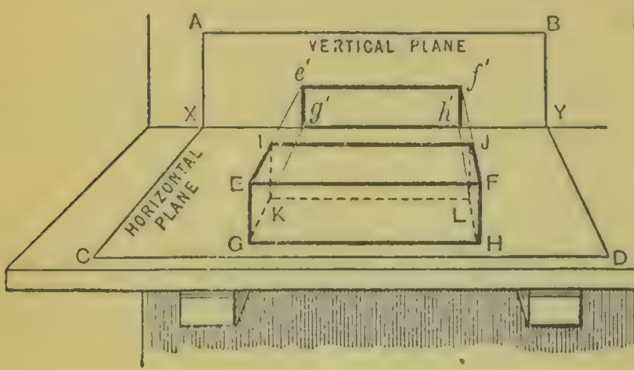


FIG. 362.—*Perspective view.*

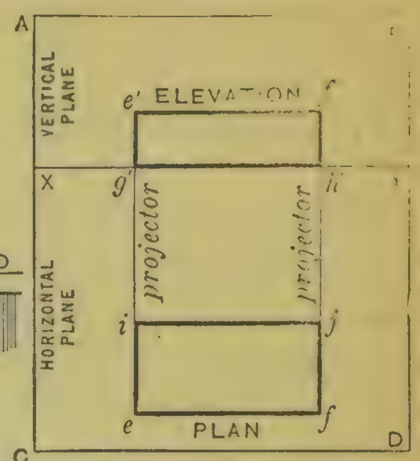


FIG. 363.—*Orthographic projections.*

257.—It is evident that it would be impossible to make drawings upon a piece of paper folded in the manner shewn in Fig. 362. If we take the paper from the wall and shelf, and pin it to a drawing-board, it becomes *one flat* surface as shewn at A B C D, Fig. 363, and the PLAN and ELEVATION, already drawn in Fig. 362, appear as indicated in Fig. 363. The operations explained in Fig. 362, can be performed upon this *flat* sheet of paper. We draw the INTERSECTING LINE, X Y (Fig. 363), at *any* convenient position, horizontally, across the paper. All *above* X Y, represents the **vertical plane of projection,** and all *below* X Y, represents the **horizontal plane of projection.** We draw ELEVATIONS (*vertical projections*) upon the VERTICAL PLANE OF PROJECTION, and PLANS (*horizontal projections*) upon the HORIZONTAL PLANE OF PROJECTION.

Note.—An *Orthographic Projection* means the correct representation of an object upon a plane. The *projectors*—the lines by which we obtain the *projection*—are drawn perpendicular to the vertical or horizontal planes, and are therefore *perpendicular* to X Y. For Figs. 362, 363, and for many of the following diagrams, the student should fold a piece of cardboard to represent the vert. and hor. planes of projection, and place the model in its proper position. This will help the student to see how the problem is to be worked.

(B) PROJECTIONS OF LINES, PLANES, AND SOLIDS.

The diagrams in this Chapter are drawn to a reduced scale. They should be copied FULL-SIZE. *The working lines are shewn as dotted lines; the student should draw them as* VERY FINE LINES. *The hidden edges of solids should be drawn as in the diagrams.*

FIG. 364.—A B C D represents the drawing-paper. X Y is the *intersecting line.* A B X Y is the *vertical plane,* for *elevations.* X Y C D is the *horizontal plane,* for *plans.*

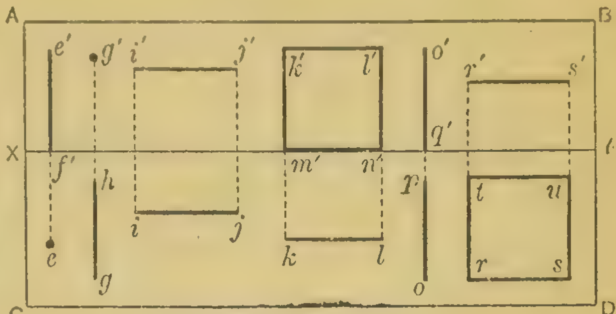


FIG. 364.

Projections of a Line, $1\frac{1}{2}''$ long, parl. to one or both planes of projection.—**I.**—Line $e'f'$ is the elev. of a line, parl. to the V. P. (*vertical plane*), and standing vertically upon the H. P. (*horizontal plane*). Get its plan. Draw $e'e$, perp. to X Y. Then e is the required PLAN of the line $e'f'$.

II.—gh is the plan of a hor. line, perp. to the V. P. Get its elev. Draw gg', perp. to X Y. Then g' is the END ELEV. of line gh.

III.—$i'j'$ is the elev. of a line parl. to the H. P. and V. P. Get its plan. Draw projecs. (*projectors*) from i', j', perp. to X Y. Draw the PLAN ij, parl. to X Y.

IV.—Projections of a Square, of $1\frac{1}{2}''$ sides.—The square, $k'l'm'n'$, stands on the H. P., and is parl. to the V. P. Get its plan. Draw projecs. from k', l'. Draw the PLAN kl, parl. to X Y.

V.—op is the plan of a square, standing vertically on one side, and perp. to the V. P. Get its elev. Draw projec. from o. Make the ELEV. $o'q' = op$.

VI.—$r's'$ is the elev. of a square, parl. to the H. P., and with 2 sides perp. to the V. P. Get its plan. Draw projecs. from r', s'. Draw rs, parl. to X Y. Make rt, $su = rs$. Join tu. Then $rstu$ is the PLAN of the square $r's'$.

Note (a).—The *plan* and *elev.* of the same point are *always* in a line drawn at RIGHT ANGLES to X Y. Thus, lines $e'e$, $g'g$, &c., are *perp.* to X Y.——No matter what the *length* of $e'f'$ or gh, the plan of $e'f'$ is a *point* e, and the elev. of gh is a *point* g'.——If e is the plan of a given point, $1\frac{1}{2}''$ above the H. P., then pt. e' is its elev.

(b)—If the vert. line $e'f'$ *touches* the V. P., its plan is *on* X Y, at f'. If gh, or ij, *rests upon* the H. P., their elevs. are *on* X Y.——Pt. f' is *underneath* e, and h is *behind* g'.——When a line is parl. to *both* planes of projection, its plan and elev. are each *parl.* to X Y, as at $i'j'$, ij.——It is customary to indicate the PLAN of a pt. by a small letter, as at e, and its ELEV. by the addition of an accent, as at e'.

FIG. 365.—A Line, $1\frac{1}{2}''$ long, **inclined** to the H. P. or V. P.—**I.**—$a'b'$ is the elev. of a line, *inclined* at 60° to the H. P., but *parl.* to the V. P.; that is, pts. a', b', are equidistant from the V. P. Get the plan. Draw projecs. from a' and b'. Draw the PLAN ab, parl. to X Y.

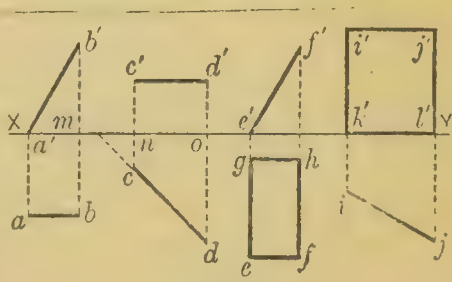


FIG. 365.

II.—cd is the plan of a hor. line, *inclined* at 45° to the V. P. Get the elev. Draw projecs. from c, d. Draw the ELEV. $c'd'$, parl. to X Y.

III.—A Square, of $1\frac{1}{2}''$ sides, **inclined** to the H. P. or V. P.—$e'f'$ is the elev. of a square, *inclined* at 60° to the H. P. Get its plan. Draw projecs. from e', f'. Draw ef, parl. to X Y. Make eg, $fh = e'f'$. Join gh, then $efgh$ is the PLAN of the square $e'f'$.

IV.—ij is the plan of a square, standing on the H. P., and *inclined* at 30° to the V. P. Get the elev. Draw projecs. from i, j. Make $k'i'$, $l'j' = ij$. Join $i'j'$; then $i'j'k'l'$ is the required ELEV.

Note (a).—a' b' and c d are seen their *real* lengths, because they are *parl.* to their planes of projection. When a line is *not parl.* to its plane of projection, we see it *foreshortened*, that is, it appears *less* than its *real* length, as at a b, or c' d'.——When an inclined line is *parl.* to the V. P., *its plan is parl. to* X Y, as at a b. When a line is *parl.* to the H. P., whatever inclination it may have to the V. P., its *elev. is parl. to* X Y, as at c' d'.——If the V. P. *contains* the line a' b' (that is, if a' b' lies *wholly* in the V. P.), the plan is *on* X Y, at a' m. If the H. P. *contains* c d, the elev. is at n o.

(b).—If a b is a *given* projection of a line, parl. to the V. P., and we are told that it makes a *given angle* with the H. P. at a, draw the projecs., and draw a' b' at the stated angle. Then a' b' gives us the *true length* of a b. Proceed on the same principle, to find the *true length* of c' d', if its angle of inclination to the V. P. be given.

(c).—The sides e' f', e g, f h, i j, i' k', j' l', are drawn their *real* lengths, because they are *parl.* to their planes of projection. Sides e f, g h, are projected on the same principle as a b; and i' j', k' l', on the same principle as c' d'.——For practice, draw the projections of an OBLONG PLANE, under similar conditions to e' f' and i j.

FIG. 366.—A Cube, of $1\frac{1}{2}''$ edges.—**I.**—a' b' c' d' is an elev. The cube stands on the H. P., and 2 faces are parl. to the V. P. Get the plan. Draw projecs. Draw a b parl. to X Y. Make a e, b $f = a$ b. Join e f. Then a b e f is the required PLAN.

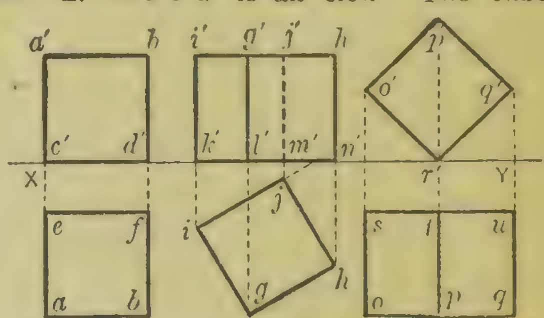


FIG. 366.

II.—In the plan g h i j, two of the vert. faces make 30° with the V. P. The cube stands on the H. P. Get the elev. Draw projecs. Make k' i', l' g', m' j', n' $h' = g$ h. Complete the ELEV. by joining i' h'. The edge m' j' is not visible, so its position is indicated by a *link-line.*

III.—The face o' p' q' r' is parl. to the V. P.; and the two faces r' o', r' q', make equal angles (45°) with the H. P. Get the plan. Draw projecs. Draw o q, parl. to X Y. Make o s, p t, q $u = o'$ p'. Complete the PLAN by joining s u.

Note (a).—Pts. e, f, are behind a', b'; and pts. c', d', are underneath a, b.——Edges i' k', g' l', h' n', are seen their *actual* length, because they are *parl.* to the V. P.——The elev. of the face, g h i j, appears as *one hor. straight line*, i' h', because each angle is the *same* distance above the H. P. Thus, if g h i j is *given* as the plan of a *square*, then i' g' j h' is its elev., $1\frac{1}{2}''$ *above* the H. P.; and k' l' m' n' is its elev. *on* the H. P.

(b).—The face o' p' q' r', being *vertical*, appears as *one* straight line at o p q. Pt. r' is *underneath* p. Pts. s, t, u, are *behind* o', p', q'.——Observe that in Fig. 366 we obtain projections of *square planes*, on the same principle as those given in Figs. 364, 365.

FIG. 367.—A Square Prism, $1''$ long, with bases of $1\frac{1}{2}''$ edges.—**I.**—a' b' c' d' is an elev. shewing a base parl. to the V. P. Obtain the hor. projection, or plan. Draw projecs. Draw a b, parl. to X Y. Make a e, b $f = 1''$. Join e f. Then a b e f is the required PLAN.

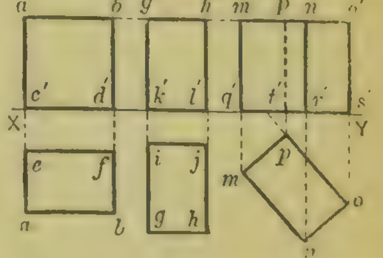


FIG. 367.

II.—g h i j is a plan, at *right angles* to a b e f. The square bases g i, h j, are perp. to both planes of projection. Get the *vertical projection*, or *elev.* Draw projecs. Make k' g', l' $h' = g$ i. Join g' h'.

III.—At m n o p the square bases o p, n m, are inclined at 45° to the V. P. Get the elev. Draw projecs. Make q' $m' = 1\frac{1}{2}''$. Draw m' o', parl. to X Y: &c.

Note (a).—If a b e f is *given*, obtain the elev. as shewn. If g' h' k' l' is *given*, obtain the plan as shewn.——g' h' k' l' is an elevation of the prism a' b' c' d', viewed at *right angles.*——Observe that the 3 *plans*, although in different positions, are alike in *shape*; also, that the 3 *elevs.* are the *same height.*

(b).—*We usually require at least* TWO PROJECTIONS *of a solid in order to ascertain its dimensions.* We must have either two elevs. at *right angles*, as at *a′ b′ c′ d′*, *g′ h′ k′ l′* ; or, a *plan* and an *elev.*, as at *a b e f*, *a′ b′ c′ d′*. Thus, *a′ b′ c′ d′* does not give us the *thickness* of the solid, but we get it at *g′ h′*, or *a e*.

FIG. 368.—**An Equilateral Triangle,** of $1\frac{1}{2}''$ sides.—**I.**—*a′ b′ c′* is an elev. parl. to the V. P. Get the plan. Draw projecs. Make the PLAN, *a b c*, parl. to X Y.

II.—*d e f* is a plan. The side *d f* rests upon the H. P. Get the elev. Draw projecs., then *d′ f′* is the elev. of the *base*. To get the position of the apex *e′*, we consider the side *d f* as a hinge, and rotate the triangle upon it until it becomes *horizontal*, as at *d* E *f*. Then *e* E is the length of the *altitude*. Make *g′ e′* = *e* E. Join *d′ e′*, *f′ e′*.

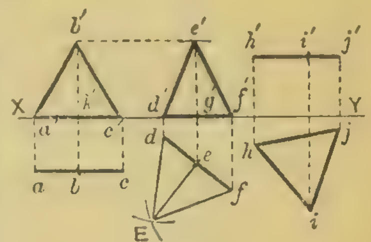


FIG. 368.

III.—The plan *h i j* is parl. to the H. P. Either side may make *any given angle* with the V. P. Get the elev., 1″ above the H. P. Draw projecs. Get the ELEV. *h′ i′ j′* as a hor. line, 1″ above X Y.

Note (a).—If *a b c* is a *given* plan, get the elev., by making *k′ b′* equal to the altitude, as at *g′ e′*.——The triangular plane *d e f* may be inclined at any *given* angle to the V. P. Observe that *a′ b′ c′*, *d′ e′ f′*, are the *same height*.

(b).—If *d′ e′ f′* is *given*, then to get the plan, make *d f* equal the side of an equilateral triangle, of which *g′ e′* is the altitude.——*h′ i′* is the elev. of *h i*; *i′ j′*, of *i j*; and *h′ j′*, of *h j*.

FIG. 369.—An Equilateral Triangular Prism, length $1\frac{3}{4}''$, base $1\frac{1}{2}''$ edges.—**I.**—*a′ b′ c′* is an end elev., with the bases parl. to the V. P. Get the plan. Draw projecs. Make *a d* = $1\frac{3}{4}''$. Draw *a b c*, *d e f*, parl. to X Y. Join *b e*, *c f*.

II.—Get a side elev. of *a′ b′ c′*, looking in direction Z. Draw a hor. line from *b′*. Make *i′ j′* = $1\frac{3}{4}''$. Raise perps. *i′ g′*, *j′ h′*.

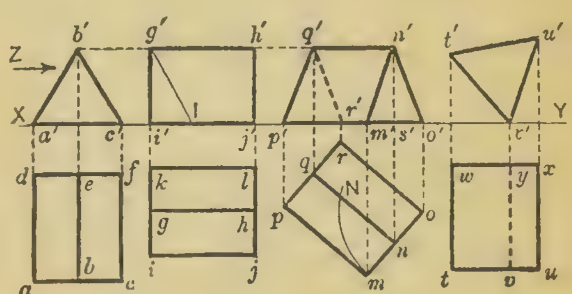


FIG. 369.

III.—Get the plan of *g′ h′ i′ j′*, without referring to *a′ b′ c′*, *d c*. Draw *g′* I, at 30° to *g′ i′*. Then *g′ i′* I represents *half* of the base. Make *g i*, *g k* = *i′* I. Draw *k l*, *g h*, *i j*, parl. to X Y.

IV.—Copy *d c*, or *k j*, in the position *m r*. Then *m r* is a plan of the prism, with one face resting on the H. P. Get the elev., without referring to *a′ b′ c′* or *g′ j′*. With centre *o*, and radius *o m*, strike the arc *m* N. Then *n* N is the altitude of the triangle. (*See e* E, *Fig.* 368.) Make *s′ n′* = *n* N. Draw *n′ q′*, parl. to X Y. Join *n′ m′*, *n′ o′* : &c.

V.—*t′ u′ v′* is an end elevation. Get the plan. Proceed as at *a′ b′ c′*.

Note (a).—*a b c*, *d e f*, are simply repetitions of *a b c*, in Fig. 368.——*j′ h′* is a side elev. of the *triangle a′ b′ c′* : it appears as a *line*, because the triangle is *perp.* to the V. P.——*j h l* is a plan of the *triangle j′ h′*.

(b).—The base *p q r*, or edge *o r*, may make any *given* angle with X Y.——*q′ n′* is a *hor.* line, because *q′* and *n′* are the *same height* above the H. P.——The *faces t′ v′*, *v′ u′*, *t′ u′*, are *perp.* to the V. P., and may be inclined at any *given* angle to the H. P. The *bases* are *parl.* to the V. P., therefore *t v u*, *w y x*, are parl. to X Y. *Proceed in the same way to get the plan, if the end elev. of* ANY PRISM *be given.*

FIG. 370.—A Regular Hexagon of 1″ sides.—**I.**—*a′ b′ c′ d′ e′ f′* is perp. to the H. P., and parl. to the V. P. Get the plan, 1½″ from the V. P. Draw projecs. Draw *a b c d*, parl. to X Y, and 1½″ from it.

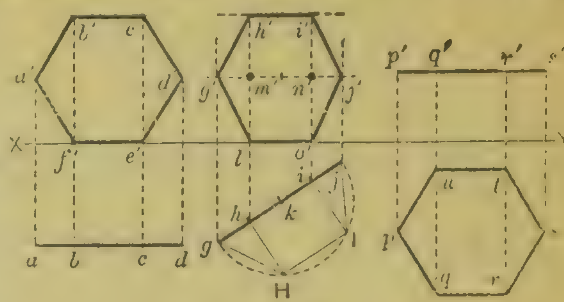


FIG. 370.

II.—Copy *a b c d* at *g h i j*. Then *g h i j* is a plan, when perp. to the H. P., and *inclined* at any given angle to the V. P. One side of the hexagon rests on the H. P. Project the elev., *without referring to a′ b′ c′ d′ e′ f′*. Bisect the *diagonal g j* at *k*. Strike a semi-circle. Draw *h* H, *i* I, perp. to *g j*. Join *g* H, H I, I *j*. On the *long* diagonal *g j*, as a hinge, we have rotated *half* the hexagon, until it becomes *parl.* to the H. P. Then *h* H equals half the length of a *short* diagonal. Make *l′ m′*, *m′ h′* = *h* H. Draw hor. lines through *h′*, *m′*. Draw the projecs., and complete the ELEV. as shewn.

III.—The hexagon *p q r s t u* is parl. to the H. P. Project the elev. when the hexagon is 1″ above the H. P. Proceed as shewn. (*See h i j, Fig.* 368.)

Note (a).—If *g j* makes 90° with X Y, the elev. is a vert. line = *h′ l′*.——If *p q r s t u* rests upon the H. P., its elev. is *on* X Y.

(b).—*The* PLAN *of a* VERTICAL PLANE, *of* **ANY shape,** *is always a* STRAIGHT LINE. *The* ELEVATION *of a* PLANE, *of* **ANY shape,** *that is* PERPENDICULAR *to the* V. P., *is always a* STRAIGHT LINE.

(c).—The rotating of a plane, as at *g* H I *j*, is a most *important operation*. It is termed the **rabattement** of a plane. (*See d f* E, *Fig.* 368.) The rotated plane must be constructed *parl.* to the H. P. in *plans*, or *parl.* to the V. P. in *elevs.* Sometimes the *whole* of the plane has to be thus constructed.——If the plan of a REGULAR PENTAGON, or OCTAGON, &c., be given, as at *g j*, with one side resting on, or parl. to, the H. P., rotate on this side the whole of the polygon, until *parl.* to the H. P. Then obtain the elev. as in Fig. 370. (*See Prob.* 52.)

FIG. 371.—A Hexagonal Prism, 2″ long, base ⅞″ edge.—**I.**—*a′ b′ c′ d′ e′ f′* is an end elev. Get the plan. Draw projecs. Make *a g* = 2″. Draw *a d*, *g h*, parl. to X Y. (*See a b c d, Fig.* 370.) Mark the centre 2, and join *b′ f′*, *c′ e′*; we then divide the *long diagonal a′ d′* into four *equal* parts, at 1, 2, 3. These equal parts are referred to further on.

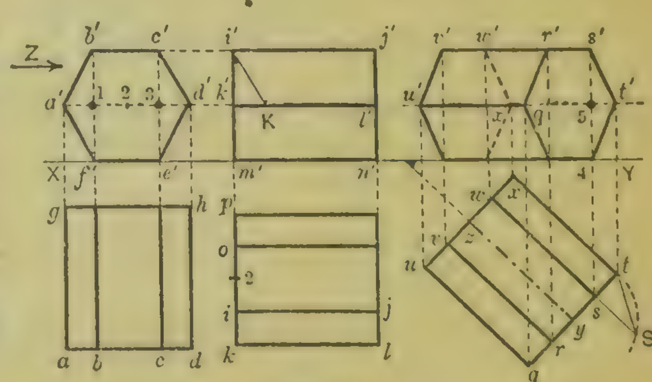


FIG. 371.

II.—Get a side elev. of the prism, looking at right angles, in direction Z, as in Fig. 369. Draw hor. lines from *c′*, *d′*. Make *m′ n′* = 2″. Raise perps. *m′ i′*, *n′ j′*.

III.—Get the plan of *i′ n′*, without referring to *b′ e′* or *g d*. Draw *i′* K, at 30° to *i′ k′*. Then *i′* K = length of edge of base; and *k′* K = ¼ of the *long diagonal*, as at 3 *d′*. Make *k i*, *i* 2, 2 *o*, *o p* = *k′* K. Draw *k l*, *i j*, &c., parl. to X Y.

IV.—*u t* is a plan of the same prism. One face rests upon the H. P. The axis is inclined to the V. P. at 45°. Draw *v z* at 45° to X Y, and copy the dimensions from *g d* or *p l*. Project the elev. Get *s* S, as at *i* I, Fig. 370. Make 4 5, 5 *s′* = *s* S. Complete the ELEV. as shewn.

Note (a).—If *g d*, *p l*, are *given* plans, get the elevs. by the same construction as at *s* S. ——For practice, get plans and elevs. of PENTAGONAL and OCTAGONAL PRISMS, placed under similar conditions.

(b).—When drawing from MODELS of HEXAGONAL PRISMS, or HEXAGONAL PYRAMIDS, *it is important to observe that a′* 1, 1 2, 2 3, 3 *d′*, *are* EQUAL *distances upon the* LONG *diagonal a′ d′*.

FIG. 372.—I.—*a b c* is the plan of an **Equilateral Triangular Prism,** $1\frac{3}{4}''$ high, base-edge $= 1\frac{1}{2}''$. It stands upon the H. P. Get its elev. Make *d′ a′*, *f′ c′* $=1\frac{3}{4}''$. Join *a′ c′*. Edge *b′ e′* is invisible.

II.—*g h i j k* is the plan of a **Pentagonal Prism,** $\frac{7}{8}''$ high, edge of base $= 1\frac{1}{4}''$. The lower base is $\frac{3}{4}''$ above the H. P. Get the elev. Draw *l′ m′*, $\frac{3}{4}''$ above X Y, and *g′ j′*, $\frac{7}{8}''$ above *l′ m′*. Complete the ELEV. as shewn. The face *a b*, *b c*, *c d*, *g h*, or *g k*, &c., may make any given angle with the V. P. *Proceed in the same way for* RIGHT PRISMS *that have* ANY *number of sides.*

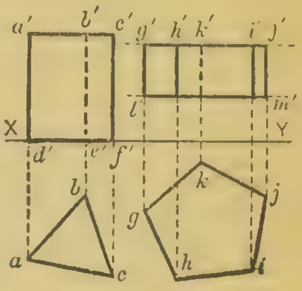


FIG. 372.

FIG. 373.—I.—*a b c d* is the plan of an irregularly shaped **Prism,** standing on the H. P. Height $= 2''$. Make *d c* $= 1\frac{1}{4}''$: get the rest in proportion. Project the elev. Draw *a′ c′*, $2''$ above X Y: etc.

II.—Project the plan of the **Prism** given at *g′ h′ k′ l′*. The shape of each base is shewn at J H N L. (Make *g′ h′* $= 2''$. Get the rest in proportion.) Produce *k′ l′* to the right. Drop perps. from J, N, H, to 1, 2, 3. Then, from 1 to 3 represents the *width* of the solid when seen in *plan*. Draw projecs. Make *j h* = 1 3. Set off *j l* = 1 L, and *l n* = L 2. Complete the PLAN.

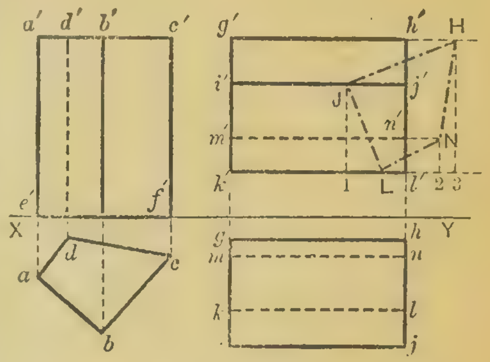


FIG. 373.

Note (a).—J H N L is a view of the solid at right angles to *g′ h′ k′ l′*. Thus, J H is the true length of *j′ k′*, but *j h*, in the plan, appears the length of 1 3.

(b).—In the same way, J L is the true length of *j′ l′*, and 1 L is the length of *j l* in the plan. Edges *m n*, *k l*, are invisible in the plan.

FIG. 374.—Pyramids with their axes vertical.—I.—*a b c d* is the plan of a **Square Pyramid,** $1\frac{3}{4}''$ high, and base of $1\frac{1}{2}''$ edges. Get the elev., standing on the H. P. Draw *e e′*, making the axis *f′ e′*$=1\frac{3}{4}''$. Get *a′*, *b′*, *c′*, *d′*. Join *e′ a′*, *e′ b′*, *e′ c′*, *e′ d′*.

II.—*g h i j k l* is the plan of a **Hexagonal Pyramid,** height $= 2''$, edge of base $= 1''$. Get the elev., standing on the H. P. Project the axis *n′ m′*$=2''$. Project *g′*, *h′*, &c. Draw *m′ g′*, *m′ h′*, &c.

Note (a).—In the *plan* of a RIGHT PRISM, standing on its base, the apex comes over the centre of the base, and the axis, no matter what length, appears a *point*, as at *e*, or *m*.

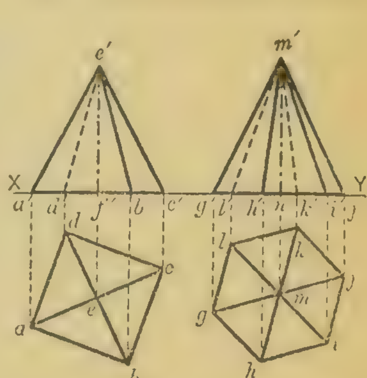


FIG. 374.

(b).—*a′ e′ b′* is the elev. of the face *a e b*, and *i′ m′ j′* is the elev. of *i m j*.

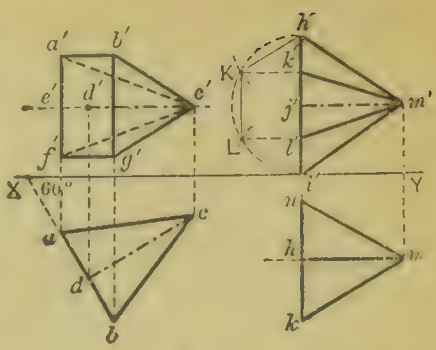


FIG. 375.

FIG. 375.—Pyramids with their axes horizontal.—I.—Draw the plan of a **Square Pyramid.** Axis ($1\frac{3}{4}''$) horizontal. Base ($1\frac{1}{2}''$ edge) inclined at 60° to the V. P., and with two of its edges vertical. Draw *a b* ($1\frac{1}{2}''$) at 60° to X Y. Bisect *a b* at *d.* Draw the axis *d c* ($1\frac{3}{4}''$). Join *c a, c b.*

II.—Obtain the elev. of *a b c.* The axis to be 1″ above the H. P. Draw *d′ c′*, 1″ above X Y. Make *e′ a′, e′ f′=d a.* Draw hor. lines *a′ b′, f′ g′.* Join *c′ a′, c′ b′, c′ f′, c′ g′.*

III.—Draw the elev. of a **Hexagonal Pyramid.** Axis = $1\frac{1}{2}''$. Base to be perp. to *both* planes of projection. A long diagonal (2″) of the base to be vertical, and rest upon the H. P. Draw a vert. line *h′ i′*=2″. Get the *rabattement* of half the base, at *h′* K L *i′*. Draw K *k′*, L *l′*. (*See j* I H *g, Fig.* 370.) Draw the axis *j′ m′* ($1\frac{1}{2}''$) parl. to X Y. Join *m′ h′, m′ k′*, &c.

IV.—Get the plan of *h′ m′ i′*, the axis to be $1\frac{1}{4}''$ from the V. P. Draw *h m*, $1\frac{1}{4}''$ from X Y. Make *h k, h n=k′* K. Join *m k, m n.*

Note.—The faces *b′ g′ c′, k′ l′ m′*, appear as *lines* at *b c, k m*, in the plans.——We can get *k′ j′ l′*, by dividing *h′ i′* into *four equal parts.* (*See a′ d′, Fig.* 371.)

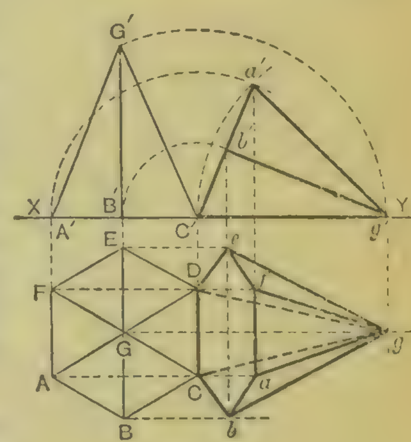


FIG. 376.

FIG. 376.—Project the elev. and plan of a **Hexagonal Pyramid,** with a triangular face resting on the H. P., and one edge of the base perp. to X Y. (Base-edge = 1″, axis = 2″.) First, draw the plan and elevation, *when standing on its base*, with C D perp. to X Y, as at A B C D E F and A′ G′ C′. Then, with the arcs, as shewn, turn A′ G′ C′ over, on the edge C′, as a hinge, until the face C′ G′ touches the H. P. at C′ *g′*. With centre *g′*, and radius *g′* C′, cut the arc A *a′* at *a′*. Complete the ELEV.——To obtain the plan :—Draw projecs. from *a′, b′, g′*, and draw lines from A, B, &c., parl. to X Y, to get *a, b, f, e, g.* Complete the PLAN.

Note (a).—C′ *a′ g′* is the same shape and size as A′ G′ C′.——The arcs G′ *g′*, A′ *a′*, B′ *b′*, are *parl.* to the V. P. Lines G *g*, A *a*, B *b*, are the *plans of these arcs.*

(b).—*Adopt the same method for* ANY PYRAMID. On the same *principle* of construction, a CONE, of given size, can be projected, when lying on its side.——If *a′* C′ *g′* be a *given* elev., find its plan by the method explained at *i′* K, Fig. 371. Thus, from *a′*, draw a line at 30° with *a′ b′*, to meet *b′ g′*, at a point. The distance from *b′* to this point should then be set off four times on the diagonal *c b* : &c.

(c).—In all projections of solids, the projectors, &c., should only be drawn, as they are required, to obtain the necessary points. This prevents unnecessary confusion of lines, especially in an intricate diagram.——When each point is obtained, it should be at once ticked and lettered as in the diagram.——Always draw the working lines of sufficient length, so that their intersection may give the position of the required point.

(d).—The student should thoroughly understand the following facts with regard to the line X Y :—1st. That it represents the LINE OF INTERSECTION, made by the horizontal with the vertical plane of projection.—2nd. That when dealing with ELEVATIONS, the line X Y also represents the *horizontal plane*, projecting forward, at right angles with the vertical plane.—3rd. That when drawing PLANS, the line X Y also represents the *vertical plane*, projecting upwards, at right angles with the horizontal plane.

(e).—When instructed to project the plan or elevation of a solid, and the distances from the H. P. or V. P are not stated, assume *any* convenient distances.

FIG. 377.—To project a Solid, with a face inclined at any given angle to the H. P.—Say a **Cube**, of $1\frac{1}{2}''$ edges.

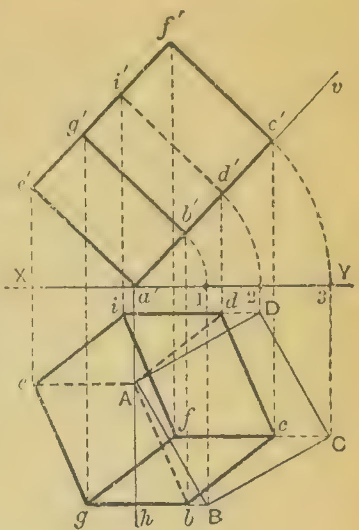


FIG. 377.

I.—Suppose A B C D to be the plan of a SQUARE, *resting upon the* H. P. Let C D make a given angle of 60° with X Y. Project the elev. of this square, when it is tilted up upon angle A, so that the plane that contains the square shall be *inclined* at 45° to the H. P., and be *perp.* to the V. P. Through A, draw *h a′*, *perp.* to X Y. At *a′* draw *a′ v* at 45° to X Y. Draw projecs. B 1, D 2, C 3. Then *a′* 1 2 3 is the elev. of the square *upon* the H. P. With *a′* as centre, transfer 1, 2, 3, to *b′ d′ c′*. Then *a′ b′ d′ c′* is the elev. of the *square* upon the required *inclined plane a′ v*.

II.—Suppose A B C D to be the plan of a CUBE. Get *a′ b′ d′ c′* as before. Draw *a′ e′*, *c′ f′*, equal to B C, and perp. to *a′ c′*. Join *e′ f′*. Draw *b′ g′*, *d′ i′*, perp. to *a′ c′*. Then *e′ c′* is the required ELEV. of the CUBE upon the *inclined plane a′ v*.

III.—Project the plan of the CUBE *e′ c′*. From B, C, D, draw lines *parl.* to X Y, and from *b′*, *g′*, *c′*, *f′*, &c., draw projecs. to meet these lines at *b*, *g*, *c*, *f*, &c. Join *e g*, *g f*, &c., then *e g b c d i* is the required PLAN of the CUBE.

Note (a).—*This method of construction can be used for* ANY *solid.* The original plan, A B C D, may be placed upon the H. P. in *any* position. One of the edges may *coincide* with *h a′*. The plane *a′ v* may be at *any* inclination.——Observe that in *any* projection of *any* solid, *the edges that are actually* PARALLEL, *remain* PARALLEL *in the projection.* Thus *a′ e′*, *b′ g′*, and *e g*, *i f*, and *e i*, *g f*, are *parallel.*

(b).—All lines or edges that are *perp.* to the original plan, A B C D, are seen their *true length* in the elev., and are *perp.* to *a′ v*. They are *parl.* to X Y in the plan.

(c).—If we join A C, B D, we have a square *pyramid.* If we draw this pyramid, the *axis* will be *perp.* to *a′ v*, and *parl.* to the V. P. in plan.

(d).—The line of intersection, made by *any* plane with the H. P. or V. P. is called the **trace** of that plane. The inclined plane, upon which the cube rests, has its **horizontal trace** at *h a′*, where it intersects the H. P.; and its **vertical trace** at *a′ v*, where it intersects the V. P.

FIG. 378.—I.—*a b c* is the plan of a **Tetrahedron** (*see* § 253) standing on one of its faces. *b c* makes 45° with X Y. Project the elev. Get *a′ b′ c′*. Consider *b d* as the plan of a *right-angled triangle.* Get its true shape at *b d* D. Thus, draw *d* D perp. to *b d*. Then because the true length of *b d*=*b a*, make *b* D=*b a*. Then *d* D=the length of the axis. Make *e′ d′*=*d* D. (The tint-lines in *b d* D need not be drawn.)

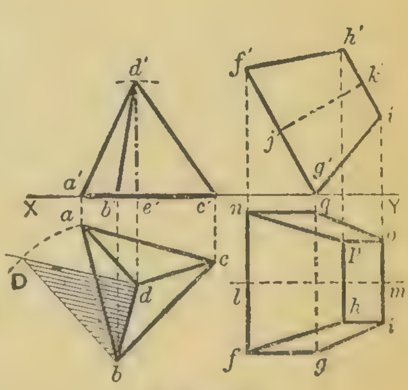


FIG. 378.

II.—*f′ g′ h′ i′* is an elev. of a **Truncated Square Pyramid.** The parl. square bases *f′ g′*, *h′ i′*, have, respectively, $1\frac{3}{4}''$ and 1″ edges. *f′ g′* makes 60° with X Y. The axis *j′ k′*=$1\frac{1}{4}''$. Get the plan, when *j′ k′* is $1\frac{1}{4}''$ from the V. P. Draw *l m*, $1\frac{1}{4}''$ from the V. P. Make *l f*, *l n*= *j′ f′*; and *m i*, *m o*=*k′ h′*. Complete the PLAN.

FIG. 379.—I.—*a′ b′ c′ d′* is an elev. of an **Oblique Prism,** with square bases of $1\frac{1}{2}''$ edges at *a′ b′*, *c′ d′*. (*See* § 252.) The axis, *r′ s′*, ($2\frac{1}{4}''$) is inclined at 68° to the base. Get the plan. Draw projecs. Make *c e*=*c′ d′*. Draw *c b*, *e f*, parl. to X Y. Complete the PLAN as shewn.

II.—*i j k l* is the plan of an **Octahedron,** of $1\frac{1}{2}''$ edges, with one of its diagonals vertical. (*See* § 253.) Get the elev. The three diagonals of an octahedron are of equal length, therefore make *n′ o′*, *o′ m′*=*i m* or *m k*. Through *o′*, draw a horizontal line, and project upon it, *i′*, *j′*, *l′*, *k′*. Join *m′ i′*, *n′ i′*, &c.

Note.—It will be seen that an *octahedron* is a solid formed, as it were, by two square pyramids placed base to base, *all* the edges being *equal.*

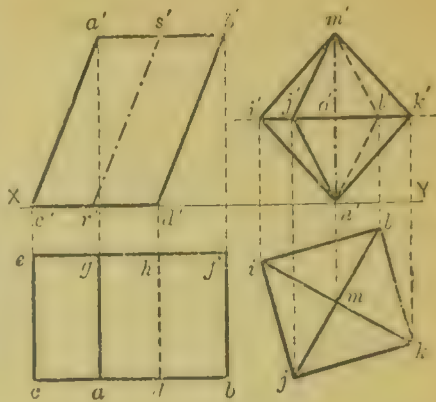


FIG. 379.

FIG. 380.—Alteration of the Intersecting Line, or Ground Line.—*a b c d*, *a′ c′ e′*, are the plan and elev. of a SQUARE PYRAMID. Height $=1\frac{1}{2}''$, base-edge$=1\frac{1}{4}''$. *a d* makes 55° with X Y. Project another elev., looking in ANY *given direction* Z. Draw a new INTERSECTING LINE, X′ Y′, in *any* convenient position, but at *right angles* to the direction Z. Draw projecs. *d d″*, *a a″*, *b b″*, &c., perp. to X′ Y′. Then *d″ b″* is the new elevation of the base. Make *f″ e″*=*f′ e′*. Join *e″ d″*, *e″ a″*, &c., and complete the new ELEVATION.

Note (a).—*This important principle of construction can be applied to* ANY *solid.* The *widths* of the new elev. are obtained from *a b c d*, and the *heights* are the *same* as at *a′ e′ c′*. All *above* X′ Y′ is the new V. P., and all *below* X′ Y′ is the new H. P.

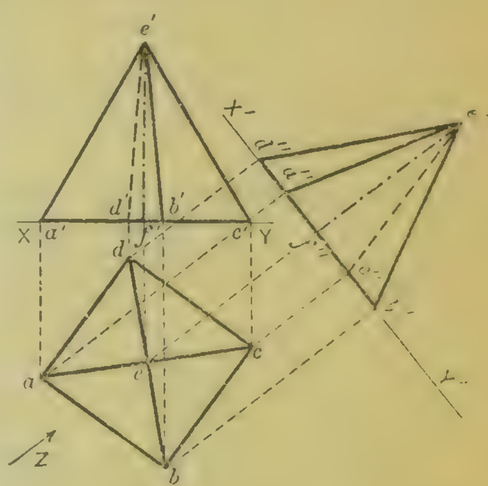


FIG. 380.

(b). Proceed in the same way if *the line* X′ Y′ *be given*, instead of the direction Z, or, if X′ Y′—or the vertical plane—is to make a *given angle* with a side or an edge of a solid.

FIG. 381.—I.—*a′ b′ c′ d′* is the given elev. of a **Square Plane** (*a′ c′*=$1\frac{1}{2}''$). *b′ d′* is nearer to the V. P. than *a′ c′*. Get the plan. Draw projecs. With centre *a*, and radius *a′ c′*, get *b*. Join *a b*.

II.—*e′ f′ g′* is a given elev. of an **Equilateral Triangle** ($2''$ sides) receding towards the V. P. *e′ f′* is parl. to the V. P., and *g′* is $1''$ above the H. P. Get the plan. Bisect *e′ f′* at *h*. Join *h′ g′*, and produce the line. With centre *f′*, and radius *f′ e′*, get G. Then *h′* G is the *true length* of the altitude *h′ g′*. Get a side elev. of the triangle, looking in direction Z. Make *e″ g″* = *h′* G. Draw *g″* 1, perp. to X Y. Then *e″* 1 is the plan-length of *h′ g′*. Draw *e f*, parl. to X Y, and make *h g* = *e″* 1 : &c.

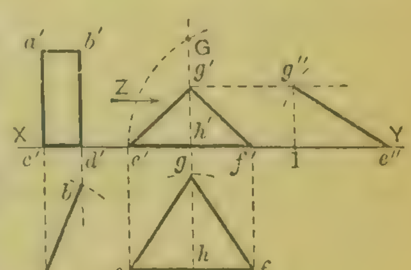


FIG. 381.

Note.—1 *e″ g″* is the angle made by the triangle with the H. P.; and 1 *g″ e″* is the angle made with the V. P. *Adopt the above construction to find the plan of a given elev. of* ANY *regular figure that is inclined to the* H. P., *under similar conditions to the triangle e′ f′ g′.*

FIG. 382.—*a b*, *a′ b′*, are the *given* plan and elev. of a LINE, *inclined to both planes of projection.* Find its **true length.** (These lines may be drawn *any* length, and at *any* angle, but the projecs. *a a′*, *b b′*, must be perp. to X Y. Two cases are given: the explanation applies to each diagram.) Draw lines from *a*, *b′*, parl. to X Y. With centre *a*, and radius *a b*, get B. Draw projec. B B′. Join *a′* B′. Then *a′* B′ is the TRUE LENGTH of *a b*, *a′ b′*.

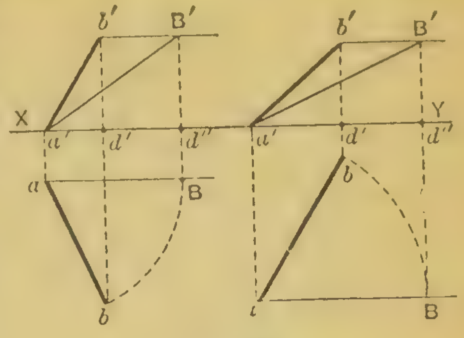


FIG. 382.

Note.—Suppose *a′ b′ d′* to be a right-angled triangle. Then, on *a*, as a pivot, we have turned the triangle round until it is *parl. to the* V. P., and we see its true shape at *a′* B′ *d″*. Therefore *a′* B′ must be the *true length* of the line *a′ b′*. B′ *a′ d″* is the true angle made by *b′ a′* with the H. P. By this prob. we can ascertain the true length of the *inclined edge of a solid;* say of *a c*, *i m*, Fig. 374; *b c*, Fig. 375; A′ G′, Fig. 376; *f′ h′*, Fig. 378.

By the same *principle*, the prob. may be worked thus:- With *a′* as centre, and radius *a′ b′*, strike an arc to meet X Y at C′. Through *b*, draw a line parl. to X Y. Get the plan of C′, on this line, at C. Join *a* C. Then *a* C is the *true length* of *a b;* and it shews the TRUE ANGLE made by *a b* with the VERTICAL plane.

FIG. 383.—I.—Draw the plan of a church. Scale, 1″ to 50′. Construct this scale. (A scale of inches, divided into fifths, will do; because each fifth part represents 10 ft. Bisect the 1st part, to represent 5 ft.) *a b* makes 35° with the V. P. The widths are given on the plan.

II.—Project the elev. The heights are as follows:—*a′* and *b′*, are 45′ above the ground; *c′* and *d′*, are 65′; *e′* is 100′; *f′* is 130′. Observe that the general form of the church is a combination of triangular and rectangular prisms, the tower being a square prism, surmounted by a square pyramid. The working-lines in the diagram sufficiently explain how the elev. is to be projected.

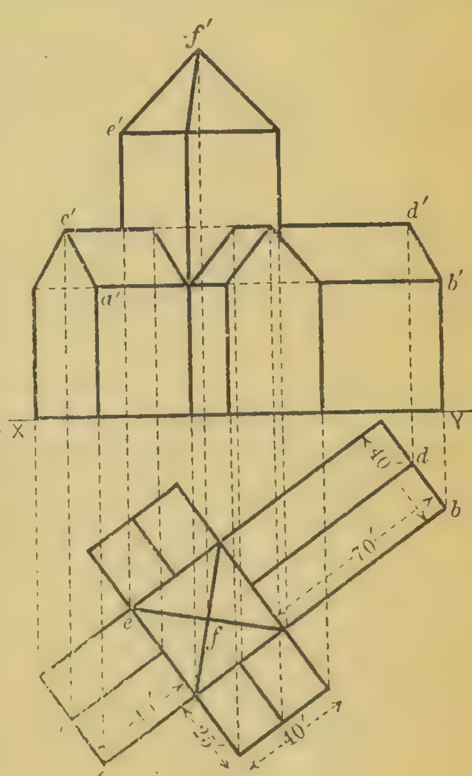


FIG. 383.

Note (a).—The edges that are *actually parl.*, are projected as *parl. lines* in the elev.—Adopt the same method if representing doors, windows, &c. The *widths* must be shewn in the *plan*, and the *heights* in the *elev.*

(b).—In this, and in most of the diagrams of this chapter, the edges of the solids are represented by very thick lines, for the sake of distinctness. *They should not be drawn quite so thick by the student.*

(c).—Find the *true lengths* of the edges *c a*, *e′ f′*, and state the angle of inclination of each of these edges to the H. P.

(d).—For practice, project an elevation of the church upon a vertical plane that is situated at the right side of the plan, and inclined to X Y, towards the right, at an angle of 78°. *(See Fig.* 380.)

(e).—For practice, project an elevation and a plan of the church, when it is placed upon an *inclined* plane. *(See Fig.* 377.)

FIG. 384.—**Projections of a Cylinder.** Diameter $= \frac{7}{8}''$; height $= \frac{1}{2}''$.—**I.**—Circle a is a plan when the axis is vert. Get the elev. Through a, draw b c, parl. to X Y. From b, c, draw projecs. Make d' b', e' c' = the height, *i.e.*, $\frac{1}{2}''$. Join b' c'. Line a' f' is the *axis*.

II.—Copy b' c' d' e' at g h i j. Then g h i j is a plan, when the axis, k l, is perp. to the V. P. Get the elev., the cylinder to rest on the H. P. Make m' k' $= k$ h. With centre k', and radius k' m', strike the circle.

III.—Get a side elev. of g' n' h' m', looking in direction Z. Draw hor. lines from n', k'. Make the axis l'' k'' $= \frac{1}{2}''$. Through k'', l'', draw vert. lines. The *plan* of l'' n'' m'', at T, is exactly the same shape and size as l'' n'' m''.

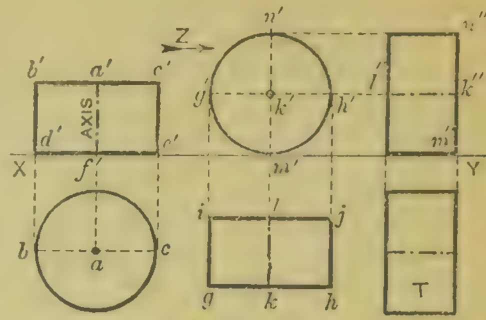


FIG. 384.

Note.—*A* CIRCLE, *when* PERP. *to the plane of projection, must always be projected as a* STRAIGHT LINE, *as at* b' c', d' e', i j, g h, n'' m''.

FIG. 385.—At o 4 q we have the plan of a semi-cylinder. Diameter $= 1\frac{1}{2}''$; height $= \frac{1}{2}''$. It has equal vertical divisions marked upon its curved surface, as at 1, 2, 3, 4 &c. Project the elev., and shew the positions of the vertical divisions, as in the diagram.

Note.—o 4 q may be the half-plan of *any* object of circular form, constructed around a central axis, such as a column, vase, cup, &c. The divisions 1, 2, 3, &c., may represent the positions of the flutings of a column, or the spacing-out of equal (or unequal) parts in a band of ornamentation on a vase, &c. Draw the half-plan of the circular band, &c., and obtain the positions of the required divisions in the elev. by this method.

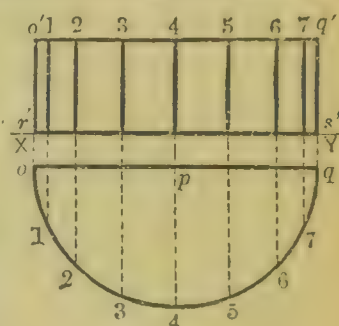


FIG. 385.

FIG. 386.—**Projections of a Cone.** Diameter of base $= \frac{3}{4}''$; axis $= \frac{7}{8}''$. **I.**—Get the elev. a' b' c'. Make a' c' $= \frac{3}{4}''$. Bisect a' c' at d'. Draw d' b' ($\frac{7}{8}''$) perp. to X Y. Project the plan. Draw projecs. With centre b, and radius d' a', strike the circle.

II.—Circle e' is an elev., with the axis perp. to the V. P. The apex, e', is further from the V. P., than the base. Get the plan. Draw projecs. Make e f = length of axis. Draw g h, parl. to X Y.

III.—Get a side elev. of g' j' h' i', looking at right angles, in direction Z. Make f'' e'' = f e: &c. The plan of j'' i'' e'' at K, is the same size and shape as j'' i'' e''. (*See Note, Fig.* 384.)

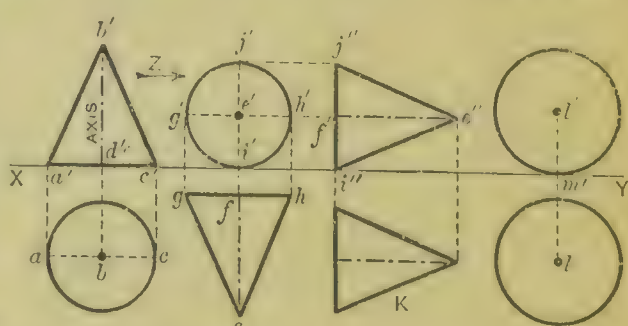


FIG. 386.

FIG. 387.

FIG. 387. Circle l' is an elev. of a **Sphere,** of $\frac{7}{8}''$ diameter. Get the plan. With centre l, and radius l' m', strike the circle.

Note.—Every projection of a *sphere*, in *plan* or *elev.*, is a CIRCLE. A sphere, in contact with a plane, touches it at *one* pt. The line from the centre to the pt. of contact, is *perp.* to the plane, as at l' m'.

FIG. 388.—A Circle, inclined to the H.P.—We have already seen that when a circle is PERP. to the plane of projection, its projection is a STRAIGHT LINE, as at $g\ h$, $j''\ i''$, Fig. 386; and that when PARL. to the plane of projection, its projection is a CIRCLE, as at b, and e', Fig. 386. When a circle is INCLINED to the plane of projection, it must be projected as an **ellipse.**

The line $a'\ b'$ (2″) is the elev. of a circle that is *perp.* to the V. P., and *inclined at* 45° to the H. P. Get the plan. Draw projecs. from a', b'. Draw $a\ b$, *parl.* to X Y. Then $a\ b$ is the plan of the *diameter* that is *parl.* to the V. P. Bisect $a'\ b'$ at c'. Get pt. 1, which is the *centre* of the circle. Make $1\ c$, $1\ d = c'\ a'$. Then $c\ d$ is the *diameter* that is *parl.* to the H. P., and it is seen its *true length.*

With centre c', and radius $c'\ a'$, strike a semi-circle, which represents half of the circle, rotated upon the *diameter* $a'\ b'$, until it is *parl.* to the V. P. Draw c' C, perp. to $a'\ b'$. On each side of C, mark E, F, *equidistant* from C, and about *midway* between C and a', b'. Transfer these pts. to $a'\ b'$, by drawing the perps. E e', F f'. Draw projecs. from e', f'. Make $2\ e$, $2\ g$, $3\ f$, $3\ h = e'$ E, or f' F. (Or, when e and g are obtained, draw $e\ f$, $g\ h$, parl. to $a\ b$.) Draw the elliptic curve, by hand, through a, e, c, f, &c.

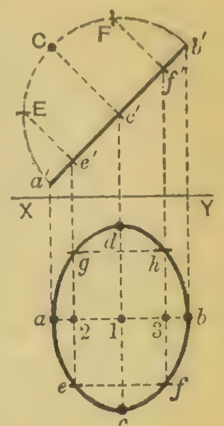


FIG. 388.

FIG. 389.—A Circle inclined to the V.P.—The line $i\ k$ (2″) is the plan of a circle that is *perp.* to the H. P., and *inclined at* 60° to the V. P. Get the elev., when standing on the H. P. The construction is the same as in Fig. 388. Get the vert. diameter, $j'\ l'$, by making $l'\ 4$, $4\ j' = j\ k$. Draw the hor. diameter, $i'\ k'$. Strike a semi-circle, and get the intermediate pts. m', n', o', p' : &c.

Notes to Figs. 388, 389.—(a).—To insure greater accuracy in the elliptic curve, a greater number of pts. should be marked on $a'\ b'$, and $j\ k$. A *French curve* may be used for the ellipse. (*See* § 12.) $2\ e$, $2\ g$, $6\ n'$, &c., are *ordinates.* (*See* § 211.)

(b).—Since $c\ d$, $a\ b$, and $j'\ l'$, $i'\ k'$, are *the major and minor axes of an ellipse*, the curves may be obtained by Probs. 141, 142, 143.

(c).—To obtain the *elev.*, if the plan $a\ c\ b\ d$ be given:—Draw projecs. from a, b. Make $a'\ b' = c\ d$. Adopt a similar method to get the *plan* if $i'\ l'\ k'\ j'$ is a *given elev.*

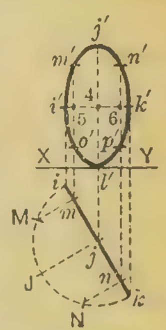


FIG. 389.

FIG. 390.—A Cylinder with its axis inclined to one of the planes of projection.—I.—Draw the elev. $a'\ b'\ o'\ k'$. Diameter = 2″. The axis (1″) is parl. to the V. P., and inclined at 60° to the H. P. Therefore the *bases* are inclined at 30° to the H. P. Make $o'\ k' = 2''$, $o'\ a' = 1''$: &c. Get the plan, the axis to be $1\frac{1}{4}''$ from the V. P. Draw $a\ k$, $1\frac{1}{4}''$ from X Y. Project the base $a'\ b'$, at $a\ c\ b\ d$, in the same way that the circle $a'\ b'$ was projected at $a\ c\ b\ d$, in Fig. 388. Draw $f'\ l'$, parl. to the axis. Draw projecs. from i', l', k'. Draw $c\ i$, $f\ l$, $b\ k$, $h\ m$, $d\ j$, parl. to the axis in the plan. Draw the curve from j, through m, k, l, to i.

II.—Draw the plan $p\ t\ z$. Diameter $= 1\frac{1}{2}''$. The axis ($1\frac{1}{4}''$) is parl. to the H. P., and inclined to the V. P. at 45°. Therefore the *bases* are inclined at 45° to the V. P. Make $p\ t = 1\frac{1}{2}''$, $r\ x = 1\frac{1}{4}''$. Get the elev., the cylinder to rest upon the H. P. Project the base $p\ t$, at $p'\ v'\ t'\ r'$, in the same way that circle $i\ k$ was projected at $i'\ l'\ k'\ j'$, in Fig. 389. Project pts. x', y', z', a', b', in the same way that pts. i, l, k, m, j, were obtained

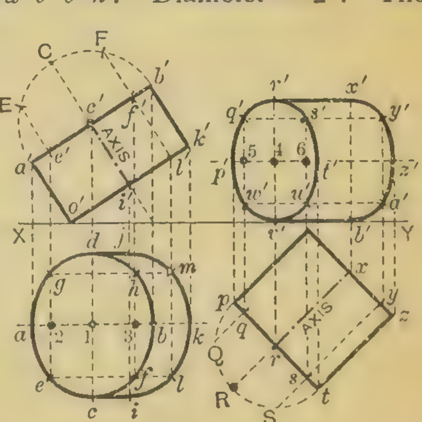


FIG. 390.

Note (a).—*d j*, *h m*, *b k*, &c., are *equal*, therefore the semi-ellipse, *j m k l i*, is precisely the same as *d h b f c*. Again, *r′x′*, *s′y′*, *t′z′*, &c., are *equal*, therefore the curve, *x′ y′ z′ a′ b′*, is the same as *r′ s′ t′ u′ v′*.

(b).—The straight lines, *d j*, *c i*, *r′ x′*, *v′ b′*, are *tangents* to the ellipses.

FIG. 391.—A Cone. Diameter of base = 2″; axis = $2\frac{1}{4}$″.—**I.**—Project an elev., when the base is perp. to the V. P., and inclined to the H. P. at 60°. Draw *a′ b′* (2″) at 60°. Bisect *a′ b′* at *c′*. Draw the axis *c′ i′* ($2\frac{1}{4}$″), perp. to *a′ b′*. Join *i′ b′*, *i′ a′*.

II.—Project the plan of *a′ b′ i′*, the axis to be $1\frac{1}{4}$″ from the V. P. Draw *b i*, $1\frac{1}{4}$″ from X Y. Get the plan of the base, and the apex *i*. From *i*, draw lines *touching* the ellipse. The lower portion of the base is hidden.

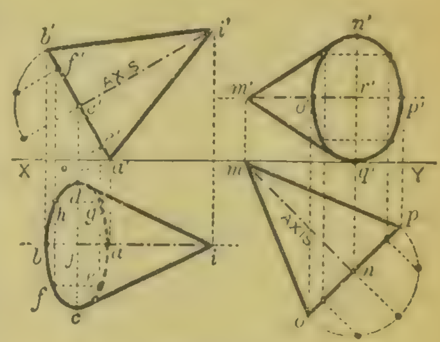


FIG. 391.

III.—Project a plan, when the axis is parl. to the H. P., and inclined to the V. P. at 45°. The apex to touch the V. P. at *m*. Draw *m n* ($2\frac{1}{4}$″) at 45° to X Y. Through *n*, draw *o p*, perp. to *m n*. Make *n o*, *n p* = 1″. Join *m o*, *m p*.

IV.—Get the elev. of *o p m*, the base to touch the H. P. Make *q′ r′*, *r′ n′* = *n o*. Draw the ellipse. From *m′*, draw *tangents* to the ellipse.

Note.—*j i* is the plan, and *r′ m′*, the elev., of the axis. *The lines from i, m′, are* TANGENTS *to the ellipse. They must* NOT *be drawn to points d, c, or n′, q′.*

FIG. 392.—Draw *a′ b′ c′ d′*, the side elev. of a LAMP-SHADE (*or* **frustum of a cone.** *See Fig.* 359). *a′ b′*, *c′ d′*, are circles of 1″ and 2″ diameters. *c′ d′* is perp. to the V. P., and inclined to the H. P. at 63°. The axis = 1″. Get the plan, with the axis $1\frac{1}{4}$″ from the V. P. Draw *a d*, $1\frac{1}{4}$″ from X Y. Project the plan of each circle. Draw the sides, *touching* the ellipses.

FIG. 393.—*i′ l′* is the elev. of a **curve,** contained in a plane that is parl. to the V. P. The plan of the curve is therefore a *straight line*, *i l*, parl. to X Y. Get an elev., when the plane of the curve is turned round at *any* given angle, as at *l m*. Mark a number of pts., as at *j′*, *k′*. Get *j*, *k*, in plan. With *l*, as centre, transfer *i*, *j*, *k*, to *m*, *n*, *o*. Draw projecs. from *m*, *n*, *o*, and hor. lines from *j′*, *k′*, to *n′*, *o′*. Draw the curve from *l′* to *m′*, through *n′*, *o′*.

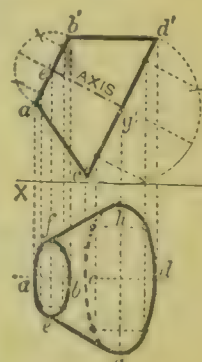


FIG. 392.

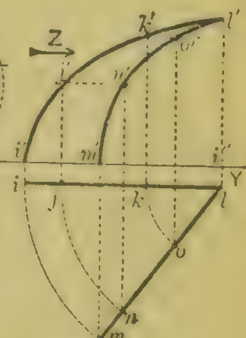


FIG. 393.

Note.—The given curve, *i′ l′*, may be *any* shape, provided it is contained in *one* plane.—*A curve, of* ANY *shape, that is contained in a plane, when viewed edgewise, is projected as a* STRAIGHT LINE. Thus, when *i′ l′* is viewed in direction Z, it appears a straight line, at *i″ l″*.

(C) SECTIONS OF SOLIDS BY PLANES.

258.—When a solid is cut into two portions, by a *plane* passing through it, the surface made by the division is called a **section.** The SECTION-PLANE may pass through a solid in *any* direction. In Fig. 394, the cube, *a b c d*, is cut by a vert. plane 1 2. The portion in front of 1 2 is supposed to be removed, and at *a′ f′* we see an elevation of the remaining portion of the cube. At *g′ h′ i′ j′* we shew the SECTION made by the plane 1 2. A section is usually distinguished by having a series of *equidistant* **section-lines** drawn across it, at 45° to X Y, whether in

plan or elev. These lines should be of *equal thickness*, and not *too close* together. Or, sections may be covered with a tint in colour.

259.—By *sections* we can shew the *internal structure* of an object, such as a box, a house, a ship, or a piece of machinery, &c. Thus, in Prob. 106, we have a SECTIONAL PLAN of a building, shewing how the walls, &c., would appear, if cut through by a hor. plane, the upper portion of the building being removed. A **sectional elevation,** or a **sectional plan,** is a projection viewed at **right-angles** to the plane of the section, by which we get the *true shape of the section.*

Sections of a Cube. Edge = $1\frac{1}{2}''$.—**FIG. 394.**—Get the elev. of $a\ b\ c\ d$, when the portion in front of the vert. *section-plane*, 1 2, is removed. Project $a'\ e'$, $c'\ f'$. Join $a'\ c'$. Project $g'\ h'$, $i'\ j'$. Draw the *section-lines* at 45°. To get the **true shape** of the section, draw a rectangle, G H I J, having G H = $g'\ h'$, and G I = $g\ i$.

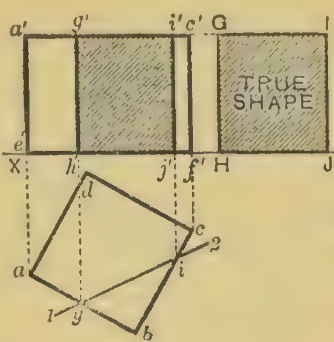


FIG. 394.

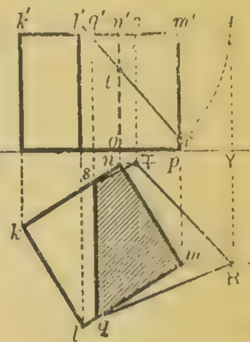


FIG. 395.

FIG. 395.—$k\ l\ m\ n$ is a plan. $k\ n$ makes 30° with X Y. Get the elev. $k'\ p'$. Then project the plan of the section made by the plane $q'\ r'$, the portion above $q'\ r'$ to be removed. Draw projec. from q' to q; then $q\ s$ shews where the plane $q'\ r'$ cuts the upper face of the cube; and $q\ s\ n\ m$ is the plan of the *section.* To get the **true shape** of the section, turn the plane, $q\ s\ n\ m$, round upon $q\ s$, until it is *parl.* to the H. P. Produce $k'\ m'$. With centre q', strike arcs t' 3, r' 4. Then line q' 3 4 is the elev. of the section, when turned round to a *hor. position.* Get the plan of q' 3 4. Draw projecs. from 3, 4, and lines from n, m, parl. to X Y, intersecting at T, R. Join s T, T R, R q. Then $q\ s$ T R is the TRUE SHAPE of the section.

Note (a).—$g'\ h'\ j'\ i'$ and $q\ s\ n\ m$ are not *true* shapes of the sections; because a section, to be seen its TRUE SHAPE, must be viewed at *right angles* to its plane.

(b).—$g'\ h'$ is *perp.* to the H. P., because it is the line of intersection between two planes, $a\ b$, 1 2, that are *perp.* to the H. P. For a similar reason, $i'\ j'$ is *perp.* to the H. P. Again, $q\ s$ is *perp.* to the V. P., since it is the line of intersection between two planes, $k'\ m'$, $q'\ r'$, that are *perp.* to the V. P.

FIG. 396.—Obtain $a'\ b'\ c'$, the elev. of a **Cube**, of $1\frac{1}{2}''$ edges, and with an opening $\frac{3}{4}''$ square, through the middle of the cube. $b'\ c'$ makes 60° with X Y. $d'\ e'$ is a *section-plane.* Get the plan of the section. Make $d\ i$, $c\ j$, &c., equal to $a'\ b'$. From f' to g' is the square opening, therefore in plan we see the *inside* edge $h\ k$.

FIG. 397.—Section of a Prism.—I.—Draw $m'\ n'\ o'\ p'\ q'$, the end elev. of a PENTAGONAL PRISM. Base edge = $1\frac{1}{4}''$; length = $1\frac{1}{2}''$. Bisect $n'\ m'$ at r'. Draw $r'\ s'$ at 65° to the ground. Get a plan of the section as shewn, when the portion $r'\ n'\ s'$ is removed.

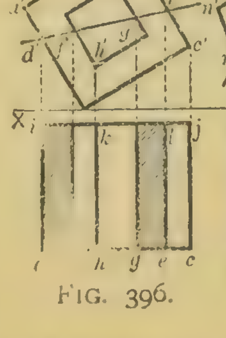


FIG. 396.

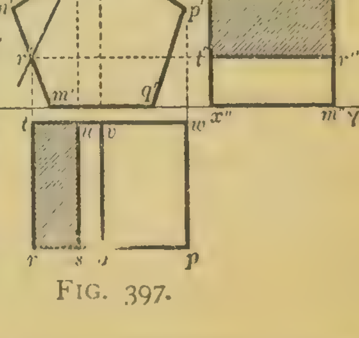


FIG. 397.

II.—Get an elev. of the section, looking in direction Z. The portion $r'\ n'\ s'$ to be removed. Then $t''\ r''\ s''\ u''$ shews the *section* made by the plane $r'\ s'$

Note (a).—Proceed as in Fig. 396, for *any hollow prism*, or *hollow cylinder.*

(b).—Proceed as in Fig. 397, for ANY *prism.* The *true form* of $t\,s$ (or $t''\,s''$) is a rectangle with sides $= r'\,s'$, $r\,t$.

(c).—The *true form* of a section, made by a plane *parl.* to the BASE of ANY *right, or oblique, prism*, is a figure, *equal and similar to the base.*——A section *parl.* to the AXIS of ANY *right prism* is a *rectangle.*

(d).—*When a section-plane, at* ANY *angle, cuts two or more* parallel planes *of a solid, the* edges *made by the section-plane, on these parallel planes, are* parallel lines, *as at* $g'\,i'$, $h'\,j'$ (*Fig.* 394); $d\,i$, f, g, $e\,l$ (*Fig.* 396); *and* $r\,s$, $t\,u$ (*Fig.* 397).

FIG. 398.—Get the plan, $a\,c$, and the elev., $a'\,e'\,f'\,c$, of an **Equilateral Triangular Prism.** Length $= 2''$; base-edge $= 1\frac{1}{2}''$. $c\,d$ makes 38° with X Y, and d touches X Y. (*See* $r\,m$, $p'\,q'\,n'\,o'$, *Fig.* 369.) Make $f\,i = \frac{1}{3}$ of $f\,e$. Through i, draw $g\,h$, *parl.* to X Y. Get the elev. of the section made by $g\,h$, the portion $g\,b\,h$ to be removed. Pt. g is on $a'\,b'$, at g'; pt. i, on $e'\,f'$; pt. h, on $f'\,c'$. Line $h'\,j'$ is perp. to the H. P. (*See note* b, *Fig.* 394.)

FIG. 399.—Get the plan and elev. of the same prism, at $m\,p$, $k'\,n'$, with $m\,n$ parl. to X Y. Get the plan of the section made by the plane $q'\,r'$. Get a *side elev.*, at *right angles*, of $l'\,n'$, by drawing l' N at 30° to $l'\,n'$. Draw hor. lines from r', q', to R, 2. Make $l\,r$, $l\,s$ $= r'$ R, and $k\,q$, $k\,t = 1\,2$. Join $q\,r$, $t\,s$. Then $q\,r\,s\,t$ is the plan of the section.

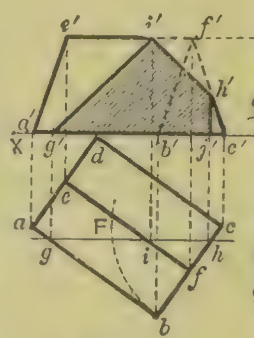


FIG. 398.

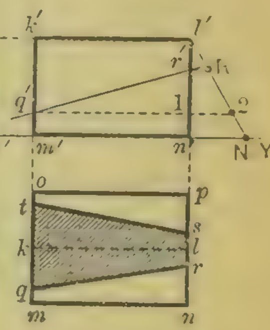


FIG. 399.

Note (a).—$g'\,i'\,h'\,j'$ is the *true* shape, because $g\,h$ is *parl.* to the V. P. The *true* shape of $q\,r\,s\,t$ is a trapezoid with its parl. sides $= q\,t$, $r\,s$, and the central line $k\,l = q'\,r'$.

(b).—Because lines $k'\,m'$, $l'\,n'$, are in planes perp. to *both* planes of projection, we have to get the side view at l' N, that we may ascertain the distances $k\,q$, $l\,r$. *This method is useful for* OTHER SOLIDS, *when a section-plane* (*in plan or elev.*) *cuts an edge that is in a* PLANE PERP. TO BOTH PLANES OF PROJECTION.

Sections of Pyramids.—FIG. 400.—Get $a\,b\,c\,d$ and $a'\,d'\,c'$, the plan and elev. of a PYRAMID, with an *equilateral triangular base.* Base-edge $= 2''$; axis $= 1\frac{3}{4}''$. $a\,c$ makes 20° with X Y. d is over the centre of the base. Get the plan of the section made by $e'\,f'$. Project e on $a\,d$, g on $b\,d$, f on $c\,d$. Join $e\,f$, $f\,g$, $g\,e$.

Fig. 401.—Get plan and elev. of a PENTAGONAL PYRAMID. Base-edge $= 1\frac{1}{4}''$; height $= 2''$. Get elev. of section $n\,o$. Get n' on $h'\,i'$, p' on $m'\,i'$, q' on $m'\,j'$, o' on $j'\,k'$: &c. Proceed in the same way as in Figs. 400, 401, for ANY PYRAMID.

Note (a).—To get the *true shape* of $e\,f\,g$:—Imagine a line, *perp.* to the V. P., passing through e'. Turn $e'\,g'\,f'$ round upon this line, until it is *parl.* to the H. P. (*See* q' 4, s T R q, *Fig.* 395.)——The *true shape* of $n'\,p'\,q'\,o'$ has the *widths* the same as at $n\,p\,q\,o$, and the *heights* are the same as at p', q'.

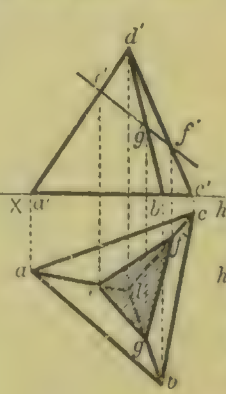


FIG. 400.

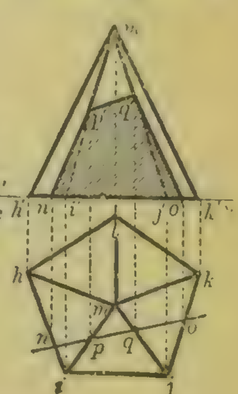


FIG. 401.

(b).—The *true shape* of a section, made by a plane *parl.* to the BASE of ANY *right, or oblique, pyramid*, is a figure similar to, but smaller than, the base.

(c).—If the edge $d\,b$, $d'\,b'$, *is in a plane* PERP. *to the* V. P. *and* H. P., get g in the same way as q, r, were obtained in Fig. 399. **Or, use the following method:**—From g', draw a line parl. to $a'\,b'$, to meet $a'\,d'$ at a pt. r'. Get the plan of r', at r, on $a\,d$. A line drawn from r, parl. to $a\,b$, gives pt. g. Adopt the same method for an ELEV. of a section.

FIG. 402.—Get $a'\ b'\ c'$, $a\ e\ c$, the elev. and plan of a **Square Pyramid.** Base-edge $= 1\frac{3}{4}''$; axis $= 2\frac{1}{2}''$. The base, $a'\ b'$, inclined to the H. P. at 65°, and a', $\frac{1}{2}''$ above X Y. The axis $1\frac{1}{4}''$ from the V. P. Make $a'\ g' = \frac{3}{8}''$, and draw $g'\ h'$ at 20° to the H. P. Project the section made by $g'\ h'$. Get $g\ i$, $h\ j$. Join $g\ h$, $i\ j$.

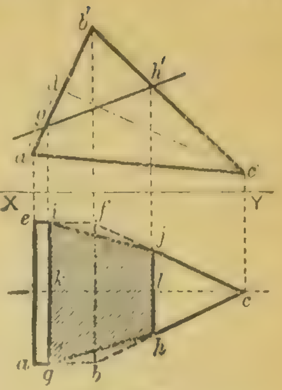


FIG. 402.

Note (a).—Adopt the same method for ANY PYRAMID. Get the *true form* of section, as in Fig. 395. We see $g\ i$, $h\ j$, their true lengths. The true length of $k\ l = g'\ h'$.

(b).—A section made by a plane that passes through the APEX of ANY *pyramid* is a *triangle.*

Sections of Cylinders.—FIG. 403.—Circle $a'\ b'$ is the end elev. of a cylinder. Diameter $= 1\frac{1}{4}''$; length $= 1\frac{1}{2}''$. Get the plan, $a\ b\ f\ e$, and shew the section made by a plane, $c'\ d'$, that is *parl.* to the H. P. Draw projecs. from c', d'. Then $c\ g$, $d\ h$, are parl. to the axis, and $c\ d\ h\ g$ is the section.

FIG. 404.—Get $i'\ j'\ k'\ l'$, $m\ n\ o\ p$, the elev. and plan of a cylinder. Length $= 2''$; diameter of base $i'\ k' = 1\frac{3}{4}''$. Axis to be $1\frac{1}{4}''$ from the V. P. Shew the section made by $q'\ r'$. The section is an ELLIPSE. Get the axes $q\ r$, $s\ t$. Strike the semi-circle, which represents half the base. Mark some pts. 2, 3, *equidistant* from 1. Draw $2\ w'$, $3\ x'$, parl. to the axis. Make $4\ w$, $4\ y$, $5\ x$, $5\ z = u'\ 2$ or $v'\ 3$. Draw the *ellipse* through q, w, s, &c.

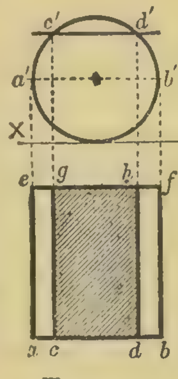


FIG. 403.

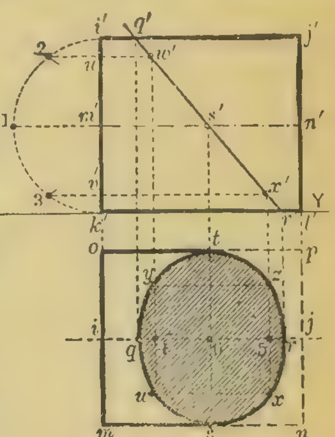


FIG. 404.

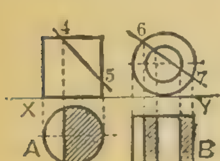


FIG. 405.

Note (a).—$c\ d\ g\ h$ is the *true shape* of the section, because it is *parl.* to the H. P. (*See note* a, *Fig.* 394.) *Any* section of a cylinder, made by a plane PARL. *to the* AXIS, *is a* **rectangle.** *Any* section made by a plane PARL. *to the* BASE, *is a* **circle.** ALL *other sections are* **ellipses.**

(b).—The hor. distance of w' from the axis $= u'\ 2$, therefore we make $4\ w = u'\ 2$. In the same way $6\ s = m'\ 1$. The *true shape* is an ellipse, with $s\ t$ for its minor, and $q'\ r'$ for its major axis.

(c).—**FIG. 405.**—Section 4 5, through a cylinder standing on its base, although a portion of an *ellipse*, appears in plan, as at A.——The plan of section 6 7, through a *hollow* cylinder, is shewn at B.

FIG. 406.—Get $a'\ b'\ c'\ d'$, the elev. of a **Cylinder,** with the axis (2″) parl. to the H. P. and V. P. Diameter of base $a'\ c' = 1\frac{3}{4}''$. Get the plan $e\ f\ g\ h$, and shew the section made by the plane $i'\ j'$. Mark an intermediate pt. k'. Strike the semi-circle on $a'\ c'$. Draw $j'\ 1$, $k'\ 2$, parl. to the axis. Get i on $a\ b$. Make $b\ j$, $b\ l = 3\ 1$, and $4\ k$, $4\ m = 5\ 2$. Draw the curve through j, k, i, m, l. The student should get *several* pts. between i' and j', and project them for the curve in the plan.

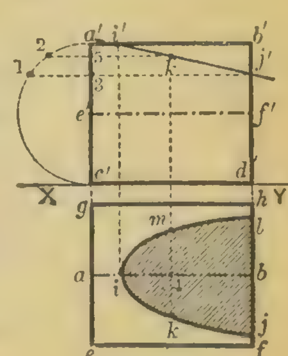


FIG. 406.

Note.—The section $i'\ j'$ is less than half of an ellipse. A section drawn from i', through f' or e', would be a *semi-ellipse.*

FIG. 407.—Get *n o p*, the plan of a **Cylinder**, with the axis (2") parl. to the H. P., and inclined at 25° to the V. P. Base-diameter $n\ o = 1\frac{1}{2}$". Get the elev., resting upon the H. P., and suppose all in front of the section *s t* to be removed. Strike the semicircle on *n o*. Mark 2, 3, equidistant from 1. Draw 2 *u*, 3 *v*, parl. to the axis. Make *x'* 4, 4 *w'* = *q* 1. Through 4 draw *s' p'*. Make 5 *u'*, 5 *y'* = 6 2, and 7 *v'*, 7 *z'* = 8 3. Draw the curve through *s'*, *u'*, *w'*, &c. Complete the diagram as shewn. At *u*, *w*, *v*, draw perps. to *s t*. Make *u* U, *v* V = 6 2, 8 3, and *w* W = *q* 1. The SEMI-ELLIPSE, *s* U W V *t*, is half the *true shape* of the section; because it has been turned round upon the axis, *s t*, and made *parl.* to the H. P.

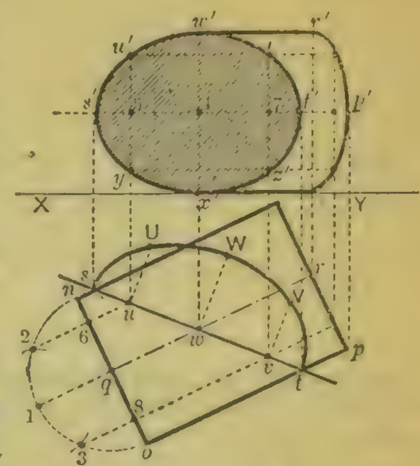


FIG. 407.

Note (a).—By the construction explained at *s* U W V *t*, we can draw the true shapes of such sections as *q' r'* (Fig. 404), and *i' j'* (Fig. 406).

(b).—By the principle explained in Figs. 404, 406, 407, we can obtain the plan or elev. of any LINE, or of a FIGURE of *any* shape, that is *drawn upon the curved surface of a cylinder.* Work out such problems for practice. Thus:—

1st.—Upon the *curved* surface of a horizontal cylinder, in elevation, draw a *square*, with none of its sides parallel to the axis. Project the shape of this figure upon the *plan* of the cylinder. Mark points in the sides of the figure, in the elevation. Project plans of these points, as in Fig. 406.

2nd.—Upon the *curved* surface of a horizontal cylinder, in plan, draw the figure of a *circle*; and then project its form when seen in *elevation.* Mark a number of pts. in the circumference, and obtain the elevations of these pts., as in Fig. 407.

(c).—**Some useful hints may be obtained if we turn some of the diagrams, in this chapter, upside down.** Thus, do so with Fig. 407, and suppose the H. P. and V. P. to be *reversed in position.* Then, *n o p* appears as an ELEV. of a cylinder, with its axis inclined to the *hor.* plane, and *s' w' t' x'* is the PLAN of the section *s t*.

Sections of Cones.—**FIG. 408.**—The *true shape* of a section made by a plane, *d' e'*, that is *parl. to the* BASE *b' c'*, is a **circle.** With centre *a*, and radius *f' d'*, strike circle *a d*.

FIG. 409.—A section made by *any* plane, *g' j'*, that passes through the APEX, *g'*, is an **isosceles triangle,** as at *j g k*. Its *true shape* has a base = *j k*, and an altitude = *j' g'*. If the plane *coincides* with the axis, we cut the cone in half, and the shape of the section is the same as *h' g' i'*.

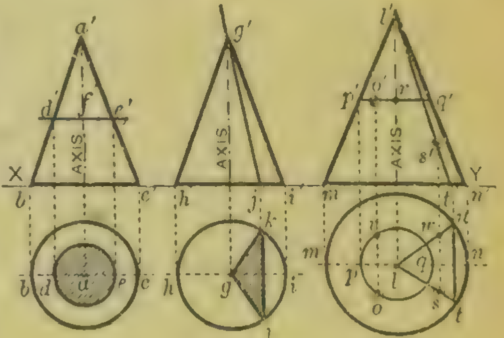


FIG. 408. FIG. 409. FIG. 410.

FIG. 410.—To project the plan or elev. of a given point that is situated upon the surface of a Cone.—1st METHOD:—Get the plan of pt. *o'*. Through *o'*, draw the plane *p' q'*, *parl. to the base.* Get the plan of the *circle p' q'*, at *p q*, as in Fig. 408. The projec. from *o'* gives the PLAN of *o'* at *o*. ——2nd METHOD:—Get the plan of pt. *s'*. From *l'*, through *s'*, draw the plane *l' t'*. Get the plan of the *isosceles triangle*, at *l t u*, as in Fig. 409. The projec. from *s'* gives the PLAN of *s'*, at *s*.

Note (a).—*v* and *w* are the plans of the pts., if *o'* and *s'* be on the *further* side of the cone. Pts. *o*, *v*, are in the *circumference* of the *circular* section *p q*. Pts. *s*, *w*, are in the *sides* of the *triangular* section.

(b).—To find the *elev.* if *o* be given:—With centre *l*, and radius *l o*, get *p*, then *p' q'*: &c. To find the *elev.* of *s*:—Get *l t* *l' t'*: &c. If Fig. 410 be understood, ANY section of a cone can be projected.

FIG. 411.—Get $a' b' c'$, the elev. of a **Cone.** Base-diameter = 2″; axis = $2\frac{1}{4}$″. Draw $e' f'$, at 30° to the H. P., and cutting the axis $1\frac{3}{8}$″ from the apex. Get the plan of the cone, and the section made by $e' f'$. Through a, draw $b\ c$, parl. to X Y. Get e, f. Bisect $e' f'$ at g'. Through g', draw $h' i'$, parl. to the base. With centre a, and radius $j' h'$, cut the projec. from g', at g, k. Get l', m', *equidistant* from g'. Through l', m', draw lines parl. to the base. With centre a, and radius $n' o'$, get l, p. In a similar way get m, q. Draw the elliptic curve through e, l, g, &c. At l', g', m', draw perps. to $e' f'$. Make l' L = 1 l, g' G = 2g, m' M = 3 m. The SEMI-ELLIPSE, e' L G M f', is half the *true shape* of the section. $e\,g\,f\,k$ is an *ellipse* (its major and minor axes are $e\,f$, $g\,k$), but it is *not the true shape* of the section.

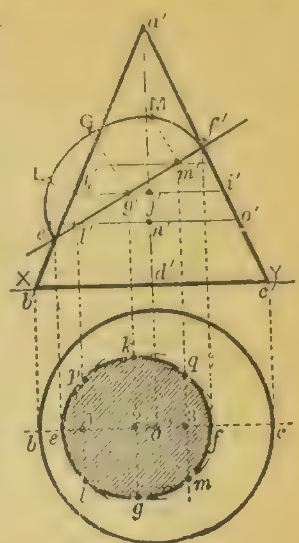
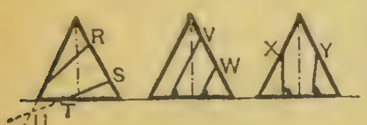


FIG. 411.

Note (a).—When obtaining such pts. as p, l, g, in Fig. 411, it is unnecessary to draw the *whole of each circle*, as was done in Fig. 410.

(b).—A section of a CONE, made by a plane that is inclined at a LESS angle to the base, than that made by the side with the base, is an **Ellipse,** as at $e' f'$ (Fig. 411), R and S T (Fig. 412). S T is a *portion* of an ellipse. We get the *entire* ellipse at S U, by producing S T to meet the opposite side, produced, at U.



FIGS. 412. 413. 414.

(c).—The curve of a section made by a plane inclined to the base at the SAME angle as the side (*i.e.*, *parl.* to the side), as at V, W (Fig. 413), is called a **Parabola.**

(d).—The curve made by a plane inclined to the base at a GREATER angle than the side (but not passing through the apex), as at X, Y (Fig. 414), is called a **Hyperbola.** Thus, a section *parl. to the axis*, as at X, gives a *hyperbolic curve*. The ELLIPSE, PARABOLA, and HYPERBOLA, are known as the **conic sections.**

(e).—By the principle of working, explained in Fig. 411, we can obtain the section made by a plane that passes through ANY solid that is based upon circular construction around a central axis, such as a *cup*, a *vase*, a *peg-top*, an *egg*, a *pulley*, &c. In each case get circular sections, as at $h' i'$, &c. Work out such problems for practice. Thus :—

1st.—Draw an elevation of a peg-top, standing vertically upon the H. P. Breadth = 3″; height of body = 4″; length of peg = $1\frac{1}{4}$″. Through a pt. in the axis, $3\frac{1}{4}$″ from the ground, draw a line, A B, at 60° to the H. P. Project the plan, and shew the section made by the section-plane A B.

2nd.—Draw a plan of the same top, standing vertically upon the H. P. Through the centre, draw a line, C D, at 40° to the V. P. Project the elevation of the top, and shew the section made by the section-plane C D.

FIG. 415.—Get $e' f' g'$, the elev. of a **Cone.** Base-diameter = 2″; axis = $2\frac{1}{4}$″. Through the middle pt. of the axis, draw $i' j'$, *parl.* to $f' e'$. Project the plan of the section made by the plane $i' j'$. Draw $f\,g$, parl. to X Y. Get $i\ k$ and j. Mark any number of pts. l', m', and get their plans, as in Fig. 411. Draw the curve from k, through n, o, &c. Draw perps. to $i' j'$, making i' I = 1 i, l' L = 2 l, m' M = 3 m. Draw the curve from j' through M, L, to I. Then i' I L M j' is half the *true shape* of the section. The entire curve is a PARABOLA. (*See note* **c,** *Fig.* 411.)

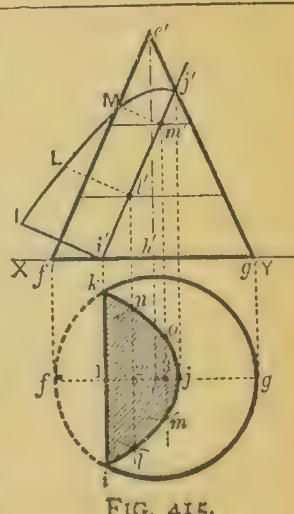


FIG. 415.

Note (a).—The pts. in the curves of the sections in Figs. 411, 415, are obtained by the 1st method, shewn in Fig. 410. They can also be obtained by the 2nd method, explained in Fig. 410. For practice, get such sections by *both* methods.

(b).—In Figs. 411, 415, 416, 417, 419, a greater number of pts. than are given, should be projected in the sections, so as to insure greater accuracy in the curves.

FIG. 416.—Get *a*, the plan of a **Cone**, standing on the H. P. Base-diameter = 2″; axis = $2\frac{1}{4}$″. Draw *d e*, parl. to X Y, and $\frac{1}{2}$″ from the axis. Get the elev., and shew the section made by the plane *d e*. Draw *a f*, perp. to *d e*. Then *g* is the highest pt. in the curve. Strike the circle *a g*, and get its elev. at *h′*. Mark *g′*. Get *d′*, *e′*. On *d e* mark *i*, *j*, equidistant from *g*. Through *i*, *j*, draw *a k*, *a l*, and get their elevs. at *a′ k′*, *a′ l′*. Get *i′*, *j′*. The curve, *d′ i′ g′ j′ e′*, is a HYPERBOLA. (*See note* **d**, *Fig.* 411.)

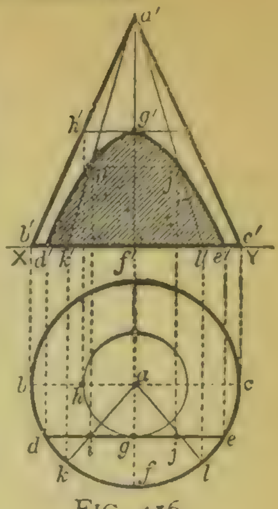


FIG. 416.

Note (a).—*d′ i′ g′ j′ e′* is the *true form* of the section, because *d e* is *parl.* to the V. P. Proceed in the same way, if the section-plane is *inclined* to the V. P. If the section-plane is perp. to X Y, the elev. of the section is a *vert. straight line.*

(b).—Pt. *g′* is obtained by the 1st method, Fig. 410, and *i′*, *j′*, by the 2nd method. The latter pts. can also be projected by the 1st method.

FIG. 417.—Get *a b c*, the plan of a **Cone**. Its base, *a c*, ($2\frac{1}{4}$″) touches the H. P., and is in a vert. plane, at 67° to the V. P. The axis = $2\frac{3}{4}$″. Make *a j* = 1″. Draw *j k*, at 25° to the V. P. Get the elev., as in Fig. 391, and shew the section made by *j k*. Get *j′ k′*, the major axis of the ellipse. Bisect *j k* at *l*. Through *l*, draw *m n*, parl. to *a c*. Strike a semi-circle on *m n*. Draw *l* L, perp. to *m n*. Then *l* L is half the length of the minor axis, so make 1 *l′*, 1 *p′* = *l* L. Get *f′ h′ b′*, the elev. of the triangular section made by the plane *f b*. (*See Fig.* 409.) Get pt. *q′* on *f′ b′*, and *r′* (the pt. underneath *q*) on *h′ b′*. Get *g′ i′ b′*, the elev. of the triangular section *g b*. Project *s′* on *g′ b′*, and *t′* on *i′ b′*. Draw the elliptic curve through *j′*, *q′*, *l′*, &c., to *touch* the sides of the cone.

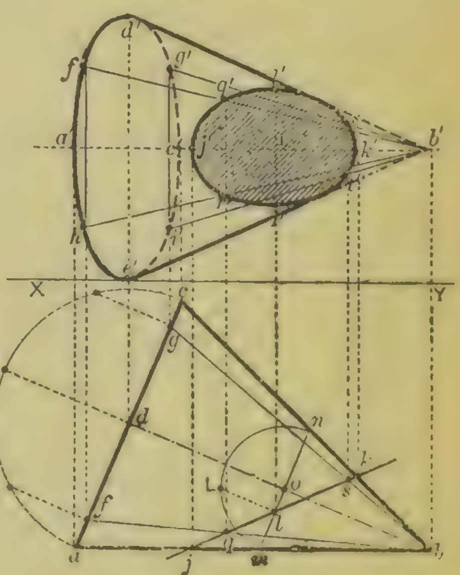


FIG. 417.

Note (a).—Pts. *l′*, *p′* are projected by the 1st method, shewn in Fig. 410. Pts. *q′*, *r′*, *s′*, *t′*, are got by the 2nd method, Fig. 410. If we draw a line from *b*, through *l*, to *a c*, we can get *l′*, *p′*, in the same way that *q′*, *r′* were obtained. By drawing lines through *q*, *s*, parl. to *a c*, we can get *q′*, *r′*, *s′*, *t′*, in the same way that we obtained *l′*, *p′*. *Any* number of pts. can be projected by *either* method.——Proceed as in Fig. 417, if the section be a PARABOLA, or a HYPERBOLA.

(b).—Get half the *true form* of the section, on *j k*, as at *e′ f′*, in Fig. 411. If the section is PARL. *to the base*, proceed as in Fig. 392.

(c).—By the principle explained in Figs. 410, 411, 415, 416, 417, we can project the plan or elev. of any LINE, or of a FIGURE of *any* shape, that is drawn *upon the curved surface of a cone.* Work out such problems for pactice. Thus :—

1st.—Upon the *curved* surface of a cone, in elevation, standing upon the H. P., draw a *circle.* Project the shape of this figure upon the *plan* of the cone.

2nd.—Upon the *curved* surface of a cone, in *any* position, in plan, draw a *compound curve.* Project the form of this curve upon the *elev.* of the cone.

(d).—To indicate different substances, the section lines are sometimes varied. Thus, the lines may be alternately thick and fine, or we can have alternate lines and strokes, &c.

(e).—When a section-plane cuts through two separate portions that are in contact, the section lines are usually drawn across the section of one portion at 45° to the *left*, and across the section of the other portion at 45° to the *right*

Sections of Spheres.—FIG. 418.—Circle *a* is the plan of SPHERE of 1¼″ diameter. Project its elev., when resting on the H. P., and shew the section made by a plane, *b c*, that is *parl.* to the V. P. Draw a hor. line through *a′*. Draw projecs. to *b′*, *c′*. With centre *a′*, and radius *a′ b′*, strike the circle. The *true form* of *every* section of a sphere, made by a plane, is a CIRCLE. Pt. *x′* is the elev. of a pt., *x*, on the sphere. Pt. *y′* is the elev. of the pt. on the sphere, underneath *x*.

FIG. 419.—Circle *d* is the plan of a SPHERE of 1″ radius, resting on the H. P. Draw the section-plane *e f*, at 60° to the V. P., and ¼″ from the centre *d*. Project the sectional elev. Get *e′*, *f′*. Bisect *e f* at *g*. Make 1 *h′*, 1 *g′* = *g e*. Then *g′ h′*, *e′ f′*, are the major and minor axes of the ellipse. Mark *i*, *j*, equidistant from *g*. Get elevs. of *i*, *j*, by means of sections parl. to the V. P., as at *x′*, *y′*, Fig. 418. Thus, through *i* draw *k l*, parl. to X Y. With centre *d′*, and radius *m k*, strike an arc, which gives us *i′*, *n′*. This arc is an elev. of a portion of the circle *k l*. In the same way get *j′*, *r′*. A projec. drawn from *s* gives the pts. of contact between the ellipse and the circle in the elev.

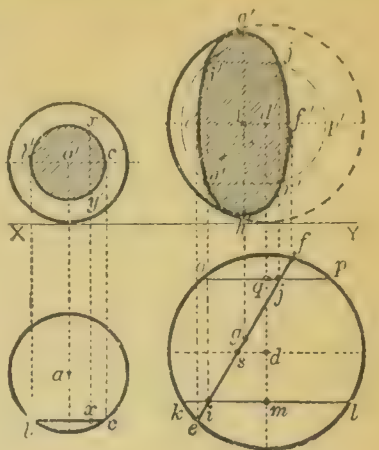


FIG. 418. FIG. 419.

A plan of a section can be obtained by the same principle of construction. For a plan of a sect., the sphere, in elevation, should be cut by a series of *horizontal* planes, passing through the line of section. The plans of these circular sections cut the projectors from the points in the line of section, and we get points in the curve, in the same way as points *i′*, *j′*, were obtained in the above elevation. (*See note* c, *Fig.* 407.)

Note (a).—Because *b c* is *parl.* to the V. P., we see the *true form* of the section in the elev. Because *e f* is *inclined* to the V. P., its projection, in the elev., is an *ellipse*. The *true form* of *e f* is a circle, with a radius = *g e*.

(b).—The section *e′ g′ f′ h′* can also be obtained as in Fig. 389.

(c).—By the method shewn in Figs. 418, 419, we can get the plan, or the elev., of any LINE, or of a FIGURE of *any* form, that is marked *upon the surface of a sphere*. Students should work out such problems as these, for practice. Thus, upon the surface of a sphere of 3 in. diameter, in plan (or elevation), draw a *compound curve*, stretching nearly across from one side of the circumference to the other side, in an oblique direction. Then project the elevation (or plan) of this curve. Indicate, by a dotted line, the position of such portion of the curve as may be hidden by the sphere.

(d).—If required to project an elev. of any of the solids given in Figs. 394 to 419, at ANY GIVEN ANGLE, OR WITH ANY GIVEN NEW LINE OF INTERSECTION, proceed as shewn in Fig. 380.

(e).—When the section-plane that cuts a sphere, or *any* other solid, is *perpendicular* to the horizontal and the vertical planes of projection, the elevation and plan of the section are *straight lines*.

(f).—We can project **curved sections**, as well as sections by *planes*. Thus, across ANY solid, in plan (or elev.), *strike an arc of a circle*, or draw *any* other curved line. Suppose this curve to be the edge of a curved surface that cuts the solid. By means of *points* in the curve, project the elev. (or plan) of the section.

(g).—It will be seen from the examples given in this chapter,—

1st.—That the shape of a section of a solid that is bounded entirely by planes, such as a cube, a prism, or a pyramid, must always be a rectilineal figure:

2nd.—That the section of a cylinder, or of a cone, may be either a rectilineal, or a curvilineal figure, or partly rectilineal and partly curvilineal:

3rd.—That a section of a sphere is always curvilineal.

Work out TEST-PAPER F, and the Exercises on SOLIDS.

CHAP. XVI. PARABOLA, HYPERBOLA, SPIRALS, IONIC VOLUTE, AND CYCLOIDS.

DEFINITIONS:—260.—A **conic (conic section)**, so called because found by cutting a cone with a plane (see *Figs.* 412, 413, 414), is a curve traced out by a pt. moving in such a way that its distance from a fixed pt., the **focus** (F, *Probs* 154, 157, 159) is in constant ratio to its perp. distance from a fixed st. line, the **directrix** (A B, *Prob.* 154; R S, *Prob.* 157; B C, *Prob.* 159).——The **focal distance** of a pt. on the curve is the st. line joining it with the focus (F P, *Prob.* 154).

261.—When the focal distance of each pt. on the curve **equals** its perp. distance to directrix, the curve is a **parabola** (P F = P U = H C, *Prob.* 154). When the focal distance is always **greater** than the distance to directrix, the curve is a **hyperbola** (F T is greater than K D, *Prob.* 159). In the **ellipse**, the focal distance is **less** than distance to directrix.

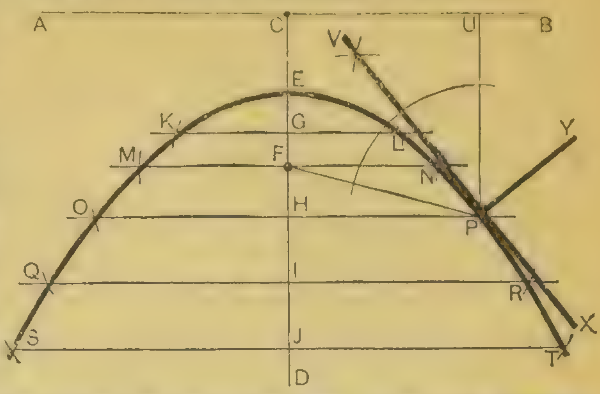
262.—The **axis** is the st. line drawn through the focus perp. to directrix; it divides the curve into two symmetrical parts, and bisects the double ordinates. ——A **chord** is any st. line drawn across the curve, as C D, I J, Prob. 157.——A **diameter** is the st. line bisecting a number of parl. chords, and, in the parabola, is parl. to the axis. Its vertex is the pt. where it meets the curve.——The **vertex** of the curve is the pt. where the curve cuts the axis, and is nearest the directrix. The vertex of the parabola bisects the distance between directrix and focus. In the hyperbola it is nearer the directrix than the focus.——The **ordinate** of any pt. on the curve is the perp. let fall from it to the axis: if produced to meet the curve on the other side of axis, it is a **double ordinate.** The **latus rectum** is the double ordinate through the focus.——The **abscissa** of a pt. on the curve is the part of the axis between the ordinate of the pt and the vertex. Thus, Prob. 154, I E is the abscissa of pt. R.——A tangent (V X, *Prob.* 154), touches the curve in one pt. P only, and bisects the angle formed by the focal distance of the pt. and its perp. distance from directrix. The tangent through the vertex is perp. to axis (A D, *Prob.* 155).——A **normal** is perp. to the tangent at its pt. of contact (P Y, *Prob.* 154).

263.—If a circle, moving along a st. line, or along the circumference of another circle, rotates about its own centre, any fixed pt. not at the centre, traces out one of a series of plane curves known as **cycloidal.** A pt. at the centre merely traces out a line parl. to the directrix. The **generator** or **generatrix** is the pt. which traces the curve. The **director** or **directrix** is the st. line or circle along which the rotating circle moves. The rotating circle is called the **generating circle.**

264.—When a circle rotates along a st. line, a pt. on the circumf. traces a complete **cycloid,** during one revolution.——When one circle rotates on the **outside** of another (in the same plane), any pt. on the circumf. traces out a complete **epicycloid.** When the circle moves **inside** the director, the curve traced out is a **hypocycloid.**——When a circle moves along a st. line, any pt. rigidly connected with the circumf., but not upon it, describes a **trochoid,** which is **inferior** or **superior** according as the generator is inside or outside the circumf. ——The **epitrochoid** is like the epicycloid, only the generator is not on the circumf. of generating circle. With the same exception, the **hypotrochoid** is like the hypocycloid.

265.—A **spiral** is a plane curve with a fixed centre, eye or pole, to which it never returns, but recedes therefrom, being traced out by the end of a radius, which increases uniformly as it revolves (*Prob.* 83).——In the **Archimedean** spiral, the difference between every two successive radii forming equal angles with each other, is the same. Thus, in Prob. 161, X Q − X P = X P − X B.

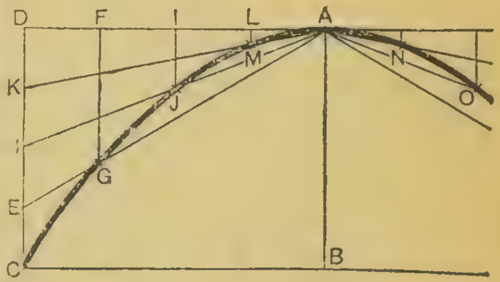
PROB. 154.—To draw a Parabola, when focus F and directrix A B are given; and through a given pt. P on the curve, to draw the tangent V X, and at P erect a normal P Y.



1. Line C D drawn through F perp. to A B and meeting it in C, is the axis: pt. E which bisects C F is the vertex.——2. Through any pts. G, H, I, etc., on C D draw perps. both ways; then these lines, G K, G L, F M, etc., are ordinates.——3. To find where the curve cuts any ordinate H P, measure distance H C and, with this as rad., and F as cent., mark P on H P. To find correspdg. pt. O on other side of C D, make H O = H P.——4. Other pts. are found in the same way, since the distance of any pt., on the curve, from A B, = its distance from F. The ordinate F N through the focus, = F C. The curve should be drawn continuously through the pts. obtained, E, K, M, O, etc.——5. To find tangent through P, join P with F, and draw P U parl. to C D. The st. line V X bisecting angle F P U is the tangent required, and the normal P Y is the perp. to V X from pt. P.

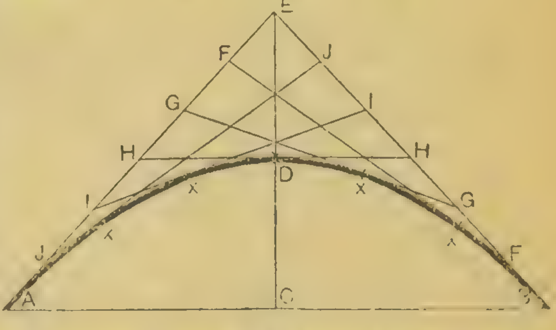
Note.—The **trajectory** of a cannon ball or other missile is the path it traces before reaching the earth, and forms part of a parabolic curve. The two tangents at the ends of a chord through the focus meet each other at right angles on the directrix. When this chord is the latus rectum, the tangents meet on the axis also at pt. C.

PROB. 155.—To describe a Parabola when the height A B, and half the base B C are given.



1. Let B C be perp. to A B, and complete rectangle A B C D; divide A D and C D each into the same number of equal parts, say 4, as at F, I, L, E, H, K.——2. Towards A draw K A, H A, E A, so that K A meets perp. from L in pt. M; H A, the perp. from I in pt. J; E A, the perp. from F in pt. G; and so on.——3. The pts. A, M, J, G, C are successive pts. on the required curve, the other half of which is found in a similar way.

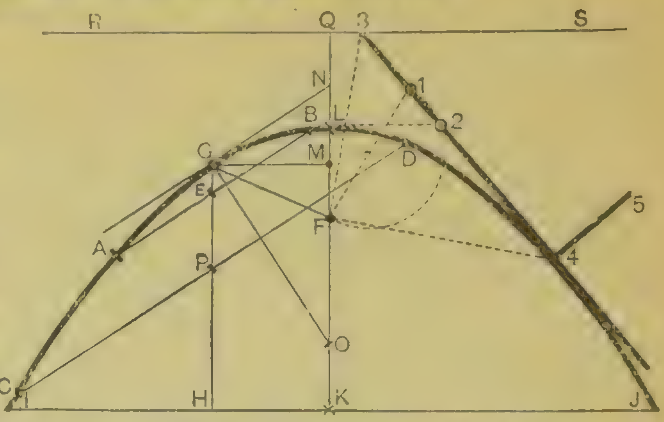
PROB. 156.—To describe a Parabola by means of intersecting tangents, having given height C D and width A B.



1. Let C D be perp. to A B from middle pt. C. Produce C D to E and make D E = D C.——2. Draw E A and E B, and divide each of them into the same even number of equal parts, say 6; and letter the pts. on E B the reverse way of E A.——3. The st. lines F F, G G, etc., joining same-lettered pts. are tangents to the curve. Curve touches them midway between their intersection pts.

Note.—By dividing the sides into an even number of parts, H H always passes through vertex D, and is parl. to base A B.

PROB. 157.—Having given the parabolic curve I L J, to find first, the axis, focus, and directrix; and 2nd, to draw a tangent from a given pt. 1 outside the curve.



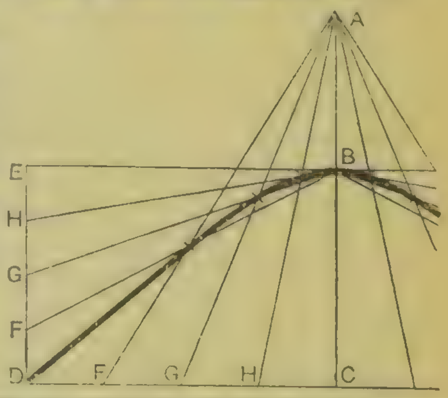
1. Draw 2 chords A B, C D parl. to each other, and through their middle pts. E, P, draw H G to meet the curve in G.——2. Through any pt. H along H G draw I J perp. to H G, and through its pt. of bisection, K, draw a perp. K L to meet the curve in L. Then K L is the AXIS and L the vertex.——3. From G drop a perp. G M to meet the axis, and make L N = M L. Join N with G and erect G O perp. to tangent G N. Make angle O G F = angle O G H. Then F, where G F meets the axis, is the FOCUS.——4. The DIRECTRIX R S is perp. to K L produced, and its distance L Q from the vertex, = L F.——5. To find tangent through 1, draw 1 F and on it describe a semicircle. From L drop a perp. to meet it at 2.——6. Draw 2 1 to meet R S at 3. Join 3 F, and from F draw F 4 perp. to 3 F. The tangent through 1 touches the curve at pt. 4. The normal 4 5 is the perp. to the tangent.

Note (a).—When diameter is perp. to the chords it bisects, it coincides with axis.

(b).—By completing circle on the other side of F 1, and proceeding in the same way, another tangent from pt. 1 can be drawn to meet the curve.

(c).—Lines E A, E B, and P C, P D, are called ordinates to diam. G H.

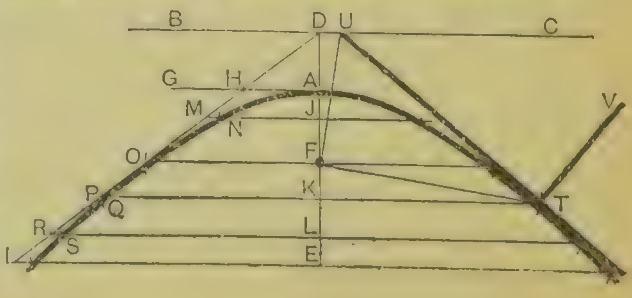
PROB. 158.—To describe a Hyperbola, given half the transverse axis A B, the abscissa B C, and the ordinate C D.



1. Lines A B, B C are in the same st. line A C, and perp. to C D. Complete rectangle E C.——2. Divide C D and D E each into the same number of equal parts, say 4; then join A with F, G, H on C D, and B with F, G, H on D E.——3. The intersection of H A and H B, G A and G B, and F A and F B, are successive pts. on the required curve. The other half of the hyperbola is found in exactly the same way.

Note.—A is called the centre of the hyperbola, and every line passing through it is called a diameter.

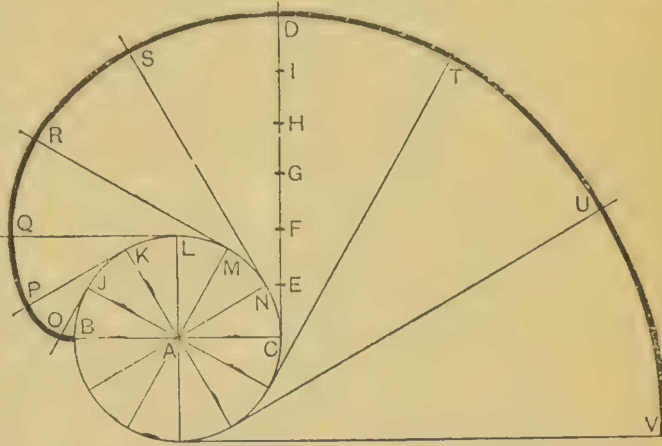
PROB. 159.—To describe the Hyperbola, having given the focus F, the vertex A, and the directrix B C; and at any given point T, to draw the tangent T U, and the normal T V.



1. Line E D drawn through F and A to meet B C in D, is the axis.——2. Through A draw A G perp. to D E, and cut off A H = A F; draw D I through H, of indefinite length. Through any pts. J, K, L, etc., on A E, draw perps. both ways, meeting D I in M, P, R, etc.——3. To

find where the hyperbola meets any perp. J M, measure distance J M, and with this as rad. from cent. F, strike off J N, and a part equal to J N along N J produced. Obtain other pts. in a similar way. ——4. Note that D I is tangent to the curve at O, one end of the latus rectum. To find tangent through any other pt. T, join T with F, and from F draw F U perp. to F T and meeting B C in U. Then U T is the tangent through T; T V, the normal, is perp. to U T.

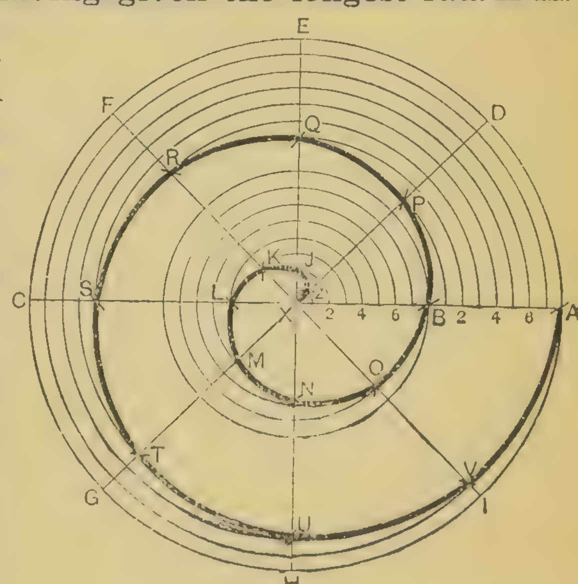
PROB. 160.—To determine the involute of a given circle A.



1. Draw any diameter B C, and divide semicircle into any number of equal parts, say 6, at J, K, L, etc. At C, the opposite end of B C to which the curve starts, draw tangent C D, equal to the semicircumf. (*see* Prob. 62), and likewise divide it into 6 equal parts at E, F, G, etc.——2. Then the curve which begins at B reaches D after evolving half the circle. Through J, K, L, etc., draw tangents, making J O=1, K P=2, L Q=3, parts of C D; and so on.——3. The tangents after C D must be made equal to 7, 8, 9, etc., parts of C D; tangent through B=12 parts of C D. The points O, P, Q, R, etc., mark the curve of the required involute.

Note (a).—See Prob. 83 for some information about the construction of simple spirals.
(b).—Where less accuracy will do, make C D=rad. multiplied by $3\frac{1}{7}$.
(c).—Circle A is called the *evolute* of the curve.

PROB. 161.—To describe an Archimedean Spiral of any number of revolutions, say 2, having given the longest rad. A X.



1. Since there are 2 revolutions, bisect A X in B, and describe a circle with centre X, rad. X A, dividing it into any number of equal parts, say 8.——2. Draw diameters A C, D G, etc., meeting the pts. of division; and divide X B into 8 equal parts; and, with centre X, through pts. of division on X B draw arcs.——3. Let arc through 1 cut X D at Z; arc through 2 cut X E at J; and so on, each succeeding arc meeting the next further diameter. Through B draw a circle. Then Z, J, K . . . B mark the path of the spiral in its first revolution.——4. The 2nd revolution is found by dividing B A (like X B) into 8 equal parts, and through the pts. of division, beginning near B, and with cent. X in each case, describing arcs to meet the same diams. as before, the 1st arc cutting X D at P, the 8th (being circle through A) cutting X A.——5. Points P, Q, R . . . A are on the 2nd revolution of the spiral, and should be joined continuously, pt. P being joined to B, the end of 1st revolution.

PROB. 162.—To draw the Ionic volute, having given the height equal to X Y.

1. Divide X Y into 16 equal parts, numbering from X, and perp. to it, from pts. 8 and 10 draw 8 D and 10 E.——2. Between these 2 lines describe a circle (the *eye* of the volute), whose centre is from X Y a distance = X 8. Draw diameter D E (*Fig.* 430) parl. to X Y, and diameter B C perp. to D E.——3. Inscribe square B E C D, bisect the sides, joining pts. 1, 2, 3, 4, to form a new square, whose diagonals 1 3, 2 4, each divided into 6 equal parts, give the successive centres (going spiral-wise from 1 to 12) for drawing the quadrants of the *outer* curve of the volute.——4. Through 4 and 1 draw 4 F, to meet X S which is perp. to X Y. Now, the radius from the end of one quadrant is always in a line with the centres of its own quadrant and the next one.——5. Hence, find F G with cent. 1, rad. 1 F; G H with cent. 2, rad. 2 G; H I with cent. 3, rad. 3 H; I J (*more* than a quadrant) with cent. 4, rad. 4 I.——6. Then, J K (*less* than a quadrant) with cent. 5, rad. 5 J; and so on, until last quadrant Q D, described with cent. 12, rad. 12 Q, meets the *eye* at D.——7. To obtain the *inner* curve of the scroll, along F 1 mark off F R = X 1. This curve, like the outer one, ends in D.——8. The centres for successive quadrants will be found along 2 1 and 3 4, at a distance, = ¼ of one of the equal divisions, from the cent. for correspdg. quadrant on outer curve: thus, cent. correspdg. to 1 is at distance from 1 = ¼th of 1 5.

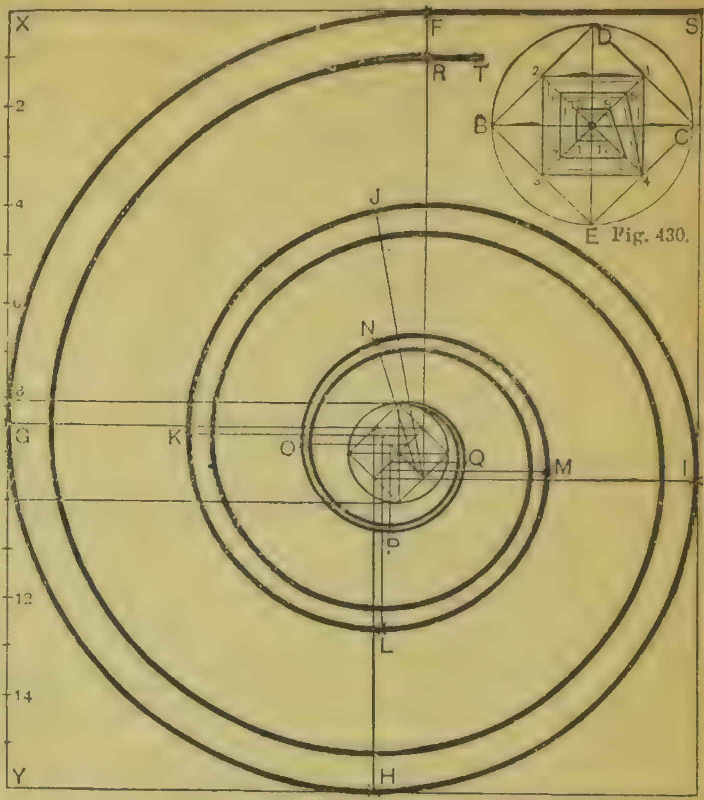


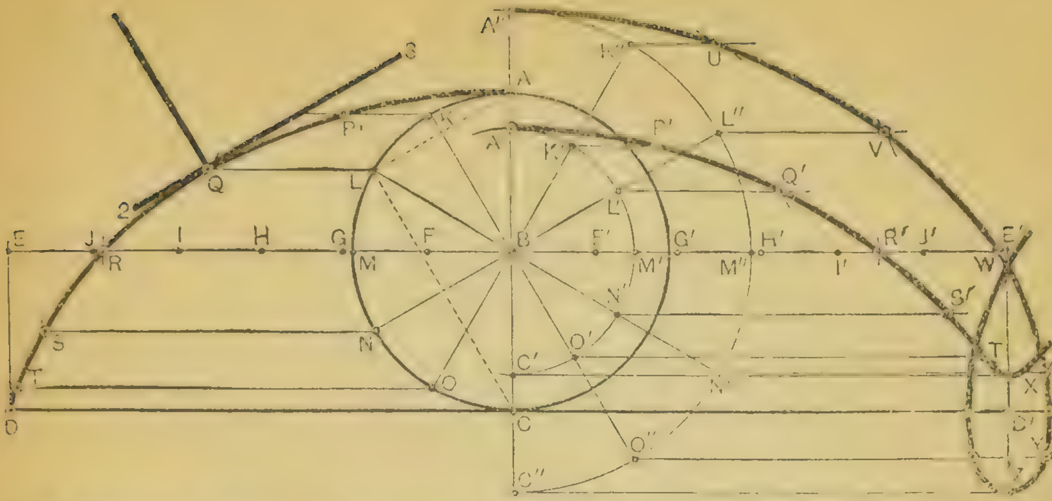
Fig. 430.

Note (a).—In an examination, candidates are advised to give only the *outer* curve of the Ionic volute, unless the *inner* curve also is asked for.

(b).—The Ionic volute is the well-known feature of the capital of the Ionic column.

PROB. 163.—To draw 1st the Cycloid, 2nd the Inferior Trochoid, 3rd the Superior Trochoid, having given generating circle B and directrix D D′; and in the case of the trochoids, the distances of A′ and A″ from centre B. Also, at a given pt. Q on the cycloid, to erect a normal, and through it draw a tangent.

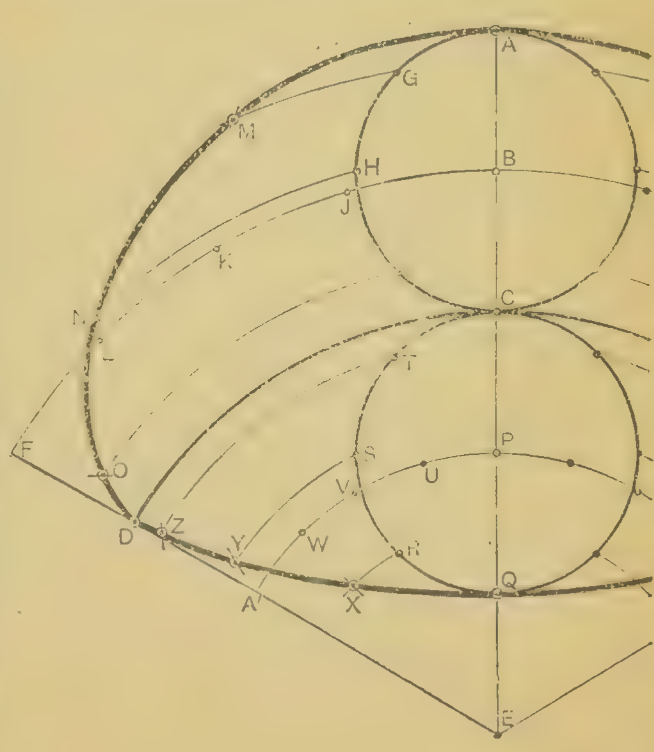
1. Through pt. C, where circle touches D D′, draw vert. diam. C A; make C D = ½ circumf. A M C (*See Prob.* 62). Then while A M C rolls from C to D, pt. A traces out half a cycloid.——2. Draw B E = and parl. to C D; then cent. B moves from B to E during ½ revolution of circle B.——3. Divide B E and A M C each into the same number of equal parts, say 6, and through K, L, N, O, draw parls.——4. With F, G, H, etc., as centres, rad. = B A in each case, cut K P in P, L Q in Q, B E in R, etc. Line K P = F B, L Q = 2 F B, M R = 3 F B, etc.——5. Then pts. A, P, Q, R, S, T, D, are on



the required curve. The other half is found in the same way.——6. Find tangent and normal to any pt. Q by drawing L Q parl. to C D to meet circle in L; join L with A and C; then the normal Q 1 is parl. to L C, tangent 2 3 is parl. to L A.——7. The trochoid curves are found in a similar way. Make C D′ = C D, B E′ = B E. Divide the circumf., not of generating circle, but of circle passing through generator (A′ or A″); and with F′, G′, etc., as centres, rad. = B A′ (or B A″) in each case, mark the parls. through K′, L′, M′, . . . (K″, L″, M″, . . .).——8. The inferior trochoid is less in height than A C, and does not touch directrix, since B A′ is less than B A. The superior trochoid, being higher than B A, dips below directrix, forming a loop which is found by choosing more centres along B E′ produced.

PROB. 164.—To draw 1st the Epicycloid, and 2nd the Hypocycloid, having given the directrix, C D, and generating circle B (or P).

1. Join E, cent. of directg. circle, with C, where circle B touches C D. Then C A (C Q), on E C (or E C produced), is vert. diam. of generatg. circle.——2. Line C D, = A H C (Q S C), is found thus:—E C : C B (C P) : : angle C E D : 180°; let $C B = \frac{E C}{3}$; then angle C E D = 60°. ——3. Now E D (produced) meets in F (A′) arc B F (P A′) drawn through B (P) with centre E. This arc is the path (or *locus*) of cent. B (P) while circle travels from C to D.——4. Divide A H C (Q S C) and B F (P A′) each into the same number of equal parts, say 4. With cent. E describe arcs through G, H, I (R, S, T).——5. Then with cents. J, K, L (U, V, W), rad. C B (C P)



in each case, cut the arcs in **M, N, O (X, Y, Z).——6.** Then **A, M, N, O, D,** are pts. on the EPICYCLOID, and **Q, X, Y, Z, D,** pts. on the HYPOCYCLOID. The other half of each of the curves is found in same way.

Note.—When diam. of generating circ. = rad. of directing circ., a pt. on the former moves along diam. of latter. By regulating the relative sizes of the two circles, an *epicycloid* may be made to go once or more times, and a *hypocycloid* three or more times, exactly around the circumf. of directing circle.

CHAPTER XVII. ARCHES AND MOULDINGS.

ARCHES.—Only the *simplest* form of most kinds of arches is given, but it must be remembered that the curves forming the arch contour, can be produced in a variety of ways, by complex curves (*See Prob.* 84), and by conic sections (*See Probs.* 142, 155, 156, 158).

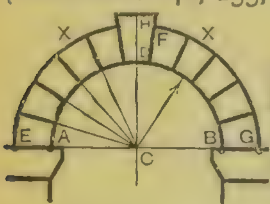


FIG. 431.

FIG. 432.

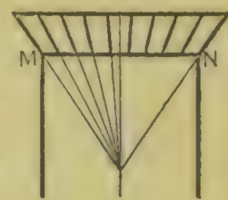


FIG. 433.

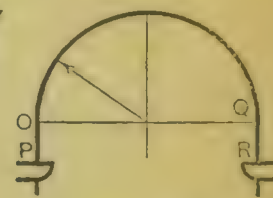


FIG. 434.

Fig. 431.—**Semicircular** arch. **A B** is the **span.** **A** and **B** are the **springing points.** The masonry, etc., against which the arch rests, is named the **abutment.** The distance **C D** is the **height** or **rise,** and **D** is the **crown** of the arch. The arch-stones are called **voussoirs, X, X,** and the one at **D H** is the **keystone.** The outside of the voussoirs is called the **extrados** or **back, E H G**; and the *under side* is the **intrados** or **soffit.**

Fig. 432.—A **segmental** arch is the segment of a circle, with cent. **I** and rad. **I J.** The height is **L K.** The arch-stones (voussoirs) being wedge-like in shape hold each other up.

Fig. 433.—This is improperly called an arch, although it has a *straight line* for the intrados **M N.**

Fig. 434 is a **stilted semicircular** arch, which stands upon vert. lines **O P, Q R,** on each side.

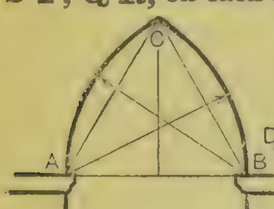


FIG. 435.

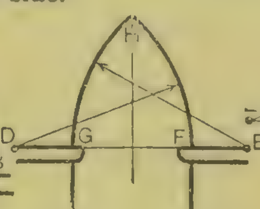


FIG. 436.

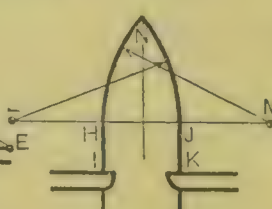


FIG. 437.

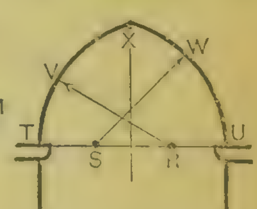


FIG. 438.

Fig. 435 is an **equilateral** arch. The span, **A B,** being the rad. used for the arcs **A C, C B.**

Fig. 436 is a **lancet** arch. The cents. **D, E,** for the arcs are outside the arch. With cents. **D, E,** and rad. **D F** and **E G,** describe the arcs.

Fig. 437 is a **stilted lancet** arch, which stands upon vert. lines **H I, J K.** The cents., **L, M,** are outside, as in the last example.

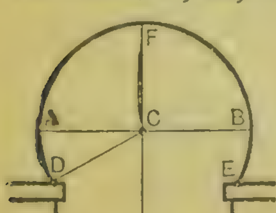


FIG. 439.

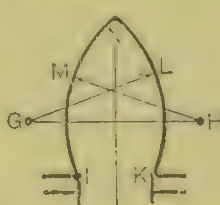


FIG. 440.

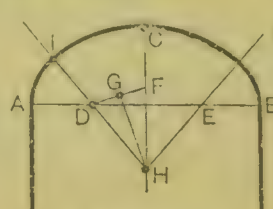


FIG. 441.

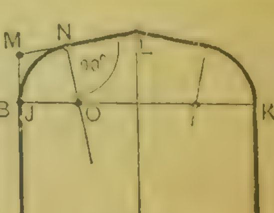


FIG. 442.

Fig. 438 is a **dropped** or **depressed** arch, with the cents. R, S, on the span T U. With rad. R V, S W, we get arcs meeting at X.

Fig. 439 is a **horse-shoe** or **Moorish** arch, with a span A B, cent. C, and the springing pts. D, E. With cent. C, and rad. C D, we get the arch.

Fig. 440 is a **horse-shoe lancet Moorish** arch. With cents. G, H, and rad. G L, H M, desc. the arcs K N, I N.

Fig. 441 is a **three-centred** arch, with the span A B and the crown C given. Mark any pts. D, E, equidistant from A and B. Make C F = A D. Draw D F. Bisect it at G and draw G H perp. to D F. Draw H I, through D, indefinitely. Then with cents. D and H draw the arcs meeting at I.

The **semi-elliptical** arch is drawn at Fig. 268, and needs no explanation.

Fig. 442 is a **straight line and arc** arch, with a given span J K, and the crown L. Make J M any length. Join M, L. Mark M N = M J. Draw N O perp. to M L, and desc. arc J N.

The **four-centred** arch is drawn and explained at Fig. 345.

The **trefoil** arch is explained at Fig. 346.

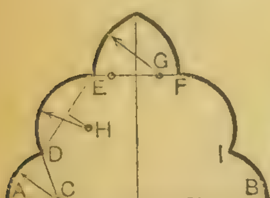


FIG. 443.

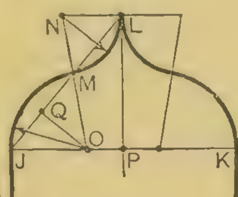


FIG. 444.

FIG. 445.

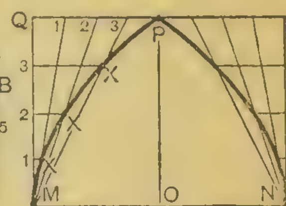


FIG. 446.

Fig. 443 is the **cinque-foil arch.** Get C D, D E, E F. Get cents. H, G. With cents. C, H, G, and the indicated rad. draw the arcs. ANY *uneven* number of foils can be used, thus a *hepta-foil*, a *nona-foil*, etc. The pts. F, I, are called CUSPS.

Fig. 444 is an **ogee** arch, or an arch of CONTRARY FLEXURE. J K is the span, and P L the height. Draw N L parl. to J K. Join J L. Mark any convenient pt. M. Bisect J M at Q. Draw the perp. Q O meeting J K. Draw O N through M. With cent. O, and rad. O J, draw the arc J M. With cent. N, and rad. N M, draw the arc M L.

Fig. 445 is a **rampant** arch, in which one of the springing pts., B, is higher than the other, A. Join A, B. Desc. a semicircle A 3 B, and divide it by any number of perps. to the diam. as C 1, D 2, etc. From C, D, E, F, G, draw vert. lines, and cut off C H = C 1, D I = D 2, etc. Draw the curve A H I J K L B. This arch is used in engineering.

Fig. 446 is an example of a pointed arch obtained by means of **parabolic curves.** M N is the span, and O P the height. Get Q and pts. 1, 2, 3; 1, 2, 3, aud obtain the curve through the pts. X, X, X. Observe that the curve P N is obtained in the same way as the curve C A, in Prob. 155, pt. C corresponding with pt. P, and N with A. If space permitted, we might give an almost endless variety of curves, but we must leave it to the ingenuity of the student to make the applications for himself.

ROMAN MOULDINGS.—These are sometimes called *simple* mouldings, because they are composed merely of ARCS OF CIRCLES. Each moulding by itself, or when it is grouped along with a number of mouldings, is called a MEMBER.

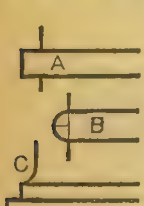


FIG. 447.

FIG. 448.

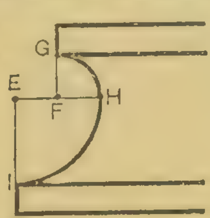


FIG. 449.

FIG. 450.

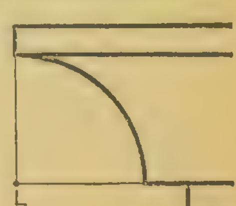


FIG. 451.

Fig. 447.—**A** is a **fillet** or **band**, a small square member, frequently seen separating mouldings; **B** is a **bead** or **astragal**, often cut into beads, berries, etc.; **C** is a **conge** or **scape** used to connect the shaft of a column with its base.

Fig. 448 is a **torus** or **half-round** moulding, composed of a semicircle.

Fig. 449 is a **scotia** composed of two quadrants **G H, H I.** The cents. **E** and **F**, and pt. **H**, must be in one straight line.

Fig. 450 is the **ovolo, echinus,** or **quarter-round,** and is composed of a quadrant struck from centre **K.** This moulding is often decorated with sculptured forms of the echinus (or egg and dart) ornament.

Fig. 451 is the **cavetto,** which is the reverse of the ovolo, with a concave quadrant struck from cent. **L.** The cent. must never be more to the right-hand side, but it may be more to the left.

Fig. 452 is a **cyma recta.** Draw line **A B.** Get *any* pt. **C.** With cents. **A, C,** and *any* convenient rad., get pt. **D.** With cent. **D,** and rad. **D C,** get arc **A C.** From **D,** through **C,** draw **D E.** Make angle **C B F** = angle **B C F.** Then with cent. **F,** and rad. **F C,** draw arc **C B.**

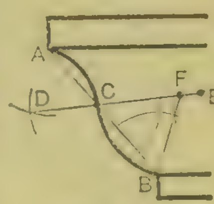


FIG. 452.

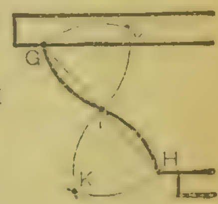


FIG. 453.

Fig. 453 is a **cyma reversa.** This can be drawn in the same way as in the last figure, only reversed, but we select another method. Draw **G H,** and bisect it at **I.** With cents. **G, I, H,** and rad. **G I,** desc. arcs meeting at **J** and **K.** With cents. **J, K,** and *same* rad., get arcs **G I, I H.** The cyma recta and cyma reversa are also called OGEE mouldings.

GRECIAN MOULDINGS are composed of CONIC SECTIONS, either the ELLIPSE, PARABOLA, or HYPERBOLA, and are hence more beautiful and varied in contour than the Roman mouldings.

Fig. 454 is an example of the application of a PARABOLA in getting the form of the **ovolo** or **echinus** moulding. The working is similar to that shewn in Prob. 155, only it is turned sideways. The axis **A B** is given. Draw **C D** parl., and **A C** perp. to **A B.** Divide **A C, C D,** each into the same number of equal parts, and draw the lines as shewn. This is sometimes termed a *quirked* ovolo.

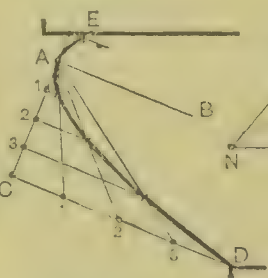


FIG. 454.

FIG. 455.

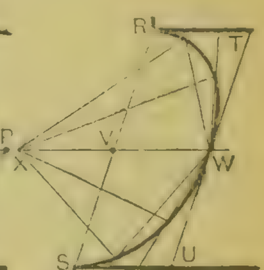


FIG. 456.

Fig. 455 is a **cyma recta** or **ogee.** From **F** draw **F H** perp. to **J H.** Draw **J I** perp. to **J H.** Draw **I F** parl. to **J H.** Bisect **J H** at **K,** and **I F** at **L.** Draw **K L** and bisect it at **M.** Through **M** draw **N P** parl. to **J H,** making **O N** = **O M,** and **Q P** = **Q M.** We get this curve by two quarter ELLIPSES **J M** and **M F** (*See Prob.* 142).

The Grecian **cyma reversa** is obtained in a similar manner. In each case, the quarter ellipses, instead of being equal, may be *unequal.*

Fig. 456 is a **scotia.** Draw **R S.** Draw **T U** parl. to **R S.** Bisect **R S** at **V.** Through **V** draw **X W** parl. **R T,** making **V X** = **V W.** Through **R, W, S,** draw the curve of half an ELLIPSE (*See Fig.* 267).

In Fig. 347 another method is given by which any moulding can be drawn.

The Roman and Grecian mouldings, sometimes enriched by carved ornament, are not only used for ARCHITECTURE, but for FURNITURE, METAL-WORK, etc.

Nisi Dominus Frustra.

EXERCISES.

When necessary, make a rough sketch before commencing to work out an exercise.—Shew all the lines of construction.—A set-square may be used when drawing a number of parallel lines. One accent (′) *after a numeral signifies* FOOT *or* FEET. *Two accents* (″) *after a numeral signify* INCH *or* INCHES: *thus,* 5′ 3″ *means* 5 *feet* 3 *inches.*

LINES AND ANGLES.

1.—Draw a vertical line A B, 2″ long. From A, draw A C, 3″ long, and at right angles to A B. Bisect A B, and divide A C into 7 equal parts.

2.—Draw D E=$2\frac{1}{2}$″. Produce D E to F, so that E F=4–11ths of D E.

3.—Draw two lines, G H, H J, each 3″ long, and meeting at an angle of 45°. Within the angle find a point K, that shall be 1″ from G H, and $1\frac{1}{4}$″ from H J. From K draw perpendiculars to G H, H J.

4.—Draw an angle, L M N, of 30°. Mark a point O, within the angle, and $2\frac{1}{4}$″ from M. From O, draw lines (each $1\frac{1}{4}$″ long) to meet L M, M N.

5.—Draw two parallel lines, P Q, R S, $1\frac{1}{2}$″ apart. Mark a point T, equidistant from P Q and R S, and a point U, $1\frac{1}{4}$″ from T, and $\frac{1}{2}$″ from P Q.

6.—Draw a horizontal line V W=3″. On the same side of V W, draw V X (2″) at 80°, and W Y ($2\frac{1}{4}$″) at 70°. From V, on V W, cut off V Z=1″. From Z, draw Z A, which shall converge towards the meeting point of V X, W Y, when that point is inaccessible.

7.—Draw seven lines, each=$1\frac{1}{2}$″. At the right-hand end of each line, in succession, make an angle equal to $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ of a right angle. (*By geometrical construction.*)

8.—Draw any four angles, each less than 45°. Then draw another angle which shall be equal to these four angles added together.

9.—Bisect a line, and trisect a right angle. (*Use set-squares only.*)

10.—Draw two right lines, each 3″ long, to make an angle of 120°. Divide this angle into 16 equal angles, by lines $2\frac{1}{2}$″ long. (*By geometrical construction.*)

11.—Draw A B (3″). On the same side of A B, draw A C (2″) at 90°, and B D ($2\frac{1}{2}$″) at 60°, to A B. Bisect A C at E. From E, draw E F to make equal angles with A C, B D.

12.—Draw a vertical line G H=3″. Find a point I, $1\frac{1}{4}$″ from G H, and equidistant from G and H. From I, draw I J to make 75° with G H.

13.—Draw ten vertical lines, 5″ long and $\frac{1}{2}$″ apart. Draw horizontal lines, across these vertical lines, and $\frac{3}{4}$″ apart. (*Use set-squares.*)

14.—State the angles made by the hands of a clock at 1, 2.30, 6.30, and 7.20.

15.—Draw a vertical line A B, 3″ long. Through a point, C, on the right-hand side of A B, draw another line, D E, parallel to A B.

16.—Draw a horizontal line F G (3″). On the same side of F G, draw F H (2″), at 110°, and G I ($2\frac{1}{2}$″), at 80°, to F G. Draw a line, J K, that would bisect the angle made by F H, G I, the angular point to be inaccessible.

17.—At one end of a given line, 3″ long, draw lines to make angles of 10°, 27°, 32°, 69°, 104°, 155°, and 172°, with the given line. (*Use a protractor.*)

18.—Draw a horizontal line K L=$2\frac{1}{2}$″. Above K L, mark a point M, 1″ from K L. Below K L, mark a point N, $\frac{1}{2}$″ from K L. Let the line connecting M, N, cut K L at 60°. Find a point, O, in K L, equidistant from M and N.

19.—Draw a horizontal line P Q, 3″ long. Below this line, mark a point R, $1\frac{1}{2}$″ from P Q, and a point S, 1″ from P Q. Points R and S are to be $1\frac{3}{4}$″ apart. Line P Q represents a wall separating two fields. R and S represent two wells. On P Q, mark the position for a gate, that shall be equidistant from the wells.

20.—Draw A B, C D, E F, parallel, and $\frac{3}{4}$″ apart. On E F, make E G=$\frac{1}{2}$″. On C D, make C H=$1\frac{1}{2}$″. From G and H, draw lines to meet A B, in one point, at equal angles.

TRIANGLES AND SCALES.

21.—Mark a point A, which indicates the position of an admiral's ship. Mark the positions of two other ships, under the following conditions:—B is to be 5 miles, and C, 2 miles from A. Ships B and C are to be stationed 6 miles apart. (*Scale,* $\frac{1}{4}$″ *to a mile.*)

22.—Construct a triangle D E F. Make D E=$2\frac{1}{2}$″, D F=$1\frac{3}{4}$″, E F=2″. Within the triangle, D E F, find a point G, from which perpendiculars to each side of the triangle shall be of equal length. On D F, construct an equilateral triangle with sides=D F. On E F, as a base, construct an isosceles triangle having a vertical angle=75°.

23.—Construct a scale shewing 5 yds. to 1 in., to measure up to 20 yds. Draw the plan of a triangular field H I J. H I=13 yds., H J=18 yds., and I J=9 yds., by scale. Find the positions of three trees, K, L, M, outside the field. K is 5 yds. from I, and 13 yds. from J. L is opposite to the centre of H J, and 6 yds. from H J. M is 10 yds. from H, and 12 yds. from K.

24.—Draw a horizontal line N O ($2\frac{1}{4}''$). N O is the altitude of an isosceles triangle, and it makes angles of 15° with the equal sides of the triangle. Construct the triangle.

25.—Draw a horizontal line P Q ($3''$). On P Q, as a base, construct a triangle, P Q R, having an angle of 40° at P, and 50° at Q. Draw the altitude of P Q R.

26.—A stick leans against a vertical wall, at an angle of 72° to the ground. What angle does the stick make with the wall?

27.—On a given base S T, $2''$ long, construct an isosceles triangle having its equal sides each three-fifths the length of S T.

28.—On a base V W, $2''$ long, construct a triangle V W X. The angle V W X to equal 120°, and the side V X to be $3''$ long.

29.—On a base Y Z, $2''$ long, construct an isosceles triangle, having its vertical angle twice the size of each angle at the base.

30.—Construct the following scales :—(a). $1\frac{1}{4}''$ to $1'$, shewing inches. Mark a distance on the scale, of $2'\ 5''$. ——(b). Half an inch to 1 yd., shewing feet. Mark a distance of 7 yds. 1 ft ——(c). $\frac{1}{8}''=1$ mile, to measure up to 50 miles.

31.—Construct an equilateral triangle, the altitude A B ($2\frac{1}{4}''$) being given.

32.—Draw a vertical line C D$=3''$. Construct a right-angled triangle, of which C D is the hypotenuse. The angle at C is to equal 20°. (*Use a protractor.*)

33.—Construct an isosceles triangle having an altitude$=2''$, and a base$=1''$.

34.—Three men, F, G, H, walk in different directions from a sign-post, X, on a moor. F goes 4 miles to the north-west, and then $1\frac{1}{2}$ miles to the east. G goes $3\frac{1}{2}$ miles to the south-west. H walks 5 miles south, and then 6 miles north-east. Draw a map shewing the tracks made by these men. (*Scale* $\frac{1}{3}''=1$ *mile.*)

35.—Construct an isosceles triangle :—(a). With base-angle and altitude given :—(b). With altitude and vertex given :—(c). With altitude and one of the equal sides given.

36.—Construct a right angled triangle. The hypotenuse$=2\frac{1}{2}''$. Another side$=1''$.

37.—Construct a triangle, the altitude$=1\frac{1}{2}''$, and base-angles$=30°$ and 45°.

38.—(a). Construct a triangle with a perimeter$=4\frac{1}{2}''$, and angles of 20° and 30°.——(b). Construct an isosceles triangle : altitude$=1''$: perimeter$=5''$

QUADRILATERALS.

39.—A diagonal of a rectangular drawing-board is $2'\ 1''$ long. One of the edges of the board makes an angle of 30° with one end of the diagonal. Draw the shape of the drawing-board, to a scale of $2''=1'$.

40.—Draw an oblong, making one side $\frac{2}{5}$ the length of an adjacent side.

41.—Draw the plan of a rectangular field, 360 yds. by 130 yds. (*Scale*, $1''=100$ *yds.*)

42.—Construct with *set-squares* :—(a). A square. Diagonal$=2''$:—(b). A rhombus. One diagonal ($1\frac{1}{4}''$) makes 60° with each side :—(c). A rhomboid. One diagonal ($3''$) makes 30° with 2 sides ($1''$ each) :—(d). An oblong. Sides$=3''$ and $1''$.

43.—Construct a trapezion A B C D. The diagonals ($3''$ and $2''$) to intersect at a point $\frac{3}{4}''$ from one end of the long diagonal. On one side, A B, construct a square. On B C, construct a rhomboid, the side B E ($\frac{1}{2}''$) to make 60° with B C.

44.—Mark a point F. Construct an oblong having F for its centre. Each of the smaller angles at the intersection of the diagonals ($3''$ long)$=30°$.

45.—Mark a point G, and construct a square of $1\frac{3}{4}''$ sides, having G for its centre. Draw a 2nd square, having the diagonals of the 1st square for its diameters.

46.—Construct a square of $2''$ sides. Draw another square, the same size, with the same centre, but with its diagonals parallel to the sides of the first square.

47.—On a base H I ($1\frac{1}{2}''$), construct a trapezium H I J K. The side H K ($2''$) makes 120° with H I. The side I J ($1''$) makes 45° with the diagonal I K.

48.—On a base L M, $2\frac{1}{3}''$ long, construct a parallelogram, having an altitude of $1\frac{1}{4}''$, and an angle of 40° at L.

49.—On a base N O ($2\frac{1}{8}''$), construct a rhombus, having an altitude of $1\frac{1}{4}''$.

50.—One angle of a parallelogram$=116°$. State the sizes of the other angles.

51.—Construct a rhomboid, having one side$=2''$, and diagonals$=3''$ and $1\frac{1}{3}''$

52.—Construct a rectangle, having one side half the length of a diagonal.

POLYGONS.

53.—Describe a circle of $2\frac{1}{2}''$ diameter. Mark a point A, on the right-hand side of the circumference. Within the circle, inscribe a regular heptagon, letting one of the angles coincide with point A.

54.—Two sides of a regular polygonal tower are each 10 ft. in length, and they meet at an angle of 135°. Draw the plan of the tower. (*Scale*, $1''$ *to* $8'$.)

55.—Describe a semi-circle of $1\frac{1}{3}''$ radius, and divide it into 12 equal sectors.

56.—On a line B C, $1''$ long, construct a regular hexagon. (*Use a set-square of* 60°.)

57.—Construct a hexagon, having a vertical line D E ($2\frac{1}{2}''$), for a diameter.

58.—Make a design for a pavement of tiles of various rectilineal shapes. Colours, light red, black, and yellow ochre. The pavement is to be for a porch, 10 ft. square, and is to have a border 1 ft. wide. (*Scale*, $1''=1'$.)

59.—Construct a regular pentagon, having its diagonals each $2\frac{1}{2}''$ long.

60.—On F G ($\frac{3}{4}''$), construct a regular nonagon. Then draw all its diagonals.

61.—Construct a scale of $1''=6'$. Draw H I $=14'$, by scale. On H I, as a base, construct an irregular pentagon H I J K L. Make the side I J $=8'$, and perpendicular to H I. Side H L to make 127° with H I. Diagonal L I to make 15° with H I. L K $=9'$. J K $=13'$. Draw another figure, equal and similar to H I J K L.

62.—On a horizontal line M N, $\frac{3}{4}''$ long, construct a regular hexagon, M N O P Q R. On each side of this hexagon, construct a hexagon, equal and similar to M N O P Q R.

63.—Construct a scale of $\frac{1}{4}''=1$ yd. Draw a horizontal line S T, 10 yds. long, by scale On S T, as a base, construct an irregular hexagon, S T U V W X. Side S X (4 yds.) to make 150° with S T. Diagonal S W (6 yds.) to make 116° with S T. Diagonal S V (9 yds.) to make 58° with S T. Diagonal S U (7 yds.) to make 25° with S T.

64.—Construct a hexagon having a horizontal line Y Z, $2\frac{1}{3}''$ long, as a long diagonal.

65.—On a given line, $1''$ long, construct an octagon, by the special method.

66.—Construct a regular heptagon, when the distance from an angle to the middle point of the opposite side is three inches.

CIRCLES AND TANGENTS.

67.—Draw two lines, A B ($2''$), B C ($1\frac{1}{2}''$), meeting at an angle of 30°. Then describe a circle to pass through the three points A, B, C.

68.—Strike a circle, $1''$ radius, passing through points, D, E, $1\frac{1}{2}''$ apart. Through D and E, draw tangents, meeting at F. With centre F, strike a circle touching the 1st circle.

69.—Draw 4 circles in a row, touching each other in succession, and a straight line, G H, $6''$ long. The circles to have diameters, successively, of $2''$, $1''$, $1\frac{1}{2}''$, $\frac{3}{4}''$.

70.—Draw a horizontal line I J, $1\frac{1}{4}''$ long. With a centre above I J, strike a circle of $1\frac{1}{2}''$ radius, passing through I and J. With a centre below I J, strike a circle of $1''$ radius, passing through I and J. Draw 2 tangents, each to touch both circles.

71.—Draw a line K L, $2\frac{1}{2}''$ long, and inclined at 45°. With centres K and L, strike circles of $\frac{3}{4}''$ radius. Draw an interior and an exterior tangent to the circles.

72.—Strike an arc M N, of $2''$ radius, and shew how the centre can be found by geometrical construction. Mark a point O, in the arc, midway between M and N. Through O, draw a tangent to the arc, when the centre is inaccessible.

73.—Draw 2 circles, P and Q. Diameters $2''$ and $1\frac{1}{2}''$. Distance between centres $=2\frac{1}{2}''$. Draw a 3rd circle, of $\frac{1}{2}''$ radius, to touch circles P and Q.

74.—From a point R, $2''$ above the centre of a circle of $1\frac{1}{2}''$ diameter, draw two tangents to the circle, each $3''$ long. Join the points of contact.

75.—Draw two lines, S T, T U, meeting at 75°. Find a point V, $\frac{3}{4}''$ from each line. With V as centre, describe a circle touching S T and T U.

76.—Construct a triangle W X Y, having sides of $2\frac{1}{2}''$, $2''$, and $1\frac{3}{4}''$. Describe three circles, touching one another, and having points W, X, and Y, for their centres.

77.—Draw a vertical line A B ($2\frac{1}{2}''$). On the left side of A B, draw A C ($2''$), at 90°, and B D ($2\frac{1}{4}''$), at 60°, to A B. Describe a circle to touch A B, A C, B D.

78.—Draw E F $=3''$. With E as centre, strike a circle of $\frac{3}{4}''$ radius, and with F as centre, a circle of $1''$ radius. Draw an interior tangent to the circles.

79.—Describe two circles (diameters $1\frac{1}{2}''$ and $2''$) touching each other. Mark a point G, on the circumference of the larger circle, and $2\frac{1}{2}''$ from the centre of the smaller circle. Describe a third circle, enclosing and touching the two circles, and passing through point G.

80.—Describe a circle H, of $1\frac{1}{2}''$ radius. Mark a point I, in the circumference. Insert a circle J, of $1''$ diameter, within the circle H, and touching it at point I. Insert two other circles, of $1\frac{1}{4}''$ diameter, each touching the circles H and J.

81.—Draw A B $=2''$. At A, draw A C ($3''$), below A B, at an angle of 30°. Bisect A B at D. Strike an arc to pass through C, and to touch A B at D.

82.—Draw E F $=1\frac{1}{2}''$. With centre E, and radius $=1''$, and with centre F, and radius $\frac{3}{4}''$, strike circles. Then describe a circle of $1\frac{3}{4}''$ radius, to touch and enclose circles E and F.

83.—Draw two concentric circles, of $1''$ and $3''$ diameters. Divide the space between these circles into twelve equal divisions, by lines radiating from the centre.

84.—Draw a horizontal line K L, $3''$ long. Bisect K L at M. Through M, draw N O, cutting K L at 45°. N is to be $\frac{1}{2}''$ above, and O, $1''$ below, K L. Strike a circle that shall pass through N and O, and have its centre in the line K L.

85.—On P Q ($2''$) as a base, construct an irregular pentagon P Q R S T. From P, through Q, R, S, to T, draw a continuous curve of tangential arcs of circles.

86.—Strike an arc U V. Radius $=2''$. Length of the chord U V $=3''$. Find 2 points in the continuation of the arc, without using the centre.

87.—From a point W, in the circumference of a circle of $1\frac{1}{4}''$ radius, draw a chord that shall cut off a segment containing an angle of 30°.

88.—On a given line X Y, $2\frac{1}{2}''$ long, describe a segment of a circle that shall contain an angle, X Z Y, equal to 110°.

RATIO AND PROPORTION.

89.—Draw A B$=2\frac{1}{4}''$, horizontally. Upon A B, mark 5 unequal divisions. From A, draw A C$=3''$, and inclined upwards at 30°. From B, draw B D$=1''$, and inclined downwards at 60°. Divide A C, B D, proportionally to the divisions on A B.

90.—Draw a horizontal line E F$=\frac{3}{4}''$. At E and F, erect perpendiculars, E G$=1\frac{1}{2}''$, and F H$=2''$. Mark 6 unequal parts upon F H. Divide E G proportionally to the divisions upon F H.

91.—Describe three concentric circles, with diameters of $2\frac{1}{2}''$, $1\frac{1}{2}''$, and $1''$. Upon the circumference of the medium-sized circle, mark 5 unequal divisions. Then divide the circumferences of the other two circles in the same proportion.

92.—Draw a vertical line J K$=3''$. Divide J K in the proportion of 4 to 9.

93.—Draw 2 parallel lines, L M, N O, $1\frac{1}{2}''$ apart, and $3''$ long. Divide L M into 4 unequal parts. Between L M and N O, and parallel to these lines, draw 3 lines, X, Y, Z, so that the distances between L M, X, Y, Z, and N O, shall be in the same proportion as the divisions marked upon L M.

94.—Draw a line P Q, $3\frac{1}{2}''$ long. Upon P Q, as a base, construct a triangle P Q R. Make P R=[illegible] of P Q, and Q R=[illegible] of P Q. Mark four unequal divisions on P R. Divide P Q and Q R proportionally to the divisions on P R.

95.—On S T ($2''$) construct a rectangle having sides, S T, T U, in the ratio of 3 : 5.

96.—On V W ($4''$) mark two points, X, Y, so that X Y=half V X, and Y W=three times X Y. On V X, Y W, strike semi-circles above the line, and on X Y, a semi-circle below the line, so as to form a continuous line of tangential arcs.

97.—On A B ($3''$) construct a parallelogram having the sides A C, B D, each$=2''$, and the angle at A$=60°$. Divide the diagonal B C into 4 unequal parts; then divide each side of the parallelogram proportionally to the divisions on B C.

98.—Draw two sectors, E F G having a radius of $2''$, and H I J, a radius of $1\frac{1}{4}''$, and each containing an angle of 60°. Divide E F G into 3 unequal sectors; then divide H I J into 3 sectors having the same proportion to one another as those in E F G.

99.—Construct an irregular pentagon K L M N O, having a perimeter equal to six inches, an angle at K$=80°$, and an angle at L$=110°$. The sides K L, L M, M N, N O, O K, to be, successively, in the proportion of 3 : 6 : 4 : 5 : 2.

100.—Find the mean-proportional to two lines, P Q$=3''$, and R S$=2''$.

101.—Find a third-proportional to two given lines, T U$=3\frac{1}{2}''$, V W$=2''$.

102.—Construct a scalene triangle having a perimeter equal to $5''$. The sides are to be in the given proportion of 8 : 5 : 11.

103.—Find a fourth-proportional to three given lines, X$=1''$, Y$=2\frac{1}{4}''$, Z$=1\frac{1}{2}''$.

104.—Draw a line, A B ($2\frac{1}{2}''$). Divide it in extreme and mean-proportion.

105.—The mean-proportional between two given lines, C D, E F, is 1[illegible] in length. C D is $2\frac{1}{2}''$ long. Find, by geometrical construction, the length of E F.

SIMILAR FIGURES.

106.—Construct a rhomboid having sides$=1\frac{1}{2}''$ and $2''$, and one diagonal A B$=3''$. Construct a similar rhomboid having the corresponding diagonal$=2\frac{1}{4}''$.

107.—On C D ($1\frac{1}{4}''$), as a base, construct a figure somewhat similar to shape B, Prob. 61. Then upon E F ($2''$) construct a figure similar to the one upon C D.

108.—Upon a horizontal line G H ($1\frac{1}{2}''$), construct a rectangle having vertical sides$=2\frac{1}{4}''$. Then on J K ($1''$), and L M ($2\frac{1}{2}''$), construct rectangles similar to the one upon G H.

109.—Upon N O ($3''$), as a base, draw an irregular figure having straight lines for 3 of its sides, and having a compound curve for one of the sides terminating at point O. Upon P Q ($2\frac{1}{2}''$), as a base, draw a figure similar to that upon N O.

110.—Upon R S ($2''$), as a base, construct a triangle having side R T$=2\frac{1}{4}''$, and side S T $=1''$. Upon V W ($3''$), as a base, construct a triangle similar to R S T.

111.—Upon X Y ($3''$), draw a triangle, X Y Z, having an angle at X$=40°$, and at Y$=70°$. Within X Y Z, draw a triangle having the side parallel to X Y$=2''$. Outside X Y Z, draw a triangle having the side parallel to X Y$=3\frac{1}{4}''$. The 3 triangles are to be concentric, and to have their corresponding (or homologous) sides parallel to one another.

112.—Draw a rhomboid A B C D. Divide it into 16 equal parts similar to A B C D.

113.—Upon a base E F ($2\frac{1}{2}''$), construct a trapezium E F G H. Make angle E$=50°$, and angle F$=110°$. Make E H$=2''$ and F G$=1\frac{1}{4}''$. Construct a similar trapezium having its base K L$=1\frac{1}{2}''$.

114.—Construct four concentric squares, with parallel sides a quarter of an inch apart. The largest square is to have sides $2\frac{1}{4}''$ long.

115.—On M N, 1″ long, construct a regular heptagon. Within this figure construct a similar polygon having sides $\frac{3}{4}''$ long. Outside the figure constructed on M N, construct a similar polygon having sides $1\frac{1}{2}''$ long. The three polygons are to have their sides parallel and equidistant.

116.—On a base O P, construct an irregular pentagon O P Q R S. Make O P=20 yds. ; P Q=15 yds. ; Q R=12 yds. ; R S=30 yds. ; S O=16 yds. The diagonals O Q, O R, are each 28 yds. long. Construct a similar figure on a base 14 yds. long. (*Scale*, 1″=10 *yds.*)

117.—On a base T V ($2\frac{1}{2}''$), construct a rectangle, T V W X, having sides T X, V W= $1\frac{3}{4}''$. Within T V W X draw a similar rectangle having the base=$1\frac{1}{2}''$. Outside T V W X draw a similar rectangle, and make the base pass through a point, Y, $\frac{1}{2}''$ below T V. The 3 rectangles are to have the same centre, and their corresponding sides are to be parallel.

118.—On A B (2″), construct an irregular heptagon. Then construct a similar figure having a perimeter=$\frac{2}{3}$ the perimeter of the figure upon A B.

119.—Construct a regular pentagon having a diagonal 2″ long. Then construct a similar figure having a perimeter two-fifths greater than that of the first figure.

120.—Draw a square of 2″ sides. Draw another square of $\frac{4}{5}$ the perimeter.

121.—Construct, faintly, a regular pentagon of $2\frac{1}{2}''$ sides. Draw all the diagonals in firm lines, and thus form what is termed a "*star polygon.*" Construct a similar figure having the distance between two of its adjacent points=$1\frac{1}{2}''$. The two figures are to be concentric, and the corresponding sides are to be parallel.

SCALES.

122.—A line A B, $2\frac{1}{2}''$ long, represents by scale a distance of 1′ 4″. Produce this line to represent 3′.

123.—On a horizontal line C D, 3″ long, construct a Scale of Chords. Draw E F, $3\frac{1}{2}''$ long, and with the scale construct the following angles at point E :—70°, 20°, 130°, 90°. Also shew how to obtain an angle E F G=150°, by *one* measurement from the scale.

124.—Draw the following plain scales :—**(a).** Of feet and inches, to measure 5 ft., when 1′ 3″ is represented by a distance of $1\frac{1}{4}''$.——**(b).** Of yards and feet, to measure 12 yds., when $\frac{1}{2}''$ represents 4′.——**(c).** Miles and furlongs, to measure 6 miles, when $1\frac{1}{2}''$ represents 2 miles.

125.—Construct a plain scale of miles and furlongs, to measure distances up to five miles, when one-tenth of an inch represents one furlong.

126.—Draw the ground-plan of a small cottage. Shew the positions of the doors, windows, &c. State the dimensions on the plan. (*Scale*, $\frac{1}{4}''$=1′.)

127.—Construct a scale in which 5″=300 yds., to shew distances of 10 yds.

128.—A tower is shewn in plan by a regular octagon of $1\frac{1}{4}''$ sides. The actual length of one of the longest diagonals is 30 feet. Construct a scale of feet from which the various dimensions of the tower can be ascertained.

129.—Two trees, A, B, 50 ft. apart, grow on the edge of a stream that has parallel sides. Another tree, C, is on the opposite edge of the stream. If A, B, C, be joined by straight lines, then B A C=70°, and A B C=40°. Draw a plan, ascertain the width of the stream, and the distances between C and the other trees. (*Scale*, $\frac{1}{2}''$=10′.)

130.—Draw an ogee curve. Make a copy of it three-fifths the size.

131.—In the plan of an estate, the space between two villages that are $1\frac{1}{4}$ miles apart is represented by a distance of $3\frac{1}{2}''$. Draw the scale to measure 2 miles. Shew furlongs.

132.—Draw a plan shewing the irregularly curved boundary of a lake. Its greatest width is 150 yds. Scale of 1″ to 50 yds. Draw another plan of the lake to a scale one-third less. Use a datum line or base line.

133.—Draw a map of an island. The greatest length, from north to south, is 23 miles. Shew two rivers, and the positions of five towns. Scale, 1″=10 miles. Draw another map of the island one-fourth greater in scale. Make the enlargement by means of squares.

134.—Draw plain scales having the following representative fractions :—**(a).** $\frac{1}{12}$, to shew feet and inches, and to measure 4 ft.——**(b).** $\frac{1}{8}$, to shew inches, only, and to measure 40″.——**(c).** $\frac{1}{36}$, to shew yards and feet, and to measure 5 yards.

135.—Construct Diagonal Scales :—**(a).** $1\frac{1}{3}''$ to a ft., to shew feet, 10ths, and 100ths of a ft. Draw lines 2·64′, 1·07′, 2·5′, 1·75′ in length, by scale.——**(b).** 1″=1 yd., to measure yds., ft., and in. Draw lines of the following scale-lengths,—3 yds. 2 ft. 8 in., 10 in., 1 yd. 7 in., 2 ft. 4 in.——**(c).** $\frac{3}{4}''$=1 mile, to shew miles, furlongs, chains. Draw lines=4 mls. 7 furs., 5 furs. 6 chs., 2 mls. 8 chs., 3 mls. 2 furs. 6 chs.

136.—Draw a scale of $\frac{1}{36000}$. Shew distances of 100 yds., to measure up to 4,000 yds.

Note.—That is, 1″=36000″. There are 1000 yds. in 36000″ : therefore, 1″=1000 yds. Set off distances=1″. Divide the 1st space of 1″ into 10 equal parts. Each 10th part represents 100 yds. The scale must be long enough to measure 4000 yds.

137.—Draw a diagonal scale of $\frac{1}{24}$, to shew ft., in., and 10ths of inches.

138.—A county map is drawn to a scale of which the representative fraction is $\frac{1}{63360}$. Construct a diagonal scale to measure miles, furlongs, and chains on this map.

INSCRIBED AND CIRCUMSCRIBED FIGURES.

139.—About a circle of 2½″ diameter, circumscribe a regular heptagon.

140.—Within a circle of 1½″ radius, inscribe an isosceles triangle having an angle of 30° at the apex. The base to be a vertical line.

141.—Construct a triangle A B C, to have sides A B=5″, A C=3¼″, and B C=2¾″. Within A B C inscribe a circle D. Inscribe two other circles, one to touch sides A B, A C, and circle D, the other circle to touch A B, B C, and circle D.

142.—Inscribe the two largest possible equal circles in a square of 2½″ sides.

143.—Within a given circle of 3½″ diameter, inscribe five equal circles.

144.—Describe a circle of one inch diameter. About this circle place six other circles of the same diameter to touch one another, successively, and the first circle.

145.—About a circle of 1″ radius, circumscribe a triangle having angles of 68° and 47°.

146.—Construct a square of 2″ sides. Circumscribe a circle and a square. Also inscribe a circle and a square. Within the smallest square inscribe a regular octagon.

147.—Within a given trapezion inscribe a circle and a square.

148.—Describe a circle, E, of ¼″ radius. Describe two circles of ½″ radius to touch each other and circle E. Circumscribe an equilateral triangle about the three circles.

149.—Draw a triangle of 3″, 2″, 2½″ sides. Inscribe a square. Circumscribe a circle.

150.—Within a regular hexagon of 1″ sides inscribe three equal circles to touch each other. Each circle to touch one side of the hexagon. Then circumscribe six equal circles to touch one another and the sides of the hexagon.

151.—In a semi-circle, 1″ radius, and in a quadrant, 2″ radius, inscribe a circle.

152.—Mark three points not in a straight line. Find a point equidistant from them.

153.—Within a square of 3″ sides, inscribe eight equal circles to touch one another successively. Each circle is to touch only one side of the square.

154.—In a square of 3″ sides, inscribe 9 equal circles to touch one another.

155.—The plan of the boundary walls of a farmyard is an irregular quadrilateral figure, F G H I. The wall F G=20 yds., G H=25 yds., H I=40 yds., and I F=15 yds., in length. Angle G=120°. Find the position of a well, J, which has to be sunk at a point equidistant from the walls F G, G H, H I. (*Scale*, 1″=10 *yds.*)

156.—Mark 2 points, K, L, 2″ apart. Mark a point M, 1¾″ from K, and 1½″ from L. From K, L, M, draw 3 lines of equal length to meet in one point N.

157.—Strike a circle of ¾″ radius. Draw a chord, O P=1″. Circumscribe an isosceles triangle about the circle, the equal sides to touch it at O and P.

158.—In an equilateral triangle of 4½″ sides, inscribe three equal tangential circles, each to touch one side of the triangle. Also inscribe three equal circles, each to touch two sides of the triangle.

159.—In a circle, 1¼″ radius, inscribe a triangle having angles of 50° and 95°.

160.—In an octagon of 1½″ sides, inscribe eight equal circles to touch one another, successively. Each circle is to touch only one side of the octagon.

161.—In a hexagon of 1½″ sides, inscribe three equal circles to touch each other. Each circle is to touch two sides of the hexagon.

162.—Within a circle of 3 inches diameter inscribe a dodecagon. Within the dodecagon inscribe an equilateral triangle, a square, and a hexagon.

163.—In a circle of 1¼″ radius inscribe an irregular pentagon having four of its sides=1″, 1½″, ¾″, 2″. In a circle of ¾″ radius inscribe a similar pentagon.

FOILED FIGURES.

164.—Describe a circle, A, of 1″ diameter. Construct a trefoil of 3 equal arcs of circles to touch one another tangentially, and to have their centres in the circumference of the circle A.

165.—Within a hexagon of 1¼″ sides inscribe a multifoiled figure of 6 equal semi-circles having adjacent diameters. Each arc is to touch 2 sides of the hexagon.

166.—Within an equilateral triangle of 3″ sides construct a pointed trefoil. Each arc is to have a radius of 9-16ths of an inch.

167.—Construct a cinquefoil of tangential arcs. Each arc is to have a radius of ¾″.

168.—In an equilateral triangle of 3″ sides, inscribe a trefoil of tangential arcs. Each arc is to touch two sides of the triangle.

169.—Draw a square having a diagonal=4″. Within the square inscribe two quatrefoils, each of equal tangential arcs. In one quatrefoil each arc must touch 1 side of the square. In the other quatrefoil each arc must touch 2 sides of the square.

170.—Make a design for an ornamental arrangement suitable for surface decoration. The forms are to be composed of circles, squares, and foiled figures.

171.—Within an equilateral triangle of 3″ sides inscribe a polyfoiled figure of six equal tangential arcs of circles.

172.—In a square of 3″ sides inscribe 2 quatrefoils, each of equal semi-circles. In one case let each arc touch 1 side, and in the other case, 2 sides of the square.
173.—In a circle of 3″ diameter inscribe six foils of equal tangential arcs
174.—Within a pentagon of 1½″ sides inscribe a pointed cinquefoil of equal arcs.
175.—Within a hexagon of 1½″ sides inscribe a six-foiled figure of equal semi-circles having adjacent diameters. Each foil is to touch only 1 side of the hexagon.
176.—On a vertical line 1″ long construct a hexagon. Within the hexagon inscribe a trefoil of equal tangential arcs. Each arc is to touch 2 sides of the hexagon.
177.—Within a circle of 2¼″ diameter inscribe a trefoil of equal semi-circles.

ELLIPSES.

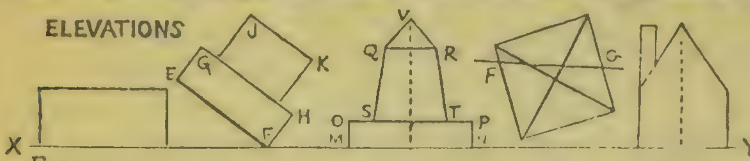
178.—Inscribe a semi-ellipse within a square of 2½″ sides, by means of intersecting lines.
179.—Draw an oblique line A B=4″. A B is the major axis of an ellipse. Make C D, the minor axis=2½″. Draw the elliptic curve by means of intersecting arcs. Through a point E, in the curve, 1″ from A, and through B, draw tangents to the ellipse. At a point F, in the curve, 1¼″ from C, and at D, draw normals (or perpendiculars) to the ellipse.
180.—Make a drawing of a semi-elliptical arch, somewhat similar to that shewn in Fig. 268, but with 9 voussoirs instead of 7. Make the keystone 3′ high. Major axis=12′. Minor axis=8′. Obtain the curve with a piece of thread. (*Scale*, 1″=3′.)
181.—Draw an ellipse when the foci are 3″ apart, and the minor axis=2½″.
182.—Draw G H=3″. Bisect G H at I by J K. Make I J=1¼″, I K=2½″. Draw a semi-ellipse having G H for its major, and I J for half its minor axis. Draw another semi-ellipse having G H for its minor, and I K for half its major axis.
183.—Draw an ellipse with a piece of thread. Within it inscribe a circle, a square, a rhombus, and an equilateral triangle. Also circumscribe a circle.
184.—Draw an ellipse. Use a thread. Shew how to find the major and minor axes.
185.—Inscribe an ellipse within any given rhomboid.
186.—Obtain the curve of an ellipse, approximately, (axes=5″ and 3¼″), by using portions of arcs of circles, and by means of a paper trammel.

AREAS.

187.—Draw an irregular heptagon. Upon a given line A B, 2″ long, construct a rectangle equal in area to the irregular heptagon.
188.—Strike a circle C, of ½″ radius. Describe 4 other circles, respectively 1½ times, 3 times, 5 times, and two-thirds the area of C.
189.—Upon a line D E, 2″ long, construct an equilateral triangle. Upon the same line construct an equivalent scalene triangle having one of its angles=20°.
190.—Draw F G, the approximate length of the arc of a quadrant of 1½″ radius. Upon F G, construct a rectangle equal in area, approximately, to the quadrant.
191.—A man had 5 square pieces of gold, $\frac{1}{10}$″ thick, of 2″, 1¼″, ½″, 3″, and ¾″ sides. He had these made into one square piece ⅛″ thick. Shew its size.
192.—Draw a trapezium having 1 angle of 90°, and=a given equilateral triangle.
193.—Construct an irregular pentagon equal to a given scalene triangle.
194.—Construct a square containing five square inches.
195.—Construct a pentagon of 1¼″ sides. Draw a square of two-thirds the area.
196.—On the same side of H I, 2″ long, construct a square and an equilateral triangle. Construct a square equal to the difference in area of these 2 figures.
197.—Construct a square of 1″ sides, and an equilateral triangle of 1½″ sides. Then construct a square equivalent to both these figures.
198.—Draw 2 circles J, K, of ⅞″ and 1″ radius. Draw a circle equal to both circles J, K, and another circle equal to the difference between J and K.
199.—From any given point L, outside, and through any given point M, inside, a given rhomboid, draw lines that shall divide the rhomboid into two equal parts.
200.—On a base N O=2″, construct a triangle having N P, O P=2¾″, 2¼″. Construct a triangle having an altitude of 1″, and equivalent to N O P.
201.—On a base Q R=3″, construct a triangle having sides Q S, R S=1¼″, 4″. Construct a right-angled triangle equal to Q R S, on a base 2″ long.
202.—Divide a pentagon, by lines drawn from its centre, into 4 equal parts.
203.—Shew the size of a pipe to discharge ⅔ as much water as a given pipe.
204.—Divide a square into 8 equal parts by lines drawn from an angle.
205.—Construct the following figures :—(a). A right-angled triangle=⅓ of an isosceles triangle.——(b). A square=¼ of a hexagon.——(c). An isosceles triangle=a pentagon.——(d). A hexagon=double the area of a given hexagon.——(e). An irregular octagon similar to, and ¾ the size of, a given irregular octagon.

SOLIDS.

(*The scale used for Figs.* 420 *to* 429 *is* 1-8*th in.*=1*ft. Copy these diagrams to a scale of* 3-4*ths in.*=1*ft. Figs.* 420 *to* 424 *are* ELEVATIONS, *and Figs.* 425 *to* 429 *are* PLANS.)



FIGS. 420. 421. 422. 423. 424.

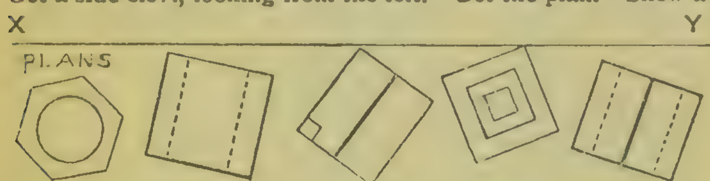
206.—Fig. 420 is the side elev. of a square prism, resting on one face. Get the end elev. and plan. Shew the plan of a section made by a plane parl. to the H.P.

207.—Fig. 421. E F G H is a square prism. E F is one of the bases. J K is a cylinder, the axis of which is a continuation of the axis of the prism. Get the side elev., when seen from the right. Shew plan of section made by a hor. plane passing through E.

208.—Fig. 422. M N O P is a square prism. Q R S T is a truncated square pyramid. Q V R is a square pyramid. The axes of the solids are in one straight line. Get a sectional plan made by a plane that passes through M at 60° to the H.P.

209.—Fig. 423 is a square pyramid. The axis=3 ft. Get an elev., at right angles, looking from the left. Get the plan of the section made by the plane F G.

210.—Fig. 424 is the end elev. of a model of a cottage, 3′ long, with a square chimney. Get a side elev., looking from the left. Get the plan. Shew a plan of any section.



FIGS. 425. 426. 427. 428. 429.

211.—Fig. 425 shews a hexagonal prism, 1′ high, with a cylinder, 3′ high, standing on the top of it. Draw the elev. of a section made by a vert. plane that passes through the centre at 45° to the V.P.

212.—Fig. 426 is a cube with a circular opening through its centre. Get the elev.

213.—Fig. 427 is a plan of a model similar to the one shewn in Fig. 424. Draw its vertical projection, and shew the section made by a plane that is parl. to the V.P., and that passes through the centre of the solid.

214.—Fig. 428 shews 3 slabs (each 6″ high) joined together. Get the elev.

215.—Fig. 429 shews a pentagonal prism standing on one face. Project its elev. Then project a side elev., looking from the left. Project the section made by a vert. plane that passes through the nearer end of the axis, and that makes an angle of 30° with the V.P., towards the right. Shew also the true shape of the section.

216.—Project a hexagonal prism, as at *u′ t′*, Fig. 371. Through *q′*, draw the edge of a section-plane at 30° to the H.P. Project the plan of this section.

217.—Project the pyramids in Fig. 374. Through *e′*, draw a line at 70°, through *g′*, one at 30°, to the H.P. Get plans of these sections. Shew their true shapes.

218.—Project the pyramids that are shewn in Fig. 375. Through *a*, draw a line at 30° to the V.P.; and through *h′*, a line at 60° to the H.P. Project these sections.

219.—Project the cylinders that are given in Fig. 390. Draw a line of section through *d′ b′*, and through *s z*. Project these sections, and shew their true shapes.

220.—Project the cones that are shewn in Fig. 391. Through *c′*, draw a line of section at 45° to the H.P., to the right. Through the middle pt. of *m n*, draw a line of section parl. to *o p*. Project these sections, and shew their true forms.

221.—Through *u′*, Fig. 404, draw a line of section at 27° to the H.P. Draw a line of section through 6 *t*, Fig. 407. Project these sections, and shew their true shapes.

222.—From *h′*, Fig. 415, draw a line of section parl. to *g′ e′*. Through *g*, Fig. 416, draw a line at 40° to the V.P. Project these sections, and shew their true forms.

223.—Draw the elev. of a sphere of 1¼″ radius. Project sectional plans made by a plane perp. to the V.P., and passing through the centre, at 45° to the H.P.; and by a hor. plane ½″ above the centre. Draw the true shapes of these sections.

224.—Draw the plan and elev. of a stool. Top, 1′ square and 2″ thick. Four legs, each 2″ square, 16″ high, perp. to the seat, and inserted 1″ from the corner. The stool is to stand upon the H.P., one side to make 30° with the V.P. (*Scale*, 2″=1′.)

225.—Draw the elev. and plan of a compound curve (or ogee) that is contained in a vertical plane, parl. to the V.P. Then project an elev. of this curve, when the plane of the curve is turned round at an angle of 30° to the V.P.

226.—Draw an elev. and plan of a hexagonal pyramid, as shewn at *a′* C *g′*, C D *g*, Fig. 376. Then project an elev. on a new ground line that makes 45° with X Y.

227.—Draw the plan and elev. of a line inclined to both planes of projection. Find its length, and the true angles made with the H.P and V.P.

Problems in Practical Plane Geometry

Book 4	Rawle	Book 5	Rawle
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7	7	18	94
8	16	19	96
9	87	20	47 or 111
10	49	21	114
11	50	22	109

Model Drawing:
Rays of light - converge
(the eye being stationary)

Solid Geometry:
Rays of light (projectors) pass from the points of the object back to the plane of projection, and at right angles with it. (eyes moves over the object, rays of projectors parallel.
Elevations are side views.
Plans are ground views.

$\frac{75}{60}$

I. Bisect a straight line ST $3\frac{1}{4}$" long.

2. Describe an arc AB with a radius of $2\frac{3}{4}$"; points AB are 4" apart. Find a point C on the arc so that AC = CB.

3. Draw a line WZ $2\frac{3}{8}$" long, on it make a point V, $1\frac{1}{4}$" from W. Draw a perpendicular $3\frac{1}{8}$" long through V.

4. Draw a line MN $3\frac{7}{8}$" long. Find a point R $1\frac{3}{4}$" ~~long~~ from N and $3\frac{1}{2}$" from M. From point R draw a perpendicular to MN.

5. A line CD is $1\frac{3}{8}$" long. Draw EF parallel with and $1\frac{1}{2}$" from CD